Methodology for Developing Reliability-Based Load and Resistance Factor Design (LRFD) Guidelines for Ship Structures

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ABSTRACT

The main objective of ship structural design is to ensure safety and functional performance requirements of a structural system for target reliability levels, for a specified period of time, and for a specified environment. As this must be accomplished under conditions of uncertainty, probabilistic analyses are necessary in the development of such probability-based design criteria of hull structural components for surface ships. A methodology for developing load and resistance factor design (LRFD) guidelines for ship structures is outlined in this paper, and demonstrated for surface ship hull girders.

Future design guidelines for hull structural components of a marine vessel are currently being developed using reliability methods and are expressed in a special format such as the Load and Resistance Factor Design (LRFD) format. Reliability of these structural elements can be defined as its ability to fulfill their design functions for a specified time period. This ability is commonly measured using probabilities. Reliability is therefore, the occurrence probability of the complementary event to failure. Based on this definition, reliability is one of the components of risk. Safety can be defined as the judgment of risk acceptability for the system making it a component of risk management.

The performance of a ship structural component is defined by a set of requirements stated in terms of tests and measurements of how well the system or element serves various or intended

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functions over its service life. Risk and reliability measures can be considered as performance measures that can be specified in the form of target reliability levels (or target reliability indices, β_0's). The selected reliability levels of a particular structural element reflect the probability of failure of that element and the risk associated with it.

In this paper, reliability methods for developing LRFD-based partial safety factors (PSF's) for ship hull structural are described. These methods include analytical procedures, such as the First-Order Reliability Method (FORM), for calculating the partial safety factors. These factors can be used in LRFD formats to account for the uncertainties in strength and in the load effects. The FORM procedure can be used to determine these factors based on prescribed probabilistic characteristic of strength and load effects.

1. INTRODUCTION

The marine transportation industry can improve its process for designing systems, subsystems, and components on which its operations depend by utilizing risk-based methods and tools. In an environment of increasingly complex engineering systems, the concern about the operational safety of these systems continues to play a major role in both their design and operation. A systematic, quantitative approach for assessing the failure probabilities and consequences of engineering systems is needed. Such an approach allows an engineer to expediently and easily evaluate complex engineering systems for safety and risk under different operational conditions with relative ease. The ability to quantitatively evaluate these systems helps reduce the cost of unnecessary and often expensive re-engineering, repair or replacement of the system. The results of risk analysis can also be utilized in decision analysis problems that are based on cost-benefit tradeoffs.

For marine systems, there are many influences that affect their safety. Numerous sources of risk include equipment failure, external events, human errors, and institutional errors. Equipment failure is the most recognized hazard on ships and can be divided into several sub-categories including independent failures and common cause failures. An example of independent equipment failure is the loss of steering due to failure of a power steering pump. An example of a common cause failure includes the loss of propulsion and steering that would result from a total loss of electrical power to the ship. Risk from external events is caused by the hazards such as collision with other ships, sea state, wind, ice, or weather factors. Humans provide another source of risk to ships due to lack of skill, mistakes, fatigue, or sabotage. Institutional failure represents risks from poor management including training, management attitude, poor communications, and poor morale.

Risk studies can be classified into risk assessment, risk management, and risk communication. These aspects of risk studies are described under subsequent sections. The objective of introducing these concepts is to prepare users and readers of these guidelines for performing risk-based analysis of marine systems. These guidelines can also be used for developing risk-based standards for system safety.

The relationship between risk and standards is not new and its definition is dependent on the point of view of an observer. To better appreciate this dilemma we must take a look at the risk and standards from a historical perspective. People have always sought to eliminate unwanted risk to health and safety, or at least control it. Great successes have been achieved in controlling risk, as evidenced by advances made in the development of building methods of skyscrapers and long span bridges or super tankers capable of withstanding powerful storms. Yet some of the familiar risks persist while others less familiar are found to escape our attention and new ones have appeared. Ironically, some of the risks that are most difficult to manage provide us with increased standards of living. The invention of the automobile, the advent of air travel and space exploration, the development of synthetic chemicals, and the introduction of nuclear power all are examples.

1.1 Risk Methods

When assessing and evaluating uncertainties associated with an event, risk is defined as the potential for loss as a result of a system failure, and can be measured as a pair of factors, one being the probability of occurrence of an event, also called a failure scenario, and the other being the potential outcome or consequence associated with the event's occurrence. This pairing can be represented by the equation:

$$Risk \equiv \left[(p_1, c_1), (p_2, c_2), \ldots, (p_x, c_x) \right] \quad (1)$$

where p_x is the probability that event x will occur, and c_x is the consequence or outcome of the event's occurrence. Risk is commonly evaluated as the product of the likelihood of an event's occurrence and the impact of the event:

$$RISK\left(\frac{Consequence}{Time}\right) = LIKELIHOOD\left(\frac{Event}{Time}\right) \times IMPACT\left(\frac{Consequence}{Event}\right) \qquad (2)$$

In Eq. 2, likelihood may also be expressed as a probability. Occurrence probabilities (that can be annual) and consequences can be plotted as a Farmer curve (Ayyub et al. 1999).

Risks to a system may result from its interaction with natural hazards, its aging and degradation, or from human and organizational factors. Consequently, risk can be classified as either voluntary or involuntary, depending on whether or not the events leading to the risk are under the control of the persons at risk. Society generally accepts a higher level of voluntary risk than involuntary. The losses associated with events may be classified as either reversible and irreversible, depending whether the loss is of property or of human life, respectively.

Risk studies should consider the population-size effect because society responds differently to risks associated with large populations than it does to small populations. For example, a risk of fatality at the rate of 1 person in 100,000 per event for an affected population of 10 results in an "intolerable" expected fatality of 10^{-4} whereas the same fatality rate per event for an affected population of 10,000,000 results in a "tolerable" expected fatality of 100 per event. While numerical impact of the two scenarios is the same on society, the size of the population at risk should be considered as a factor is setting the acceptable risk level.

Risk methods may be classified as either risk management, which includes risk assessment and risk control, or risk communication, as shown in Figure 1.

Risk assessment is a technical and scientific process by which the risk of given situations for a system are modeled and quantified. Risk assessment provides qualitative and quantitative data to decision-makers for later use in risk management.

Risk assessment includes risk analysis and risk evaluation, where risk analysis consists of hazard identification, event-probability assessment, and consequence assessment, and risk evaluation requires the definition of acceptable risk and a comparative evaluation of options and/or alternatives. Risk control is achieved through monitoring and decision analysis. Risk communication is classified according to its target audience; either the media and the public or the engineering community.

The reliability of a system can be improved or decreased by the combination of individual elements in a system; therefore, occurrence probability and consequence are used to determine the risk associated with the system. When applying risk-based technology methods to system safety analysis, the following interdependent primary activities are considered: (1) risk assessment, (2) risk management, and (3) risk communication. These activities, when applied consistently provide a useful means for developing safety guidelines and requirements to the point where hazards are controlled at predetermined levels.

A risk assessment answers three questions: (a) What can go wrong? (b)What is the likelihood that it will go wrong? (c) What are the consequences if it does go wrong? In order to perform risk assessment several methods have been created including:

- Safety and Review Audits,
- Check List,
- What-if,
- Hazard and Operability Study (HAZOP),
- Probabilistic Risk Analysis (PRA),
- Preliminary Hazard Analysis (PrHA),
- Failure Modes and Effects Analysis (FMEA),
- Failure Modes Effects and Criticality Analysis (FMECA),
- Fault Tree Analysis (FTA), and
- Event Tree Analysis (ETA).

Each method of is suitable in certain stages of a system's life cycle.

The characteristics of commonly used methods are shown in Table 1. Other methods for reliability and consequence analysis and assessment are described by Kumamoto and Henley (1996).

Risk assessment methods can also be categorized according to whether the risk is determined by quantitative or qualitative analysis. Qualitative risk analysis uses expert opinion to identify and evaluate the probability and consequence of a hazard; quantitative analysis relies on statistical methods and databases. Safety Review/Audit, Checklist, What-If, Preliminary Hazard Analysis, and HAZOP are normally considered qualitative techniques. Probabilistic Risk Analysis, Failure Modes and Effects Analysis, Fault Tree, and Event Tree are generally considered quantitative risk assessment techniques. Whether to select a quantitative or a qualitative risk assessment method depends upon the availability of data for evaluating the hazard and the level of comfort of those performing the risk assessments.

Risk management incorporates all the processes by which system operators, managers, and owners make safety decisions and regulatory changes, and choose system configurations based on the data generated in the risk assessment; risk management involves using information from the risk assessment stage to make educated decisions about different configurations and operational parameters of a system. Its aim is to maintain the safety of the system and to control the risks involved in operating the system.

Risk management facilitates the making of decisions based on risk assessment and other factors including economic, political, environmental, legal, reliability, producibility, and safety.

Despite society's attempt to prevent accidents, government agencies can be reactive in the development of regulations. The answer to the question, "how safe is safe enough?" is difficult to reach because of changing perceptions and understandings of risk. Unfortunately, it often takes a disaster to stimulate action for safety issues. Although communication is necessary, it is important that risk management be separated from risk assessment to lend credibility to the risk assessment without biasing the evaluation in consideration of other factors. Especially in a qualitative assessment of risk, where "expert judgment" plays a role in decisions, it is important to allow the risk assessors to be free of the political pressures that managers encounter; however, there must be communication linking the risk assessors and risk managers. The risk assessors need to assist the risk managers in making decisions. While the managers should not be involved in making risk assessments, they should be involved in presenting the assessors with questions that need to be answered.

Several steps that should be considered in order to determine acceptable risk (Ayyub et al. 1999): (1) define alternatives, (2) specify the objectives and measures for effectiveness, (3) identify consequences of alternative, (4) quantify values for consequences, and (5) analyze alternatives to select the best choice. Risk managers need to weigh various other factors, for example, a manager might make a decision based on cost and risk using decision trees (Ayyub and McCuen 1997).

Risk communication can be defined as an exchange of information and opinion among individuals, groups, and institutions. This definition of risk communication contrasts it to risk-message transmittal from experts to non-experts. Risk communication should be interactive (NRC 1989); however, simply constructing a process as two-way does not make it an easy process. Technical information about controversial issues needs to be skillfully related by risk managers and communicators who may be viewed by the public as adversaries. Risk communication between risk assessors and risk managers is necessary to fully understand and effectively apply risk assessments in decision-making. Risk managers must participate in determining the criteria for determining acceptable and unacceptable risks.

While risk communication vitally links risk assessors, risk managers, and the public, it does not necessarily lead to harmony among the parties. Risk communication is a complex, dynamic process that must be handled with extreme care by experts, especially after disasters. Risk managers must establish contingency plans for risk communication about disasters. Added pressure by the media and the public following a disaster can create miscommunication that might be difficult to undo or remedy.

Reliability of a system can be defined as the system's ability to fulfill its design functions for a specified time. This ability is commonly measured using probabilities. Reliability is, therefore, the probability that the complementary event will occur to failure, resulting in

$$\text{Reliability} = 1 - \text{Failure Probability} \tag{3}$$

Based on this definition, reliability is one of the components of risk. Safety can be defined as the judgment of a risk's acceptability for the system safety, making to a component of risk management.

After risk and safety analyses are performed, system improvement in terms of risk can be achieved in one or more ways: (1) consequence reduction in magnitude or uncertainty, (2) failure-probability reduction in magnitude or uncertainty, and (3) reexamination of acceptable risk. Commonly in engineering, attention is given to failure-probability reduction in magnitude or uncertainty because it offers more system variables that can be controlled by analysts than the other two cases. As a result, it is common to perform a reliability-based design of systems. However, the other two cases should be examined for possible solution because they might offer some innovative options for system improvement.

1.2 Design of Ship Structural Components

The design of ship hull structural components needs to be performed within the framework of system design of ships that can be based on risk methods. In recent years, ship structural design has been moving toward a more rational and probability-based design procedure referred to as limit states design. Such a design procedure takes into account more information than deterministic methods in the design of structural components. This information includes uncertainties in the strength of various structural elements, in loads and load combinations, and modeling errors in analysis procedures. Probability-based design formats are more flexible and rational than working stress formats because they provide consistent levels of safety over various types of structures. In probability-based limit-state design, probabilistic methods are used to guide the selection of strength (resistance) factors and load factors, which account for the variability in the individual resistance and loads and give the desired overall level of reliability. The load and resistance factors (or called partial safety factors) are different for each type of load and resistance. Generally, the higher the uncertainty associated with a load, the higher the corresponding load factor; and the higher the uncertainty associated with strength, the lower the corresponding strength factor.

Ship designers can use the load and resistance factors in limit-state equations to account for uncertainties that might not be considered properly by deterministic methods without explicitly performing probabilistic analysis. For this reason, design criteria can be kept as simple as possible. Moreover, they should be developed in a form that is familiar to the users or designers, and should produce desired levels of uniformity in reliability among different types of structures, without departing drastically from an existing practice. There is no unique format for a design criterion. A criterion can be developed on probability bases in any format. In general, the basic approach to develop reliability-based design guidelines is first to determine the relative reliability of designs based on current practice. This relative reliability can be expressed in terms of either a probability of failure or a reliability index. The reliability index for structural components normally varies between 2 and 6 (Mansour et al. 1984). By performing such reliability analyses for many structures, representative values of target reliability (or safety) index can be selected reflecting the average reliability implicit in current designs. Based on these values and by using reliability analysis again, it is possible to select partial safety factors for the loads and the strength random variables that can be used as a basis for developing the design requirements.

For design code provisions, the most commonly used format is the utilization of load amplification factors and resistance reduction factors (partial safety factors), as represented by

$$\phi R \geq \sum_{i=1}^{n} \gamma_i L_i \tag{4}$$

where ϕ = the resistance R reduction factor; γ_i = the partial load amplification factor; and L_i = the load effect. In fact, the American Institute of Steel Construction (AISC) and other industries in this area have implemented this format. Also, a recommendation for the use of this format is given by the National Institute of Standards and Technology (Ellingwood et al. 1980). The First-Order Reliability Method (FORM) is commonly used to estimate the partial safety factors ϕ and γ_i for a specified target reliability index β_0. This method was used to determine the partial safety factors associated with the recommended strength models for ship hull girders as demonstrated in this chapter.

2. RELIABILITY-BASED DESIGN METHODS

Reliability-based design of ship structures requires the consideration of the following three components: (1) loads, (2) structural strength, and (3) methods of reliability analysis. These three components are shown in Figure 2 in the form of several blocks for each. Also, the figure shows their logical sequence and interaction.

There are two primary approaches for reliability-based design: (1) direct reliability-based design and (2) load and resistance factor design (LRFD) as shown in Figure 2 (Ayyub et al. 1995). The direct reliability-based design approach can include both Level 2 and/or Level 3 reliability methods. Level 2 reliability methods are based on the moments (mean and variance) of random variables and sometimes with a linear approximation of nonlinear limit states, whereas, Level 3 reliability methods use the complete probabilistic characteristics of the random variables. In some cases, Level 3 reliability analysis is not possible because of lack of complete information on the full probabilistic characteristics of the random variables. Also, computational difficulty in Level 3 methods sometimes discourages their uses. The LRFD approach is called a Level 1 reliability method. Level 1 reliability methods utilize partial safety factors (PSF) that are reliability based; but the methods do not require explicit use of the probabilistic description of the variables.

2.1 Reliability-Based Design Philosophy

The design of any ship structural system or element must provide for adequate safety and proper functioning of that system or element regardless of what philosophy of design is used. The structural systems or elements must have adequate strength to permit proper functioning during their intended service life. The performance of a hull structural element as presented in this paper is defined by a set of requirements stated in terms of tests and measurements of how well the hull girder serves various or intended functions over its service life. Reliability and risk measures can be considered as performance measures, specified as target reliability levels (or target reliability indices, β_0's). The selected reliability levels of a particular structural element reflect the probability of failure of that element. These levels can be set based on implied levels in the currently used design practice with some calibration, or based on cost-benefit analysis. The following three methods can be used to select a target reliability value: (1) agreeing upon a reasonable value in cases of novel structure without prior history, (2) calibrating reliability levels implied in currently used design codes, and (3) choosing a target reliability level that minimizes total expected costs over the service life of the marine structure when dealing with design for which failures result in only economic losses and consequences.

The reliability-based design approaches for a system start with the definition of a mission and an environment for a ship. Then, the general dimensions and arrangements, structural member sizes, scantlings, and details need to be estimated or assumed. The weight of the structure can then be estimated to ensure its conformance to a specified limit. Using an assumed operational-sea profile, the analysis of the ship produces stochastic stillwater and wave-induced responses. The resulting responses can be adjusted using modeling uncertainty estimates that are based on any available results of full-scale or large-scale testing.

The reliability-based design procedure also requires defining performance functions that correspond to limit states for significant failure modes. In general, the problem can be considered as one of supply and demand. Failure of a structural element occurs when the supply (i.e., strength of the element) is less than the demand (i.e., loading on the element). On the other hand, the reliability of this element is achieved when the supply is greater than the demand. A generalized form for the performance function for a structural component is given by

$$g = R - L \tag{5}$$

where g = performance function, R = strength (resistance), and L = loading on the structural element. The failure in this case is defined in the region where g is less than zero or R is less than L, that is

$$g < 0.0 \text{ or } R < L \tag{6}$$

whereas the reliability is defined in the region where g is greater than zero or R is greater than L, that is

$$g > 0.0 \text{ or } R > L \tag{7}$$

The reliability-based design approach as given assumes the strength R and the load L to be random variables. Typical frequency distributions of such random variables are shown in Figure 3. If R is greater than L, there will be a margin of safety. However, unless R is greater than L by a large amount, there is always a probability that L may exceed R. This possibility is illustrated as the shaded area in Figure 3 where the two curves for R and L overlap. Due to the variability in both strength and loads, there is always a probability of failure that can be defined as

$$P_f = P(g < 0.0) = P(R < L) \tag{8}$$

The reliability of a system or a component can be defined as the probability that the system or the component meets some specified demands for a specified time frame. Mathematically, it is given by the following expression:

$$R_c = P(g > 0.0) = P(R > L) \tag{9}$$

where P_f = probability of the system or component and R_c = reliability of the system or component.

The many advantages and benefits of using reliability-based design methods include the followings:

1. They provide the means for the management of uncertainty in loading, strength, and degradation mechanisms.
2. They provide consistency in reliability.
3. They result in efficient and possibly economical use of materials.
4. They provide compatibility and reliability consistency across materials, such as, steel grades, aluminum and composites.

5. They allow for future changes as a result of gained information in prediction models, and material and load characterization.
6. They provide directional cosines and sensitivity factors that can be used for defining future research and development needs.
7. They allow for performing time-dependent reliability analysis that can form the bases for life expectancy assessment, life extension, and development of inspection and maintenance strategies.
8. They are consistent with other industries as presented by AISC, ASHTO, ACI, API, and ASME.
9. They allow for performing system reliability analysis.

2.2 Direct Reliability-Based Design

The direct reliability-based design method uses all available information about the basic variables (including correlation) and does not simplify the limit state in any manner. It requires performing spectral analysis and extreme analysis of the loads. In addition, linear or nonlinear structural analysis can be used to develop a stress frequency distribution. Then, stochastic load combinations can be performed. Linear or nonlinear structural analysis can then be used to obtain deformation and stress values. Serviceability and strength failure modes need to be considered at different levels of the ship, i.e., hull girder, grillage, panel, plate and detail. The appropriate loads, strength variables, and failure definitions need to be selected for each failure mode. Using reliability assessment methods such as FORM, reliability indices β's for all modes at all levels need to be computed and compared with target reliability indices β_0's. The relationship between the reliability index β and the probability of failure is given by

$$P_f = 1 - \Phi(\beta) \tag{10}$$

where $\Phi(.)$ = cumulative probability distribution function of the standard normal distribution, and β = reliability (safety) index. It is to be noted that Eq. 10 assumes all the random variables in the limit state equation to have normal probability distribution and the performance function is linear. However, in practice, it is common to deal with nonlinear performance functions with a relatively small level of linearity. If this is the case, then the error in estimating the probability of failure P_f is very small, and thus for all practical purposes, Eq. 10 can be used to evaluate P_f with sufficient accuracy (Ayyub and McCuen 1997).

2.3 Load and Resistance Factor Design (LRFD)

The second approach (LRFD) of reliability-based design consists of the requirement that a factored (reduced) strength of a structural component is larger than a linear combination of factored (magnified) load effects as given by the following general format:

$$\phi R_n \geq \sum_{i=1}^{m} \gamma_i L_{ni} \tag{11}$$

where ϕ = strength factor, R_n = nominal (or design) strength, γ_i = load factor for the ith load component out of n components, and L_{ni} = nominal (or design) value for the ith load component out of m components.

In this approach, load effects are increased, and strength is reduced, by multiplying the corresponding characteristic (nominal) values with factors, which are called strength (resistance) and load factors, respectively, or partial safety factors (PSF's). The characteristic value of some quantity is the value that is used in current design practice, and it is usually equal to a certain percentile of the probability distribution of that quantity. The load and strength factors are different for each type of load and strength. Generally, the higher the uncertainty associated with a load, the higher the corresponding load factor; and the higher the uncertainty associated with strength, the lower the corresponding strength factor. These factors are determined probabilistically so that they correspond to a prescribed level of reliability or safety. It is also common to consider two classes of performance function that correspond to strength and serviceability requirements.

The difference between the allowable stress design (ASD) and the LRFD formats is that the latter uses different safety factors for each type of load and strength. This allows for taking into consideration uncertainties in load and strength, and to scale their characteristic values accordingly in the design equation (Assakkaf 1998). ASD (sometimes called working stress) formats cannot do that because they use only one safety factor as seen by the following general design format:

$$\frac{R}{\mathrm{FS}} \geq \sum_{i=1}^{m} L_i \tag{12}$$

where R = strength or resistance, L_i = load effect, and FS = factor of safety. In this design format, all loads are assumed to have average variability. The entire variability of the strength and the loads is placed on the strength side of the equation. The factor of safety FS accounts for this entire variability.

In the LRFD design format, ship designers can use the load and resistance factors in limit-state equations to account for uncertainties that might not be considered properly by deterministic methods (i.e., ADS) without explicitly performing probabilistic analysis. The LRFD format as described herein is concerned mainly with the structural design of ship hull girder structural components under combinations of different load effects. The intention herein is to provide naval architects and ship designers with reliability-based methods for their use in both early and final design stages and for checking the adequacy of the scantlings of all structural members contributing to the longitudinal and transverse strength of ships. Eq. 11 gives the general form of the LRFD format used in this paper.

The probabilistic characteristics and nominal values for the strength and load components can be determined based on statistical analysis, recommended values from other specifications, and by professional judgment. The LRFD general design formats for hull structural components are given by one of the following two main cases, limit sate 1, and limit sate 2, respectively:

<u>Limit State 1</u>:

$$\phi R_n \geq \gamma_{SW} L_{SW} + k_{WD} \gamma_{WD} L_{WD} \tag{13}$$

<u>Limit State 2</u>:

$$\phi R_n \geq \gamma_{SW} L_{SW} + k_W \left(\gamma_W L_W + k_D \gamma_D L_D \right) \tag{14}$$

where ϕ = strength factor, R_n = nominal (or design) strength such as the ultimate stress, γ_{SW} = load factor for stillwater load effect such as bending moment, L_{SW} = nominal (or design) value for stillwater load effect such as bending moment, k_{WD} = combined wave-induced and dynamic bending moment factor, and γ_{WD} = load factor for combined wave-induced and dynamic bending moment, L_{WD} = nominal (or design) value for wave-induced and dynamic bending moments effect, k_W = load combination factor, γ_W = load factor for waves bending moment load effect, L_W

= nominal (or design) value for waves bending moment load effect, k_D = load combination factor, γ_D = load factor for dynamic load effect such as bending moment, and L_D = nominal (or design) value for dynamic load effect such as bending moment.

The strength and load factors are collectively called partial safety factors (PSF's). These factors are determined using structural reliability methods based on the probabilistic characteristics of basic random variables for materials, geometry and loads including statistical and modeling (or prediction) uncertainties. The factors are determined to meet target reliability levels that were selected based on assessing previous designs. This process of developing LRFD guidelines to meet target reliability levels that are implicit in current practices is called code calibration.

2.4 Reliability Checking

The LRFD methods also provide formats for reliability (safety) checking for various types of hull structural elements. In order to perform a reliability checking on these elements, the computed reliability safety index β resulting from reliability assessment using for example FORM, should not be less than the target safety index β_0 as given by the following expression:

$$\beta \geq \beta_0 \tag{15}$$

Reliability checking for different classes of ship structural elements can also be performed using the general form of the load and resistance factor design format of Eq. 11. Depending on the limit state, the nominal strength R_n of the structural component shall meet one of following two main requirements for limit states 1 and 2, respectively:

$$R_n \geq \frac{\gamma_{SW} L_{SW} + k_{WD} \gamma_{WD} L_{WD}}{\phi} \tag{16}$$

$$R_n \geq \frac{\gamma_{SW} L_{SW} + k_W \left(\gamma_W L_W + k_D \gamma_D L_D \right)}{\phi} \tag{17}$$

2.5 First-Order Reliability Method (FORM)

The First-Order Reliability Method (FORM) is a convenient mathematical tool to assess the reliability of a ship structural element. It also provides a means for calculating the partial safety factors ϕ and γ_i that appear in Eq. 11 for a specified target reliability level β_0. The simplicity of the first-order reliability method stems from the fact that this method, beside the requirement that the distribution types must be known, requires only the first and second moments; namely the mean values and the standard deviations of the respective random variables. Knowledge of the joint probability density function (PDF) of the design basic variables is not needed as in the case of the direct integration method for calculating the reliability index β. Even if the joint PDF of the basic random variables is known, the computation of β by the direct integration method can be a very difficult task.

In design practice, there are usually two types of limit states: the ultimate limit state and the serviceability limit state. Both types can be represented by the following performance function:

$$g(\boldsymbol{X}) = g(X_1, X_2, ..., X_n) \tag{18}$$

in which $\boldsymbol{X}$ is a vector of basic random variables (X_1, X_2, ..., X_n) for the strengths and the loads. The performance function $g(\boldsymbol{X})$ is sometimes called the limit state function. It relates the random variables for the limit-state of interest. The limit state is defined when $g(\boldsymbol{X}) = 0$, and therefore, failure occurs when $g(\boldsymbol{X}) < 0$ (see Figure 4). The reliability index β is defined as the shortest distance from the origin to the failure surface in the reduced coordinates at the most probable failure point (MPFP) as shown in Figure 4.

As indicated earlier, the basic approach for developing reliability-based design guidelines requires the determination of the relative reliability of designs based on current practices. Therefore, reliability assessment of existing structural components of ships such as the hull girder is needed to estimate a representative value of the reliability index β. The first-order-reliability method is very well suited to perform such a reliability assessment. The following are computational steps as described in Ayyub and McCuen (1997), and in Ang and Tang (1990) for determining β using the FORM method:

1. Assume a design point x_i^* and obtain $x_i^{'*}$ in the reduced coordinate using the following equation:

$$x_i^{'*} = \frac{x_i^* - \mu_{X_i}}{\sigma_{X_i}} \tag{19}$$

where $x_i^{'*} = -\alpha_{\mathrm{i}}^* \beta$, μ_{X_i} = mean value of the basic random variable, and σ_{X_i} = standard deviation of the basic random variable. The mean values of the basic random variables can be used as initial values for the design points. The notation x^* and $x^{'*}$ are used respectively for the design point in the regular coordinates and in the reduced coordinates.

2. Evaluate the equivalent normal distributions for the non-normal basic random variables at the design point using the following equations:

$$\mu_X^N = x^* - \Phi^{-1}\left(F_X(x^*)\right)\sigma_X^N \tag{20}$$

and

$$\sigma_X^N = \frac{\left(\Phi^{-1}\left(F_X(x^*)\right)\right)}{f_X(x^*)} \tag{21}$$

where $\mu_X^{\mathrm{N}} =$ mean of the equivalent normal distribution, $\sigma_X^N =$ standard deviation of the equivalent normal distribution, $F_X(x^*) =$ original (non-normal) cumulative distribution function (CDF) of X_i evaluated at the design point, $f_X(x^*)$ = original probability density function (PDF) of X_i evaluated at the design point, $\Phi(\cdot)$ = CDF of the standard normal distribution, and $\phi(\cdot)$ = PDF of the standard normal distribution.

3. Compute the directional cosines at the design point (α_i^*, i = 1,2, ..., n) using the following equations:

$$\alpha_i^* = \frac{\left(\frac{\partial g}{\partial x_i'}\right)_*}{\sqrt{\sum_{i=1}^{n}\left(\frac{\partial g}{\partial x_i'}\right)_*^2}} \qquad \text{for } i = 1, 2, ..., n \tag{22}$$

where

$$\left(\frac{\partial g}{\partial x_i'}\right)_* = \left(\frac{\partial g}{\partial x_i}\right)_* \sigma_{X_i}^N \tag{23}$$

4. With α_i^*, $\mu_{X_i}^N$, and $\sigma_{X_i}^N$ now known; the following equation can be solved for the root β:

$$g\left[(\mu_{X_1}^N - \alpha_{X_1}^* \sigma_{X_1}^N \beta), ..., (\mu_{X_n}^N - \alpha_{X_n}^* \sigma_{X_n}^N \beta)\right] = 0 \tag{24}$$

5. Using the β obtained from step 4, a new design point can be obtained from the following equation:

$$x_i^* = \mu_{X_i}^N - \alpha_i^* \sigma_{X_i}^N \beta \tag{25}$$

6. Repeat steps 1 to 5 until a convergence of β is achieved. The reliability index is the shortest - distance to the failure surface from the origin in the reduced coordinates as shown in Figure 4.

The important relation between the probability of failure and the reliability (safety) index is given by Eq. 10.

2.5.1 Procedure for Calculating Partial Safety Factors (PSF) Using FORM

The first-order reliability method (FORM) can be used to estimate partial safety factors such those found in the design format of Eq. 11. At the failure point ($R^*, L_1^*, ..., L_n^*$), the limit state of Eq. 11 is given by

$$g = R^* - L_1^* - ... - L_n^* = 0 \tag{26}$$

or, in a general form

$$g(\boldsymbol{X}) = g(x_1^*, x_2^*, ..., x_n^*) = 0 \tag{27}$$

For given target reliability index β_0, probability distributions and statistics (means and standard deviations) of the load effects, and coefficient of variation of the strength, the mean value of the resistance and the partial safety factors can be determined by the iterative solution of Eqs. 19 through 25. The mean value of the resistance and the design point can be used to compute the required mean partial design safety factors as follows

$$\phi = \frac{R^*}{\mu_R} \tag{28}$$

$$\gamma_i = \frac{L_i^*}{\mu_{L_i}} \tag{29}$$

The strength factors are generally less than one, whereas the load factors are greater than one.

2.5.2 Determination of a Strength Factor for a Given Set of Load Factors

In developing design code provisions for ship hull girders, it is sometimes necessary to follow the current design practice to insure consistent levels of reliability over various types of ship structures. Calibrations of existing design codes is needed to make the new design formats as simple as possible and to put them in a form that is familiar to the users or designers. Moreover, the partial safety factors for the new codes should provide consistent levels of reliability. For a given reliability index β and probability characteristics for the resistance and the load effects, the partial safety factors determined by the FORM approach might be different for different failure modes for the same structural component. Therefore, the calculated partial safety factors (PSF's) need to be adjusted in order to maintain the same values for all loads at different failure modes by the strength factor ϕ for a given set of load factors. The following algorithm can be used to accomplish this objective:

1. For a given value of the reliability index β, probability distributions and statistics of the load variables, and the coefficient of variation for the strength, compute the mean strength needed to achieve the target reliability using the first-order reliability method as outlined in the previous sections.

2. With the mean value for R computed in step 1, the partial safety factor can be revised for a given set of load factors as follows:

$$\phi` = \frac{\sum_{i=1}^{n} \gamma`_i \mu_{L_i}}{\mu_R} \tag{30}$$

where $\phi`$ = revised strength factor, $\mu_{L_i}$ and $\mu_R$ are the mean values of the loads and strength variables, respectively; and $\gamma`_i$, i = 1, 2, ..., n, are the given set of load factors.

3. EXAMPLE: UNSTIFFENED PLATE PANEL UNDER UNIAXIAL COMPRESSION

Plates are important components in ship structures, and therefore they should be designed for a set of failure modes such as yielding, buckling, and fatigue of critical connecting components. This example consider only a simply supported rectangular plate of size a by b under uniaxial compressive stress. The limit state for this case is given by

$$g = F_u - f_{SW} - f_W \tag{31}$$

where F_u = the strength of the plate (stress), f_{SW} = external stress due to stillwater bending, and f_W = external stress due to wave bending. The strength F_u is given by one of the following two cases:

1. For $a/b \geq 1.0$

$$F_u = \begin{cases} F_{yp}\sqrt{\dfrac{\pi^2}{3(1-\nu^2)B^2}} & \text{if } B \geq 3.5 \\ F_{yp}\left(\dfrac{2.25}{B} - \dfrac{1.25}{B^2}\right) & \text{if } 1.0 \leq B < 3.5 \\ F_{yp} & \text{if } B < 1.0 \end{cases} \tag{32}$$

2. For $a/b < 1.0$

$$F_u = F_{yp}\left[\alpha C_u + 0.08(1-\alpha)\left(1 + \frac{1}{B^2}\right)^2\right] \leq 1.0 \tag{33}$$

where F_{yp} = yield strength (stress) of plate, a = length or span of plate, b = distance between longitudinal stiffeners or width of plate, and in which $B = \frac{b}{t}\sqrt{\frac{F_{yp}}{E}}$, $\alpha = \frac{a}{b}$, t = thickness of the plate, E = modulus of elasticity, ν = Poisson's ratio, and

$$C_u = \begin{cases} \sqrt{\dfrac{\pi^2}{3\left(1-\nu^2\right)B^2}} & \text{if } B \geq 3.5 \\ \dfrac{2.25}{B} - \dfrac{1.25}{B^2} & \text{if } 1.0 \leq B \leq 3.5 \\ 1.0 & \text{if } B < 1.0 \end{cases} \tag{34}$$

3.1 Probabilistic Characteristics of the Strength F_u

The probabilistic characteristics of the strength F_u was assessed based on the underlying basic random variables that define F_u. These variables are a, b, t, F_{yp}, and E. Monte Carlo simulation was utilized to assess the probabilistic characteristics of the strength, F_u by generating a, b, t, F_{yp} , and E, and then feeding the generated values in the strength equation to obtain F_u values. This process was repeated for ranges of selected key parameters as shown in Table 2a. Additional information and assumptions were needed for the probabilistic characteristics of the basic random variables (Assakkaf and Ayyub 1995). This information and assumptions are provided in Table 2b. Poisson's ratio ν was assumed to be deterministic and thus, a value of 0.3 was considered in this example.

The above strength basic random variables were assumed to have normal probability distributions. The results of the simulation were expressed in the form of mean to nominal ratio of F_u, the coefficient of variation (COV) of F_u, and the distribution type of F_u. The number of simulation cycles was set at 100, which is adequate for all practical purposes based on the charts provided in Figure. 5, for a typical set of an estimated mean, coefficient of variation, and the coefficient of variation of the sample mean for F_u. The results of the simulation of F_u are summarized in Tables 3, and 4. The distribution type for F_u was determined to be either normal or lognormal. A lognormal probability distribution for R was used in this study. The strength F_u has a mean to nominal ratio of about 1.03. This ratio will be needed to revise the resulting strength reduction factor by multiplying it by 1.03. The maximum and minimum strength ratios were found to be 1.043, and 1.006, respectively. The maximum and minimum coefficients of variation (COV) of strength were found to be 0.08, and 0.04, respectively.

3.2 Calculation of Partial Safety Factors

The partial safety factors for the limit state equation (Eq. 31) were developed using a target reliability index β of 3.0. This equation provides strength minus load effect expression of the limit state. The First-Order Reliability Method (FORM) as discussed in Section 2.5 requires the probabilistic characteristics of F_u, f_{SW}, and f_W. The stillwater load effect f_{SW} is due to stillwater bending that can be assumed to follow a normal distribution with a coefficient of variation of 0.2. The wave load effect f_W is due to waves that can be assumed to follow an extreme value distribution (Type I, largest) with a coefficient of variation of 0.1. The mean values of stillwater and waves are considered in the study in the form of a ratio of wave/stillwater loads that ranges from 1.5 to 1.7.

The simulation results of F_u were used to develop the partial safety factors based on the limit state equation. The partial safety factors were computed for several selected cases that cover the assumed ranges of the parameters *a, b, t,* F_{yp} and *E*. The ratios of means for strength/stillwater load and the partial safety factors for a target reliability of 3.0 are summarized in Tables 5 and 6, respectively, and in Figure 6. Based on these results, the following preliminary values for partial safety factors are recommended for demonstration purposes:

Strength reduction factor (ϕ) = 0.85(1.03) = 0.88

Stillwater load factor (γ_S) = 1.3

Wave load factor (γ_W) = 1.25

3.3 Calculation of Strength Factor For a Given Set of Load Factors

As indicated in Section 2.5.2, for a given β and probabilistic characteristics for the strength and the load effects, the partial safety factors determined by the FORM approach might be different for different failure modes. For this reason calibration is often needed on the strength factor ϕ to maintain the same values for all load factors γ's. The following numerical example illustrates the procedure of Section 2.5.2 for revising the strength factor for a given set of load factors. For instance, given $\gamma_S = 1.3$, $\gamma_W = 1.2$, and the probabilistic characteristics of the random variables as shown Table 7, the corresponding strength factor ϕ was calculated for a target reliability level $\beta = 3.0$. Using FORM as outlined in Section 2.5.2, the mean of F_u was found to be 3.66. With the mean value known, Eq. 30 gives

$$\phi = \frac{\gamma_S \mu_S + \gamma_W \mu_W}{\mu_{F_u}}(1.03) = \frac{1.3(1) + 1.2(1.6)}{3.66}(1.03) = 0.91$$

Since the strength F_u has a mean to nominal ratio of 1.03, this ratio was needed to revise ϕ by multiplying it by 1.03.

4. SUMMARY AND CONCLUSIONS

Reliability of a system or component can be defined as its ability to fulfill its design functions for a specified time period. This ability is commonly measured using probabilities. Reliability is therefore, the occurrence probability of the complementary event to failure as given by Eq. 3. Based on this definition, reliability is one of the components of risk. Safety can be defined as the judgment of risk acceptability for the system making it a component of risk management.

The performance of ship structural components is defined by a set of requirements stated in terms of tests and measurements of how well the system or element serves various or intended functions over its service life. Risk and reliability measures can be considered as performance measures that can be specified in the form of target reliability levels (or target reliability indices, β_0's). The selected reliability levels of a particular structural element reflect the probability of failure of that element and the risk associated with it.

An important consideration in the choice of LRFD criteria is the consequence of failure. Clearly the target reliability levels relative to the collapse of the hull girder should be larger than that of a non-critical welded detail relative to fatigue. The following three methods (Ayyub et al. 2000) can be used to select a target reliability value: (a) agreeing upon a reasonable value in the case of novel structures without prior history using expert opinion elicitation, (b) calibrating reliability levels implied in currently and successfully used design codes, and (c) choosing target reliability level that minimizes the costs over the service life of the structure for dealing with design for which failure results in only economic losses an consequences.

The First-Order Reliability Method (FORM) can be used to assess the reliability of a structural system as well as to develop and establish partial safety factors. In this study, the FORM method was used to develop partial safety factors for a simply supported plate (unstiffend panel) under uniaxial compressive stress. The strength model for the plate F_u for this case was established. Then Monte Carlo simulation was utilized to assess the probabilistic characteristics of the strength F_u by generating the basic random variables that define the strength and then feeding the generated values in the strength model for the plate to obtain F_u values. The distribution type of F_u was determined to be lognormal. The maximum and minimum *COV* values of F_u were found to be 0.08 and 0.04, respectively. The prescribed probabilistic characteristics of the load effects and the results of the strength simulation were used to develop the partial safety factors based on a linear limit state. The partial safety factors were computed for several selected cases that cover the assumed ranges of key parameters that define the strength F_u. Based on these results and for a target reliability level β of 3.0, the following values for partial safety factors were selected for the purpose of demonstration:

Strength reduction factor $\phi = 0.88$

Stillwater load factor $\gamma_{SW} = 1.30$

Wave load factor $\gamma_W = 1.25$

The resulting partial safety factors can be used to design plates under uniaxial compressive stress to meet a strength limit state given by the following design format:

$$\phi F_u \leq \gamma_{SW} f_{SW} + \gamma_W f_W \tag{35a}$$

or

$$0.88 F_u \leq 1.3 f_{SW} + 1.25 f_W \tag{35b}$$

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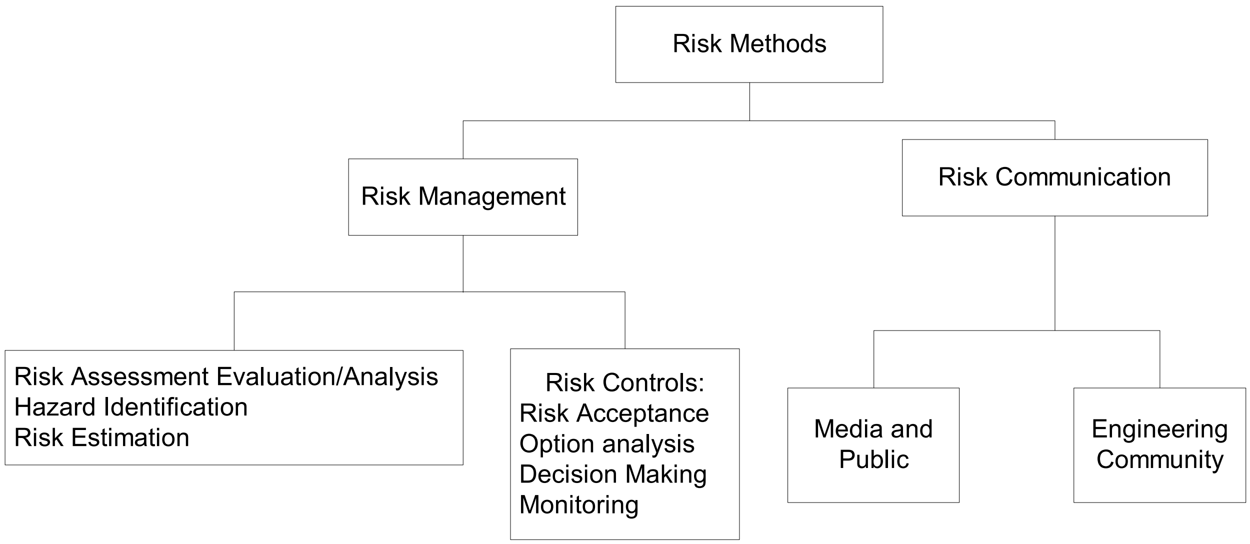

Figure 1. Risk Methods

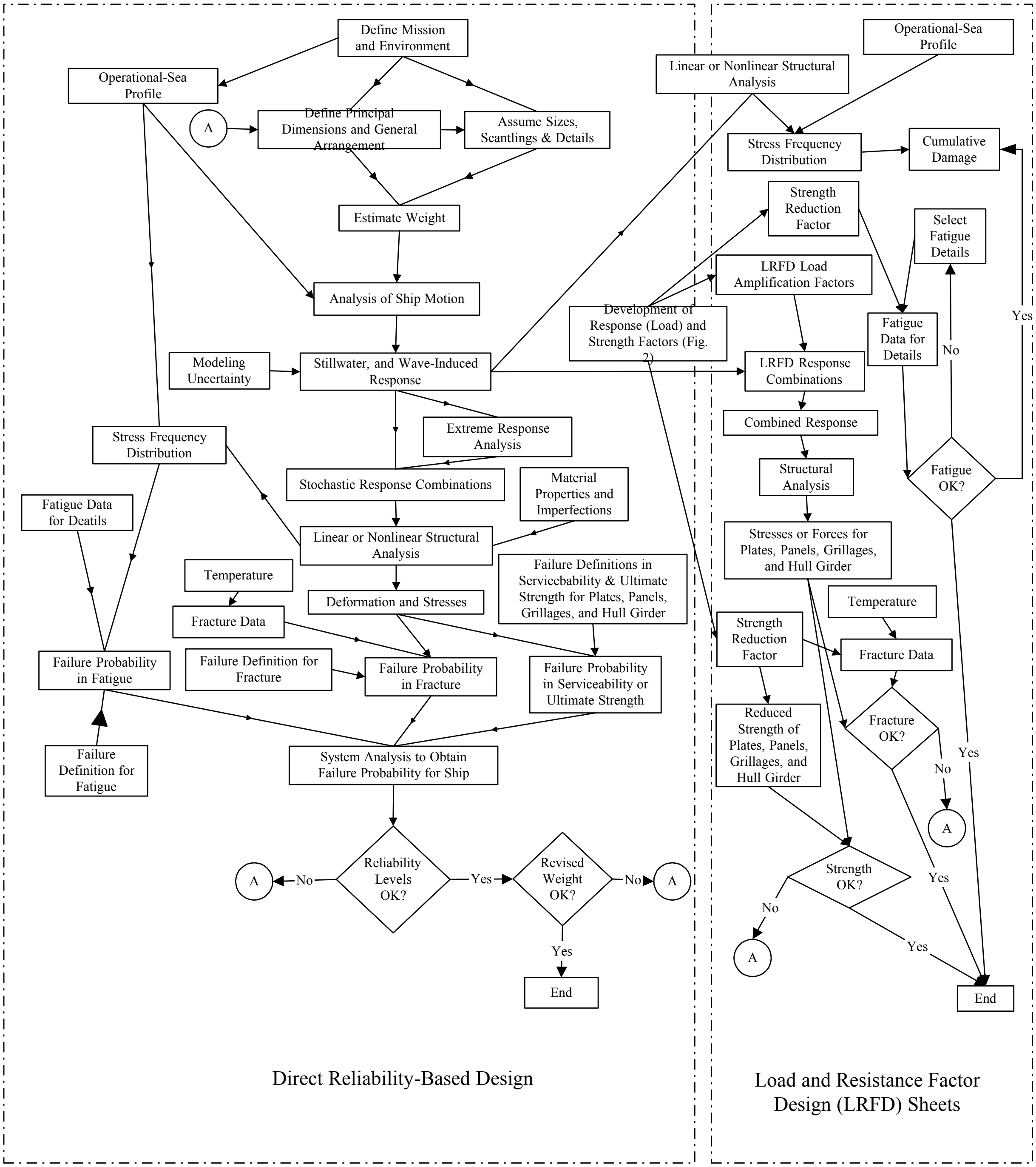

Figure 2. Reliability-based Design of Ship Structures (Ayyub et al 1995)

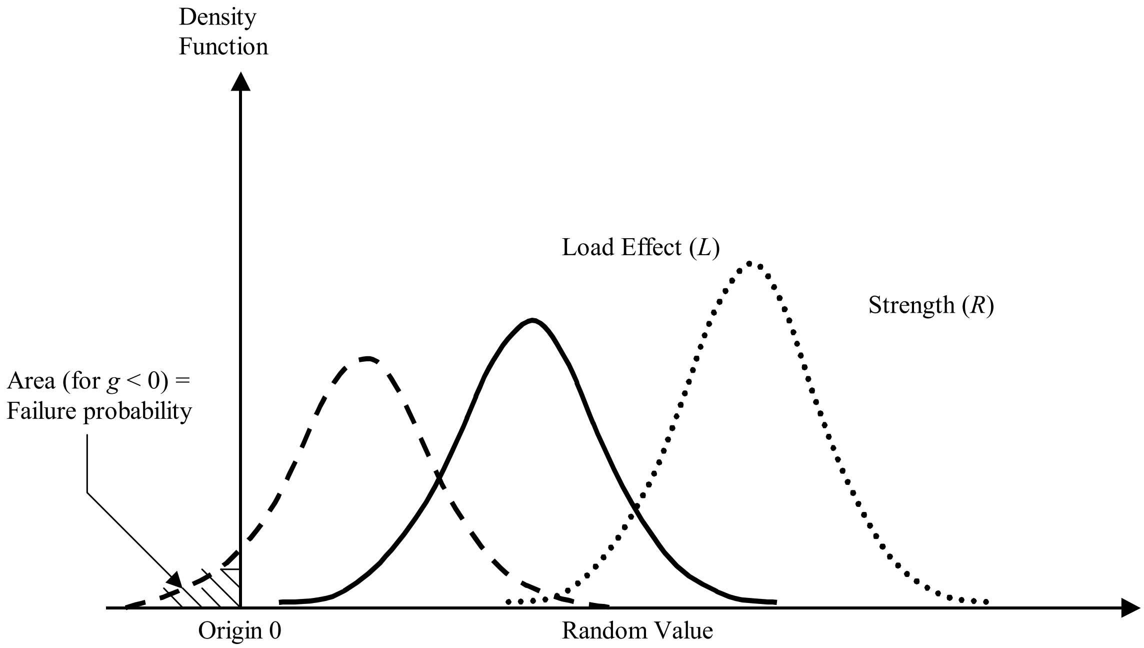

Figure 3. Reliability Density Functions of Resistance R and Load L

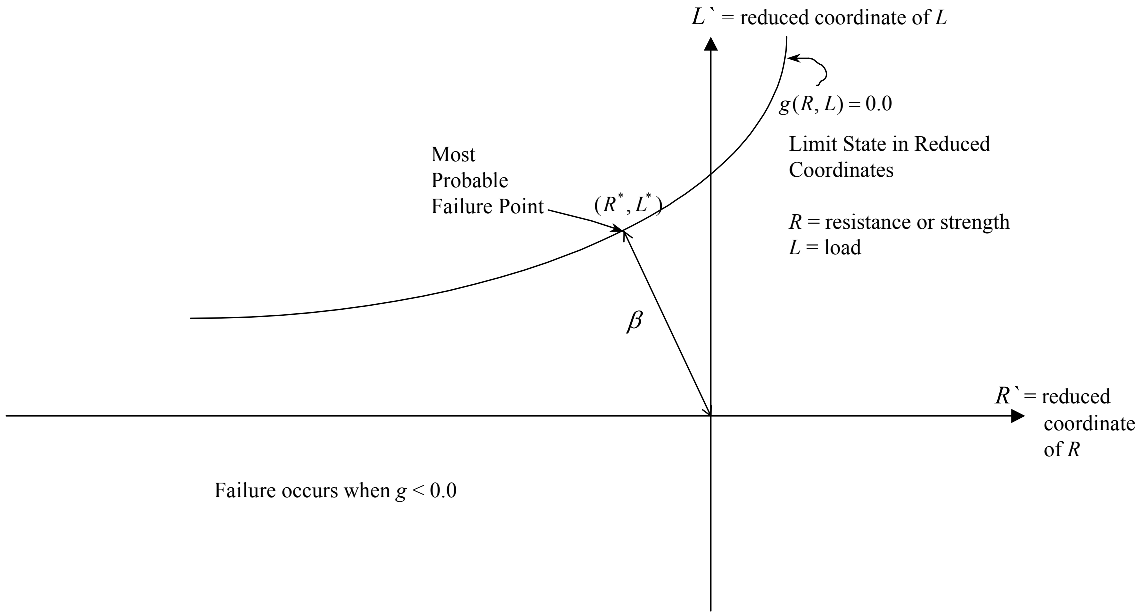

Figure 4. Space of Reduced Random Variables Showing the Reliability Index and the Most Probable Failure Point

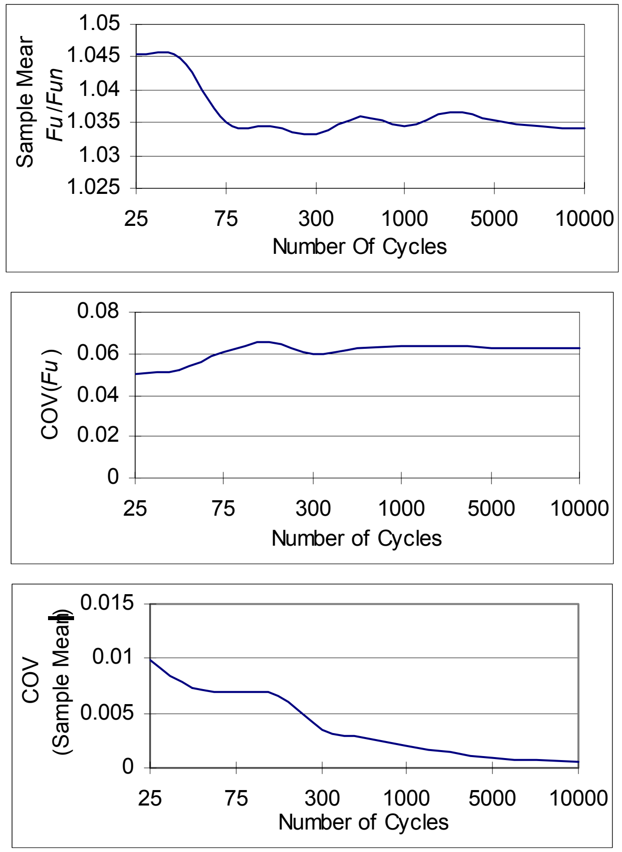

Figure 5. Effect of Simulation Cycles on Sample Mean for F_u / F_{un}, *COV* of F_u, and *COV* of Sample Mean for F_u / F_{un}

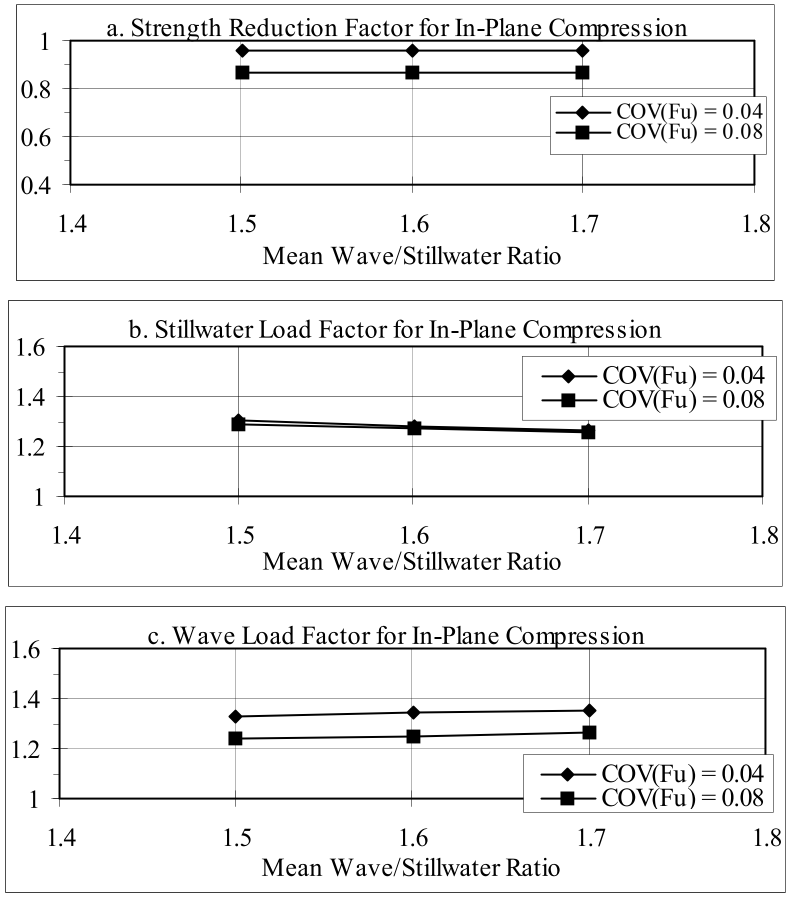

Figure 6. Partial Safety Factors for Plates Under Uniaxial Compressive Stress

Table 1. Risk Assessment Methods

Safety/Review Audit
Identify equipment conditions or operating procedures that could lead to a casualty or result in property damage or environmental impacts.
Checklist
Ensure that organizations are complying with standard practices.
What-If
Identify hazards, hazardous situations, or specific accident events that could result in undesirable consequences.
Hazard and Operability Study (HAZOP)
Identify system deviations and their causes that can lead to undesirable consequences. Determine recommended actions to reduce the frequency and/or consequences of the deviations.
Failure Modes and Effects Analysis (FMEA)
Identifies the components (equipment) failure modes and the impacts on the surrounding components and the system.
Failure Modes Effects, and Criticality Analysis (FMECA)
Identifies the components (equipment) failure modes and the impacts on the surrounding components and the system, and criticality of failures.
Fault Tree Analysis (FTA)
Identify combinations of equipment failures and human errors that can result in an accident.
Event Tree Analysis (ETA)
Identify various sequences of events, both failures and successes, that can lead to an accident.
Preliminary Hazard Analysis (PrHA)
Identify and prioritize hazards leading to undesirable consequences early in the life of a system. Determine recommended actions to reduce the frequency and/or consequences of prioritized hazards.
Consequence Assessment and Cause Consequence Diagrams
Assess consequences and scenarios leading to them.

Table 2a. Ranges of Key Parameters

Mean	Range
a/*b*	0.4, 0.6, 0.8, 2, 3, and 4
b/*t*	50, 100, and 150
t (inch)	0.25, 0.375, and 0.5

Table 2b. The probabilistic Characteristic of the basic random variables

Variable	Value	Nominal		Statistical Information		Bias or Error Information	
		Mean	COV	Dist. Type	Mean	Std. Dev.	Dist. Type
t (inch)					0	0.01563	Normal
b (inch)					0	0.125	Normal
a (inch)					0	0.125	Normal
F_{yp} (ksi)	34	35.7	0.07	Normal	1.05		
E (ksi)	29500	29500	0.05	Normal			

Table 3. Mean to Nominal Strength Ratio (F_u/F_{un}) using 100 Simulation Cycles

a/*b*	*t* (in)	*b*/*t*		
		50	100	150
2	0.250	1.0329	1.018011	1.024373
	0.375	1.038469	1.02323	1.026731
	0.500	1.041583	1.029762	1.027882
3	0.250	1.032268	1.024927	1.02176
	0.375	1.040157	1.025057	1.009896
	0.500	1.041212	1.021123	1.02741
4	0.250	1.043122	1.006146	1.030023
	0.375	1.035976	1.029633	1.020356
	0.500	1.031797	1.03514	1.021249
0.4	0.250	1.031613	1.036789	1.037933
	0.375	1.028689	1.032298	1.027118
	0.500	1.037029	1.031344	1.031872
0.6	0.250	1.029244	1.024514	1.028243
	0.375	1.03174	1.032877	1.032408
	0.500	1.040443	1.031721	1.034645
0.8	0.250	1.023216	1.012801	1.019138
	0.375	1.040286	1.011904	1.014104
	0.500	1.039777	1.034836	1.020961

Table 4. Coefficient of Variation of Strength (Fu) using 100 Simulation Cycles

a/b	*t* (in)	*b/t*		
		50	100	150
2	0.250	0.058425	0.079082	0.069403
	0.375	0.060794	0.051048	0.057236
	0.500	0.052735	0.047537	0.055338
3	0.250	0.057636	0.07937	0.069333
	0.375	0.054287	0.053333	0.058584
	0.500	0.048914	0.05461	0.051153
4	0.250	0.066812	0.076344	0.070726
	0.375	0.060021	0.047904	0.059547
	0.500	0.055633	0.050637	0.054919
0.4	0.250	0.070527	0.074448	0.070684
	0.375	0.05726	0.058802	0.053054
	0.500	0.052342	0.053527	0.056163
0.6	0.250	0.057405	0.050443	0.048501
	0.375	0.055282	0.055728	0.061751
	0.500	0.054886	0.057613	0.04678
0.8	0.250	0.062148	0.070153	0.071715
	0.375	0.059722	0.051749	0.058896
	0.500	0.052693	0.046299	0.059177

Table 5. Ratios of Means for Strength/Stillwater Load

COV(F_u)	Ratios of Means for Wave/Stillwater Load		
	1.5	1.6	1.7
0.04	3.43035	3.5695	3.70977
0.08	3.6375	3.7817	3.9271

Table 6. Partial Safety Factors (for COV(Fu) of 0.04 and 0.08, respectively)

Partial Safety Factors	Ratios of Means for Wave/Stillwater Load		
	1.5	1.6	1.7
Strength Reduction Factor (ϕ)	0.960338 0.863684	0.961079 0.86526	0.961747 0.86679
Stillwater Load Factor (γ_S)	1.301221 1.28566	1.283616 1.270806	1.267817 1.257081
Wave Load Factor (γ_w)	1.328696 1.237262	1.341832 1.250783	1.352955 1.262827

Table 7. Probabilistic Characteristics of Random Variables

Random Variable	Mean	COV	Distribution Type
F_u	not provided	0.06	Lognormal
f_S	1	0.2	Normal
f_W	1.6	0.1	Type I (Largest)