CHAPTER

5

SENSITIVITY ANALYSIS

A. J. Clark School of Engineering •Department of Civil and Environmental Engineering

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By
Dr . Ibrahim. Assakkaf

ENCE 627 – Decision Analysis for Engineering

Department of Civil and Environmental Engineering

University of Maryland, College Park
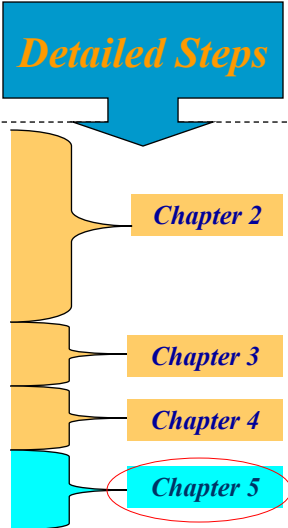
Methodology of Modeling Decisions

The Methodology of Modeling Decisions is to:

ᵛ   Understand the problem under study

ᵛ   Introduce quantitative modeling

- -

ᵛ   Discuss the elements of a decision.

  ➢ Values and Objectives

  ➢ Decisions to be made

  ➢ Upcoming uncertain events, and

  ➢ Consequences

ᵛ   Build the decision Model and identify a set of feasible alternatives.

ᵛ   Evaluate the alternatives and make a choice of a feasible alternative.

ᵛ   **Re-evaluate the alternatives using sensitivity analysis to refine the solution.**

Detailed Steps

Chapter 2

Chapter 3

Chapter 4

Chapter 5

Sensitivity Analysis

Contents

- Sensitivity Analysis: A Modeling Approach

- Problem Identification and Structure

- Sensitivity Analysis in Example: Eagle Airlines

- One-Way Sensitivity Analysis

- Tornado Diagrams

Contents (cont'd)

- Dominance Considerations
- Two-Way Sensitivity Analysis
- Sensitivity to Probabilities
- Sensitivity Analysis by Computer
- Sensitivity Analysis: A Built-In Irony
- Questions and Problems

Sensitivity Analysis

- The idea of sensitivity analysis is central to the structuring and solving of decision models using decision-analysis techniques.

  1. We will discuss sensitivity-analysis issues,

  2. How sensitivity analysis relates to the overall decision-modeling strategy,

  3. Introduce a variety of graphical sensitivity-analysis techniques.

Sensitivity Analysis Issues

- Don't always know values for parameters (probabilities, costs, etc.) with complete certainty.

- Need to see how the problem outcome can change relative to the values for key parameters.

Sensitivity Analysis: A Modeling Approach

Sensitivity analysis answers the question, "What makes a different in this decisions?"

- Recall:

  The issues that are addressed in a requisite decision model are the ones that matter, and **those issues left out are the ones that do not matter.** Determining what matters and what does not requires incorporating sensitivity analysis **throughout the modeling process. No "optimal" sensitivity-analysis procedure exists for decision analysis. To a great extent, model building is an art.**

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Problem Identification and Structure

- **Sensitivity analysis can lead the decision maker to reconsider the very nature of the problem.**

- **The question that we ask in performing sensitivity analysis at this level is, "Are we solving the right problem?"**

- **Do we have an appropriate decision context? Solving the wrong problem sometimes is called an, "<u>error of the third kind</u>."**

- **An error of the third kind, or Type III error, implies that the <u>wrong question was asked;</u>**

- **In terms of decision analysis, the implication is that <u>an inappropriate decision context was used</u>, and hence the <u>wrong problem was solved</u>.**

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Problem Identification and Structure

Sensitivity analysis can lead the decision maker to reconsider the very nature of the problem.

The question that we ask in performing sensitivity analysis at this level is, "Are we solving the right problem?"

Do we have an appropriate decision context? **Solving the wrong problem sometimes is called an, "<u>error of the third kind</u>." An error of the third kind, or Type III error, implies that the <u>wrong question was asked</u>; in terms of decision analysis, the implication is that <u>an inappropriate decision context was used</u>, and hence the <u>wrong problem was solved</u>.**

Examples of Type III errors: Think of times when a <u>symptom was treated instead of a cause</u>. How can one avoid a Type III error? The best solution is simply:

1. **Keep asking whether the problem on the surface is the real problem.**
2. **Is the decision context properly specified?**
3. **What exactly is the "unscratched itch" that the decision maker feels?**

Sensitivity analysis can aid the resolution of the problem of <u>multiple representations</u> by helping to <u>identify the appropriate perspective on the problem</u> as well as by <u>identifying the specific issues that matter to the decision maker</u>.

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Sensitivity Analysis

Example: Eagle Airline

Currently

- **Eagle Airlines own and operate 3 twin-engine aircraft**
  - **Charter flight (50% of flights)**
  - **Scheduled commuters service (50% of flights)**

The Decision Offer (The Decision Context)

- **A Piper Seneca plane for sale**
  - **Could help with need to expand charter flights**
  - **Asking sale price $95,000**
  - **Reasonable range according to President of EA, $85,000-$90,000 (say $87,500)**

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Sensitivity Analysis

Example: Eagle Airline (cont'd)

- **Costs for new plane**
  - **Operating costs $245/hour (fuel, maintenance and pilot salary)**
  - **Annual fixed costs**
  - **Insurance $20,000**
  - **Finance charges (borrow 40% of the money; rate = prime + 2% = 11.5%)**
- **Revenue for new plane**
  - **Charters for $300 to $350 per hour or**
  - **$100/person on scheduled flights (50% full on the average)**
- **Max of 1000 hours/year for the plane**
  - **More likely 800 hours**

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Sensitivity Analysis
Example: Eagle Airline (cont'd)

- **The Alternatives:**
  1. **Owner sell the airplane outright to Eagle airlines**

  Or

  2. **Owner of the plane could also offer an option to buy the plane within a year at a specified price *(The current owner would continue to own and operate the plane during this time).* Costs of option are between $2500 and $4000**

  Or

  3. **EA Could also invest cash (60% of purchase price = 0.6 x 87,500 = $52,500) in a money market at 8% return.**

- **What to do?**

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Sensitivity Analysis
Example: Eagle Airline (cont'd)

- What is the main (fundamental) objective in this problem?

- What is this objective affected by?

- What are the main decisions to be made?

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Sensitivity Analysis

Example: Eagle Airline (cont'd)

- **The main (fundamental) objective in this problem is:**
  - **Profit for Eagle Airlines is the main objective**
  - **What kind of a Influence Diagram Node will profit be?**
- **This objective affected by:**
  - **The total cost and the total revenue**
  - **What kinds of nodes will total cost and total revenue be?**
- **The main decisions to be made is to:**
  - **Purchase Seneca plane, yes or no?**
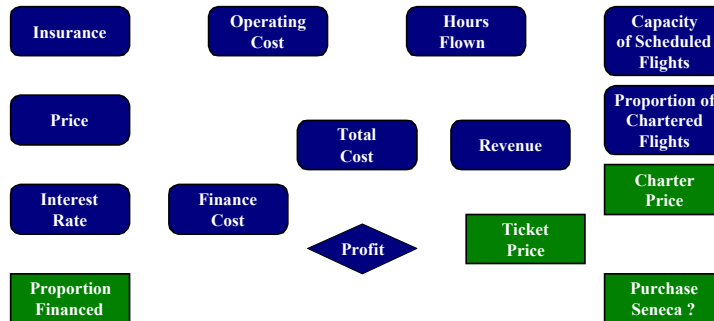 - **Proportion to be financed if yes?**

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Sensitivity Analysis

Example: Eagle Airline (cont'd)

- **What are the elements of total cost? for example:**
  - **Operating costs**
  - **Insurance**
  - **Financing of loan**
  - **What kind of nodes will these costs be?**
- **What are the elements of revenue? for example:**
  - **Hours flown**
  - **Capacity of scheduled flights**
  - **Proportion of chartered flights**
  - **Charter price**
 - **What kind of nodes will these costs be?**

Influence Diagram of Eagle Airlines Decision

| Insurance | Operating Cost | Hours Flown | Capacity of Scheduled Flights |

| Price | | Total Cost | Revenue | Proportion of Chartered Flights |

| | | | | Charter Price |

| Interest Rate | Finance Cost | | Ticket Price |

Profit

| Proportion Financed | | | Purchase Seneca ? |

- *Influence Diagram representing the Eagle Airlines decision*
- *Which way do the arcs go?*

Influence Diagram of Eagle Airlines Decision (cont'd)
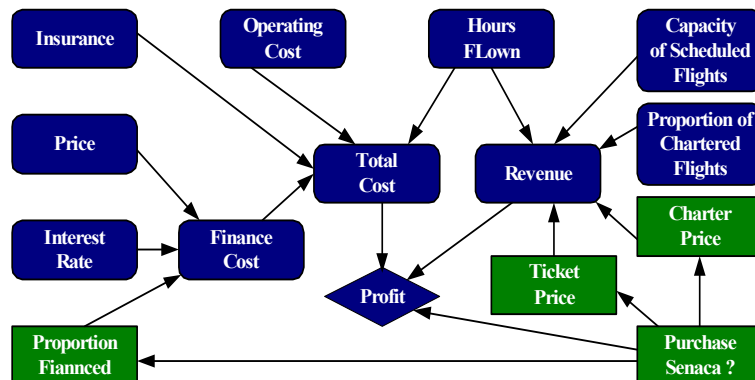
Influence Diagram representing the Eagle Airlines decision
Hint: You need to study these arcs carefully!!!

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Variables of Eagle Airlines Decision

Variable	Base Value	Lower Bound	Upper Bound
Hours Flown	800	500	1000
Charter Price/Hour	$325	$300	$350
Ticket Price/Hour	$100	$95	$108
Capacity of Scheduled Flights	50%	40%	60%
Proportion of Chartered Flights	0.50	0.45	0.70
Operating Cost/Hour	$245	$230	$260
Insurance	$20,000	$18,000	$25,000
Proportion Financed	0.40	0.30	0.50
Interest Rate	11.5%	10.5%	13%
Purchase Price	$87,500	$85,000	$90,000

Input variables and ranges of possible values for Eagle Airlines aircraft-purchase decision

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Calculations of Revenues of Eagle Airlines Decision

Total Revenue= Revenue from Charters + Revenue from Scheduled Flights

= (Charter Proportion × Hours Flown × Charter Price) + [(1 - Charter Proportion) ×

Hours Passenger   **Flown × Ticket Price) × Number of Seats × Capacity of Scheduled Flights]**

= (0.5 × 800 × $325) + (0.5 × 800 × $100 × 5 × 0.5)

= $230,000

Calculations of Costs of Eagle Airlines Decision (cont'd)

Total Cost = (Hours Flown × Operating Cost) + Insurance + Finance Cost

= (Hours Flown × Operating Cost) + Insurance
+ (Price × Proportion Financed × Interest Rate)

= (800 × $245) + $20,000 + ($87,500 × 0.4 × 11.5%)

= **$220,025**

Calculations of Profit of Eagle Airlines Decision

■ **Thus, using the Base Values,**

■ **EA Annual Profit is estimated to be**

■ **Profit = Revenues – Costs**

■ **Profit = $230,000 - $220,025 = $9,975.**

■ **This represents a return of approximately 19% on an investment of $52,500 (60% of the purchase price).**

■ **At 8% then the profit of investing in money market = 0.08 x 52,500 = $4,200.**

One-Way Sensitivity Analysis
(Eagle Airlines Example)
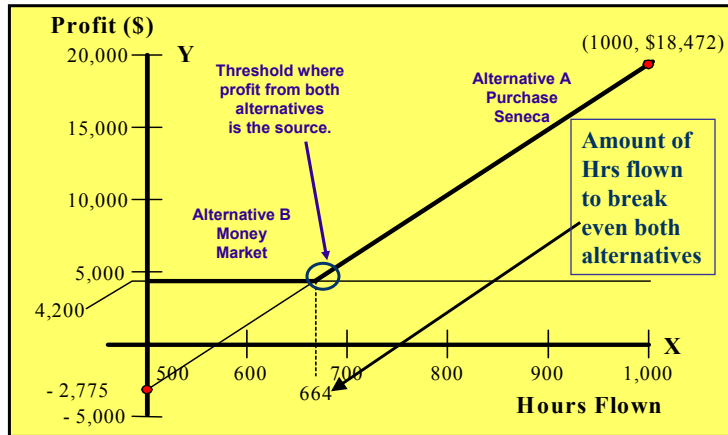
*One way sensitivity analysis of hours flown*

Tornado Diagrams

■A *tornado diagram* allows us to **compare one-way sensitivity analysis for many input variables at once.**

■The length of the bar for any given variable represents the **extent to which annual profit is sensitive to this variable.**

■The graph is laid out so that the **most sensitive variable, the one with the longest bar** is at the top, and the least sensitive is at the bottom.

■The tornado diagram tells us which variables we need to **consider more closely** and which ones we **can leave at their base values.**

Tornado Analysis (Eagle Airlines Example)
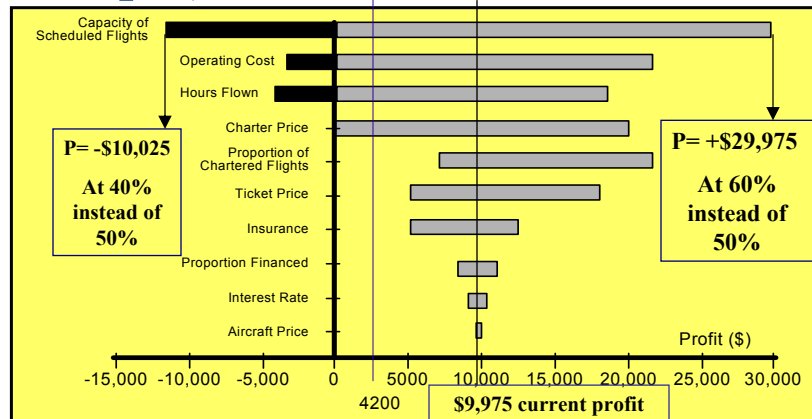
The bars represent the range for the annual profit when the specified quantity is varied from one end of its range to the other, keeping all other variables at their base values.

Dominance Considerations

- **Alternatives can be screened** on the basis of deterministic and stochastic dominance, and inferior alternatives can be eliminated.

- Identifying **dominant alternatives** can be viewed as a version of **sensitivity analysis** for use early in an analysis.

- In sensitivity-analysis terms, analyzing alternatives for dominance amounts to asking whether there is **any way that one alternative could end up being better than a second**. If no, then the first alternative is dominated by the second and can be ignored.

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Two-Way Sensitivity Analysis

■ **The value of the two-way sensitivity graph is to provide guidance in determining how much effort is needed to model uncertainty in a decision problem.**

■ **Looking at it another way, the graph can reveal whether the decision is sensitive to the uncertainty in the problem and to the modeling of that uncertainty.**

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Two-Way Sensitivity Graph
# (Eagle Airlines Example)
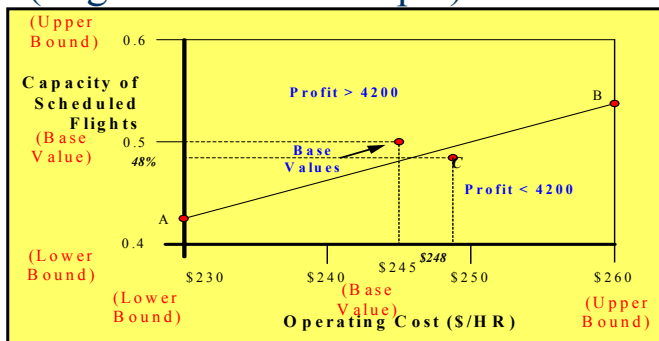
Two-way sensitivity graph for Eagle Airlines.

The Line AB represents the points where profit would be $4200.

• To perform a two-way sensitivity analysis for <u>operating cost and capacity</u>, calculate the following equation :

• <u>Total Revenue < Total cost + 4200</u>

• Don't input values of capacity and operating cost variable.

• Therefore: Capacity < 0.004 x Operating Cost – 0.509 then Substitute with extreme values of operating cost ($230 and $260) to get capacity (0.411 and 0.531)

• Hence on graph, point A (230, 0.411) and point B (260, 0.509) and line AB can be drawn.

Sensitivity To probabilities (Eagle Airlines Example
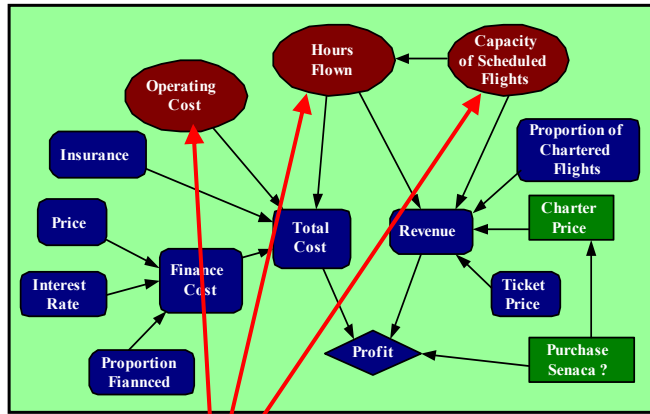
Influence diagram of Eagle Airlines decision

Note that only <u>three variables</u> (check these!!! Became chance nodes instead of intermediate rounded rectangle nodes) are considered to be uncertain, and that Hours Flown and Capacity are considered to be probabilistically dependent.
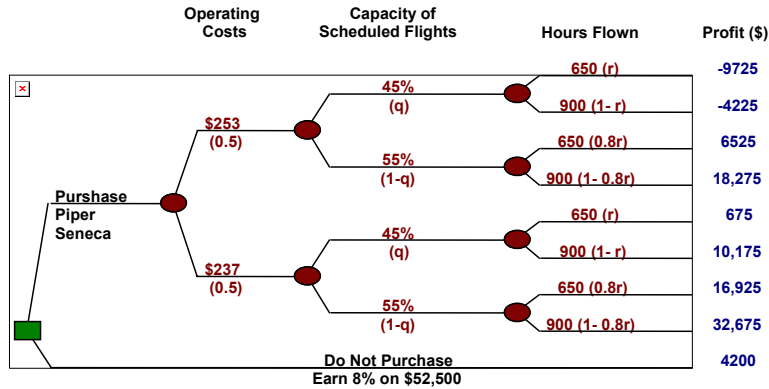

Decision Tree of Eagle Airlines Decision

Operating Costs	Capacity of Scheduled Flights	Hours Flown	Profit ($)
		650 (r)	-9725
	45% (q)	900 (1-r)	-4225
$253 (p)	High 55% (1-q)	650 (s)	6525
		900 (1-s)	18,275
Purchase Piper Seneca		650 (r)	675
	45% (q)	900 (1-r)	10,175
$237 (1-p)	High 55% (1-q)	650 (s)	16,925
		900 (1-s)	32,675
	Do Not Purchase Earn 8% on $52,500		4200

Probability p = probability (of operating cost = $253)

Probability q = probability (of capacity)

Probability r = P (Low Hours | Low capacity)

Probability s =P (Low Hours | High capacity). And It is expected that r > s

Decision Tree of Eagle Airlines Decision

Operating Costs	Capacity of Scheduled Flights	Hours Flown	Profit ($)

650 (r) — -9725
45% (q) — 900 (1- r) — -4225
$253 (0.5)
55% (1-q) — 650 (0.8r) — 6525
900 (1- 0.8r) — 18,275

Purshase Piper Seneca

650 (r) — 675
45% (q) — 900 (1- r) — 10,175
$237 (0.5)
55% (1-q) — 650 (0.8r) — 16,925
900 (1- 0.8r) — 32,675

Do Not Purchase
Earn 8% on $52,500 — 4200

Probability p = probability (of operating cost = $253) = 0.5

Probability q = 1-p = probability (of operating cost = $237) = 0.5

Probability r = P (Low Hours | Low capacity) = r

Probability s =P (Low Hours | High capacity) = 0.8r i.e. r > s. Now we have to calculate q and r.

Two-Way Sensitivity Graph for *q* and *r* of Eagle Airlines Example

B   Invest in Money Market

r = 1
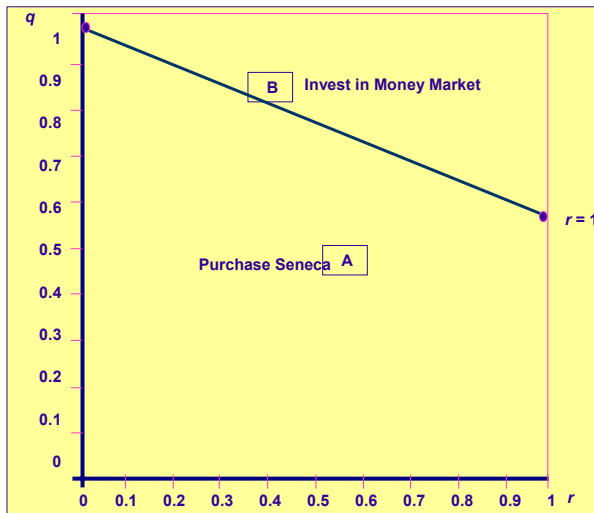
Purchase Seneca   A

Rolling back the decision tree so that EMV (Purchase) > 4200

Therefore:

$$\frac{21,275 - 11,000\ r}{22,500 - 3500\ r} > q$$

For Example:

If q is between 0.4 and 0.5 and r between 0.5 and 0.65. These are represented by points inside rectangle A. Hence, purchase Seneca, otherwise for other values rectangle B can be the result.

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Sensitivity Analysis Using Precision Tree

Class Exercise to Show How Sensitivity Analysis is made for a Decision Model

Run-Demo

Sensitivity Analysis for Oil Drilling Problem

▶

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Sensitivity Analysis: A Built-In Irony

- There is a strange irony in sensitivity analysis and decision making.

- The best way to view sensitivity analysis is as a source of guidance in modeling a decision problem.

- It provides the guidance for each successive iteration through the decision-analysis cycle.

Sensitivity Analysis:
A Built-In Irony

- The ultimate objective of this cycle of modeling and analysis is to arrive eventually at a requisite decision model and to analyze it just enough to understand clearly which alternative should be chosen.

- By the time the decision maker reaches this point, all important issues will be included in the decision model, and the choice should be clear.