Abstract

Marine and offshore structures are subjected to fatigue primarily due to the action of seawater waves and the sea environment in general. The load cycles in such an environment can be in the order of million cycles per year. The objective of this paper is to develop design methods for fatigue of structural details for conventional displacement type surface monohull ships. The methods are based on structural reliability theory and can be either as direct reliability-based design or in a load and resistance factor design (LRFD) format. The resulting design methods are to be referred to as the LRFD fatigue rules for marine structures. They were developed according to the following requirements: (1) spectral analysis of wave loads, (2) building on conventional codes, (3) nominal strength and load values, and (4) achieving target reliability levels. The first-order reliability method (FORM) was used to demonstrate the development of partial safety factors for a selected limit state.

Introduction

In recent years, a great deal of attention has been focused on general fatigue cracking of ship structural details because the phenomenon is so vital that marine engineers must consider fatigue strength in their designs, especially for those structural components that are exposed to cyclic loading.

Fatigue cracking of structural details in ship and offshore steel structures due cyclic loading has gained considerable attention in the past few years. Numerous research studies have been conducted in this field on both the theoretical and practical aspects. Consequently, a great deal of papers has been published resulting in various topics relating to fatigue assessment and prediction. In these papers, the macroscopic behavior of materials as well as models for its description is investigated. Due to the extreme complexity in modeling the process of material cracking at the microscopic level, solutions from the microscopic aspect are rarely available or not practically feasible. This is mainly due to the complexity of the damaging process under cyclic loading and the scatter of material properties. Ship and offshore structures are subjected to fatigue primarily due to the action of seawater waves (Byers et al, 1997) and the sea environment in general. The load cycles in such an environment can be in the order of million cycles per year. Fatigue failures in ship and offshore structures can take place at sites of high stress concentration that can be classified into two major categories: (1) baseplate and (2) weldments. The former includes locations of high stress concentration such as openings, sharp re-entry corners, and plate edges. In general, the mechanisms behind these failures are described by the general approaches to fatigue life prediction as discussed in this paper.
Fatigue Analyses and Design Approaches

There are two major technical approaches for fatigue analysis and design of welded joints: (1) the fracture mechanics approach and (2) the characteristic $S$-$N$ approach. Both of these approaches are discussed briefly in the subsequent sections with the emphases on the latter approach.

The Fracture Mechanics Approach

The fracture mechanics (FM) approach is based on crack growth data. For welded joints it is assumed that the initiation phase is negligible and that life can be predicted using the fracture mechanics method. The fracture mechanics approach is more detailed and it involves examining crack growth and determining the number of load cycles that are needed for small initial defects to grow into cracks large enough to cause fracture. The growth rate is proportional to the stress range. It is expressed in terms of a stress intensity factor $K$, which accounts for the magnitude of the stress, current crack size, and weld and joint details.

The Characteristic $S$-$N$ Approach

The Characteristic $S$-$N$ approach is based on fatigue test data ($S$-$N$ curves) and on the assumption that fatigue damage accumulation is a linear phenomenon (Miner’s rule). According to Miner’s rule, the total fatigue life under a variety of stress ranges is the weighted sum of the individual lives at constant stress $S$ as given by the $S$-$N$ curves, with each being weighted according to fractional exposure to that level of stress range (Hughes 1988). Upon crack initiation, cracks propagate based on the fracture mechanics (FM) concept as shown in Figure 1.

The fatigue behavior of different types of structural details is generally evaluated in constant-cycle fatigue tests and the results are presented in terms of the nominal applied stresses and the number of cycles of loading that produce failure. The resulting $S$-$N$ curves are usually presented as straight lines on a log-log paper as shown in Figure 2. The basic equation that represents the $S$-$N$ curve is given by

$$N = \frac{A}{S^m}$$  \hspace{1cm} (1)

where $N$ = number of cycles to fatigue initiation (failure), $A$ = the intercept of the $S$-$N$ curve at $S$ equals to one, $S$ =constant amplitude stress range at $N$, and $m$ = slope of the $S$-$N$ curve. Eq. 1 can also be expressed as

$$\log N = \log A - m \log S$$  \hspace{1cm} (2)

where log is to the base 10. The fatigue strength can be computed over a range of lives covered by the straight line if the slope of the line and one point on the line are known.
However, only one type of stress cycle and one detail are represented on an individual S-N curve (Munse et al. 1983). In general, a least-squares analysis of log \( N \) given \( S \) is used to produce the S-N curve.

### Uncertainty Analysis

Uncertainty in fatigue strength is evidenced by the large scatter in fatigue S-N data. The scatter of the data about the mean fatigue line is not the only uncertainty involved in the S-N analysis (White and Ayyub, 1987). For this reason, a measure of the total uncertainty in the form of a coefficient of variation (COV) in fatigue life is usually developed to include the uncertainty in data, errors in fatigue model, and any uncertainty in the individual stresses and stress effects.

### Reliability-based Design Methods

**Direct-Reliability-Based Design**

A direct reliability-based design requires performing spectral analysis for the loads. The spectral analysis can be used to develop lifetime fatigue loads spectra by considering the operational conditions and the characteristics of a ship in the sea. The operational conditions are divided into different operation modes according to the combinations of ship speeds, ship headings, and wave heights. The ship characteristics include the length between perpendicular (LBP), beam (B), and the bow form as shown in Figure 3. With the proper identification of the hull girder section modulus (Z), the bending moment histograms (moment range versus number of cycles) can be converted to mean stress range spectra to compute the equivalent stress range \( \bar{S}_e \) according to the following equation:

\[
\bar{S}_e = m \sqrt[n_h]{\sum_{i=1}^{n_h} f_i S_i^m}
\]  

(3)
where \( S_e \) = Miner’s mean equivalent stress range, \( S_i \) = stress in the \( i^{th} \) block, \( f_i \) = fraction of cycles in the \( i^{th} \) stress block, \( m \) = slope of \( S-N \) curve, \( n_b \) = number of stress blocks in a stress (loading) histogram.

**The Load and Resistance Factor Design (LRFD)**

The load and resistance factor (LRFD) approach consists of the requirement that a factored (reduced) strength of a structural component is larger than a linear combination of factored (magnified) load effects. In this approach, load effects are increased, and strength is reduced, by multiplying the corresponding characteristic (nominal) values with factors, which are called strength (resistance) and load factors, respectively, or partial safety factors (PSF’s). The characteristic value of some quantity is the value that is used in current design practice, and it is usually equal to a certain percentile of the probability distribution of that quantity. The load and strength factors are different for each type of load and strength. The higher the uncertainty associated with a load, the higher the corresponding load factor. These factors are determined probabilistically so that they correspond to a prescribed level of safety. Designers can use the load and resistance factors in limit-state equations to account for uncertainties that might not be considered properly by deterministic methods without explicitly performing probabilistic analysis.

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**Figure 3. Direct Reliability-based Design and Analysis for Fatigue (Assakkaf, 1998)**

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Calculation of partial safety factors (PSF’s) for fatigue variables in the limit state function can be accomplished using the first-order reliability methods (FORM). The partial safety factors are defined as the ratio of the value of a variable in a limit state at its most probable failure point (MPFP). Reliability-based design formats for fatigue can be expressed in the following form:

\[
g(\Delta, A, k_s, S_e, N_t) = \frac{\Delta A}{k_s^m S_e^m} - N_t
\]  

where \(S_e\) as given by Eq. 3, \(\Delta = \) fatigue damage ratio, \(A = \) intercept of the \(S-N\) curve, \(m = \) slope of the \(S-N\) curve, \(S_e = \) Miner’s mean equivalent stress, \(k_s = \) fatigue stress uncertainty factor, \(N_t = \) number of loading cycles expected during the life of a structural detail, \(n_b = \) number of stress blocks in a stress (loading) histogram, \(f_i = \) fraction of cycles in the \(i^{th}\) stress block, and \(S_i = \) stress in the \(i^{th}\) block. By equating the reliability index, \(\beta\), with the target reliability index, \(\beta_o\), the partial safety factors are computed. The strength variables in the limit-state at the design point (MPFP) is given by

\[
S_e^* = \left[ \frac{\Delta^* A^*}{k_s^m N_t} \right]^{1/m}
\]  

By treating \(S_e, \Delta, A,\) and \(k_s\) as random variables, the partial safety factors are computed based on probabilistic methods. The variable \(N_t\) was treated as a deterministic quantity. However, it can be treated as a random variable, and its partial safety factor can be evaluated accordingly. The uncertainty in \(A\) can be attributed to the regression standard error.

**Example: Partial Safety Factors for Fatigue**

In this example, partial safety factors calculation for one class of structural detail is illustrated. The probabilistic characteristics of the random variables pertaining to this detail are shown in Table 1. The first-order reliability method (FORM) was used to develop the partial safety factors. The following performance function is used as defined by Eq. 4:

\[
g = \frac{\Delta A}{k_s^m S_e^m} - N_t
\]  

where \(A, S_e, \Delta,\) and \(k_s\) are random variables, \(m = \)slope of \(S-N\) curve (deterministic), and \(N_t = 10^5\). The partial safety factors are defined as the ratio of the value of a variable in the performance function at its most probable failure point (MPFP) to the nominal value. Summary of the partial safety factors for detail B of the British standards are shown in Table 2.
Table 1. Statistics of Random Variables for Category B of the British Standards (BS 5400, 1980)

<table>
<thead>
<tr>
<th>Random Variable</th>
<th>Mean</th>
<th>COV</th>
<th>Distribution Type</th>
</tr>
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<tr>
<td>$S_e$</td>
<td>27.54 ksi</td>
<td>0.1</td>
<td>Lognormal</td>
</tr>
<tr>
<td>$\Delta$</td>
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<td>0.48</td>
<td>Lognormal</td>
</tr>
<tr>
<td>$A$</td>
<td>4.47E11</td>
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<td>Lognormal</td>
</tr>
<tr>
<td>$k_s$</td>
<td>1.0</td>
<td>0.1</td>
<td>Normal</td>
</tr>
<tr>
<td>$m$</td>
<td>4.0</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>$N_t$</td>
<td>$10^5$</td>
<td>n/a</td>
<td>n/a</td>
</tr>
</tbody>
</table>

Table 2. Partial Safety Factors for Category B of the British Standards (BS 5400, 1980)

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\phi_A$</th>
<th>$\phi_A$</th>
<th>$\gamma_s$</th>
<th>$\gamma_s$</th>
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</thead>
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<td>0.60</td>
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<tr>
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<td>0.53</td>
<td>1.11</td>
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</tr>
<tr>
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<td>0.48</td>
<td>1.13</td>
<td>1.15</td>
</tr>
<tr>
<td>3.5</td>
<td>0.37</td>
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<td>1.18</td>
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<td>1.21</td>
</tr>
</tbody>
</table>

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References

Assakkaf, I.A., (1998), "Reliability-based Design of Panels and Fatigue Details of Ship Structures," A dissertation submitted to the Faculty of the Graduate School of the University of Maryland, College Park in partial fulfillment of the requirements for the degree of Doctor of Philosophy.


