LECTURE

8

Chapter 3.7 – 3.8 3.13

The McGraw-Hill Companies Third Edition

SHAFTS: POWER, STRESS CONCENTRATION, THIN-WALLED

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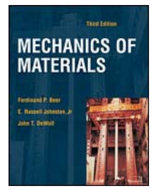

by

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ENES 220 – Mechanics of Materials

Department of Civil and Environmental Engineering

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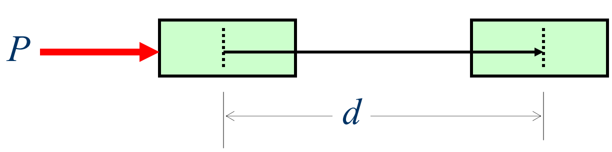
Power Transmission

- Work of a Force
  – A force does work only when the particle to which the force is applied moves.

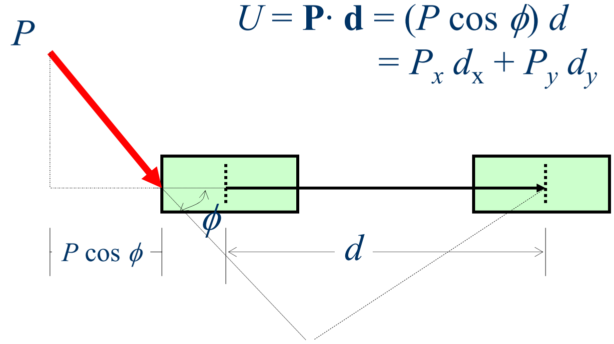
$$\text{Work} = U = Pd \qquad (1)$$



Power Transmission

- Work in Two and Three Dimension

$$U = \mathbf{P} \cdot \mathbf{d} = (P \cos \phi)\, d$$
$$= P_x d_x + P_y d_y \qquad (2)$$



Power Transmission

- Work of a Couple
  - The work of a couple is defined as the magnitude of the couple **C** times the angular movement of the body.

$$U_{1\rightarrow 2} = C\Delta\theta \qquad (3)$$

$$dU = \vec{C} \cdot d\vec{\theta}$$

Power Transmission

- Power Transmission by Torsional Shaft

  The power is defined as the time rate of doing work, that is

$$\frac{dU}{dt} = C \cdot \frac{d\theta}{t} = T\frac{d\theta}{dt} = T\omega \qquad (4)$$

ω = angular velocity of the shaft in radians per minute

Power Transmission

- Power Transmission by Torsional Shaft
  - But $\omega = 2\pi f$, where f = frequency. The unit of frequency is 1/s and is called *hertz* (Hz).
  - If this is the case, then the power is given by

$$P = 2\pi f T$$

or

$$T = \frac{P}{2\pi f} \qquad (5)$$

Power Transmission

- Power Transmission by Torsional Shaft
  - Units of Power

SI	US Customary
watt (1 N·m/s)	hp (33,000 ft·lb/min)

Power Transmission

- Power Transmission by Torsional Shaft
  - Some useful relations

$$1\ \text{rpm} = \frac{1}{60} s^{-1} = \frac{1}{60}\ \text{Hz}$$

$$1\ \text{hp} = 550\ \text{ft} \cdot \text{lb/s} = 6600\ \text{in} \cdot \text{lb/s}$$

rpm = revolution per minute

Power Transmission

- Example 5

  What size of shaft should be used for a rotor of 5-hp motor operating at 3600 rpm if the shearing stress is not to exceed 8500 psi in the shaft?

$$1\text{ hp} = 6600\text{ in}$$
$$5\text{ hp} = P$$
$$\longrightarrow \quad P = 5(6600) = 33{,}000\text{ in}\cdot\text{lb/s}$$

Power Transmission

- Example 5 (cont'd)

$$f = (3600\text{ rpm})\frac{1\text{ Hz}}{60\text{ rpm}} = 60\text{ Hz} = 60/\text{s}$$

$$T = \frac{P}{2\pi f} = \frac{33{,}000}{2\pi(60)} = 87.54\text{ lb}\cdot\text{in}$$

Let *J* denotes the polar moment of area, and *c* the maximum radius, therefore,

$$\tau = \frac{Tc}{J} \Rightarrow \frac{J}{c} = \frac{T}{\tau} \qquad (6)$$

Power Transmission

- Example 5 (cont'd)
  - Evaluating the term *J/c* in Eq. 6, yields

$$\frac{J}{c} = \frac{\frac{1}{2}\pi c^4}{c} = \frac{1}{2}\pi^2 c^3$$

  - Therefore,

$$\frac{1}{2}\pi^2 c^3 = \frac{J}{c} = \frac{T}{\tau} = \frac{87.54}{8500} = 0.001030\ \text{in}^3$$

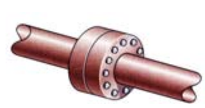
$$\Rightarrow c = 0.1872\ \text{in} \Rightarrow \text{Shaft size (dia)} = 2c = \underline{0.375\ \text{in}}$$

Stress Concentrations in Circular Shafts

- The derivation of the torsion formula,

$$\tau_{\max} = \frac{Tc}{J} \qquad (7)$$

assumed a circular shaft with uniform cross-section loaded through rigid end plates.

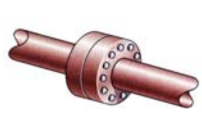


Stress Concentrations in Circular Shafts

- The use of flange couplings, gears and pulleys attached to shafts by keys in keyways, and cross-section discontinuities can cause stress concentrations.
- Experimental or numerically determined concentration factors are applied as

$$\tau_{\max} = K\frac{Tc}{J} \qquad (8)$$

Stress Concentrations in Circular Shafts



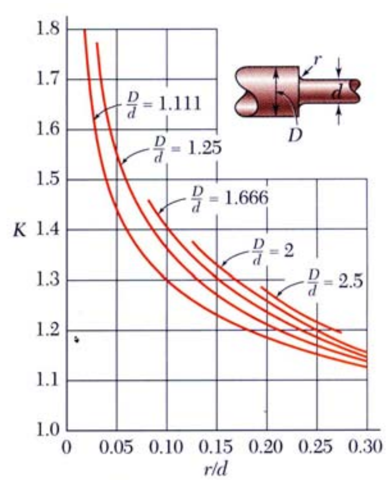


Fig. 3.32 Stress-concentration factors for fillets in circular shafts.†

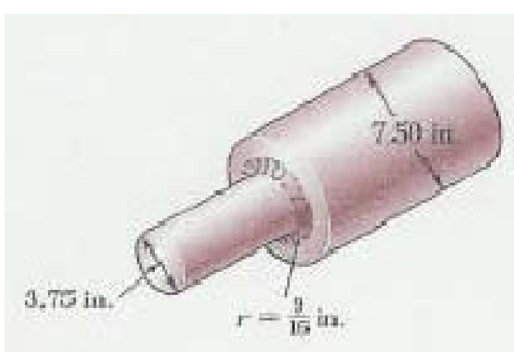
Stress Concentrations in Circular Shafts

- Example 6

The stepped shaft shown is to rotate at 900 rpm as it transmits power from a turbine to a generator. The grade of steel specified in the design has an allowable shearing stress of 8 ksi. (a) For preliminary design shown, determine the maximum power that can be transmitted. (b) If in the final design the radius of the fillet is increased so that r = 15/16 in., what will be the percent change , relative to the preliminary design in the power?

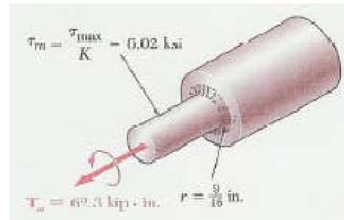
Stress Concentrations in Circular Shafts

- Example 6 (cont'd)



Stress Concentrations in Circular Shafts

- Example 6 (cont'd)
  - (a) Preliminary Design:
    Using Fig. 3.32, and
    knowing that the following are given:
    $D = 7.50$ in., $d = 3.75$ in., $r = 9/16$ in. = 0.5625 in.
    Therefore,



$$\frac{D}{d} = \frac{7.50}{3.75} = 2 \quad \text{and} \quad \frac{r}{d} = \frac{0.5625}{3.75} = 0.15$$

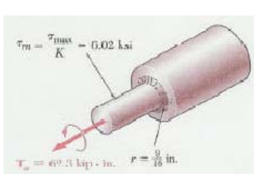
- Hence, from Fig. 3.32

$$K = 1.33$$

Stress Concentrations in Circular Shafts

- Example 6 (cont'd)

$$K = 1.33$$



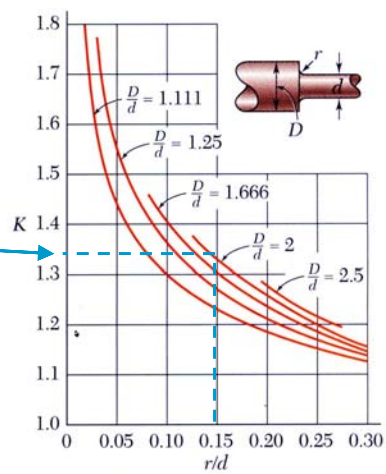
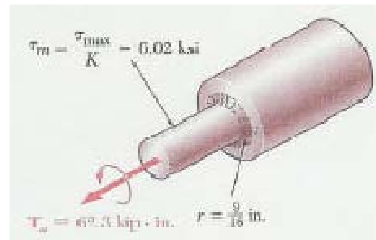


Fig. 3.32 Stress-concentration factors for fillets in circular shafts.†

Stress Concentrations in Circular Shafts

- Example 6 (cont'd)

  Using Eq. 40,



$$\tau_{max} = K\frac{Tc}{J} \quad \text{or} \quad T = \frac{J}{c}\frac{\tau_{max}}{K}$$

$$J = \frac{1}{2}\pi c^4 = \frac{1}{2}\pi\left(\frac{3.75}{2}\right)^4 = 19.414 \text{ in}^4$$

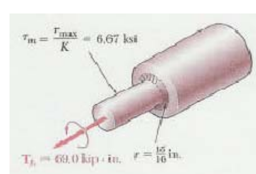
$$\therefore T = \frac{J}{c}\frac{\tau_{max}}{K} = \frac{19.414(8)}{(3.75/2)1.33} = 62.3 \text{ kip-in}$$

From Eq. 37, the power is given by

$$P = 2\pi f T = 2\pi\left(\frac{900}{60}\right)(62.3) = 5{,}869.9 \text{ in-kip/s} = 5.87\times10^6 \text{ in-lb/s}$$

$$P_a = \frac{5.87\times10^6}{6600} = 889.3 \text{ hp}$$

Stress Concentrations in Circular Shafts



- Example 6 (cont'd)
  - Final Design:

    Using Fig. 3.32, and knowing that the following are given:

    $D = 7.50$ in., $d = 3.75$ in., $r = 15/16$ in. $= 0.9375$ in.

    Therefore,

$$\frac{D}{d} = \frac{7.50}{3.75} = 2 \qquad \text{and} \qquad \frac{r}{d} = \frac{0.9375}{3.75} = 0.25$$

- Hence, from Fig. 3.32

$$K = 1.20$$

Stress Concentrations in Circular Shafts

- Example 6 (cont'd)

$K = 1.20$

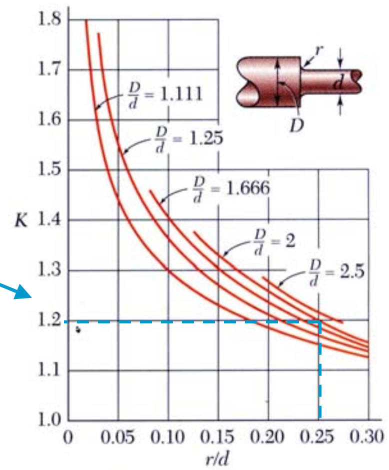
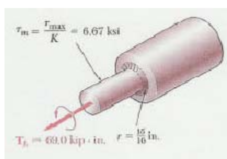


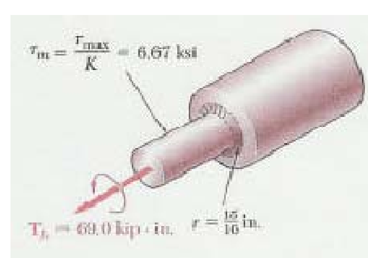
Fig. 3.32 Stress-concentration factors for fillets in circular shafts.†



Stress Concentrations in Circular Shafts

- Example 6 (cont'd)

Using Eq. 40,



$$\tau_{max} = K\frac{Tc}{J} \quad \text{or} \quad T = \frac{J}{c}\frac{\tau_{max}}{K}$$

$$J = \frac{1}{2}\pi c^4 = \frac{1}{2}\pi\left(\frac{3.75}{2}\right)^4 = 19.414 \text{ in}^4$$

$$\therefore T = \frac{J}{c}\frac{\tau_{max}}{K} = \frac{19.414(8)}{(3.75/2)1.20} = 69 \text{ kip-in}$$

From Eq. 37, the power is given by

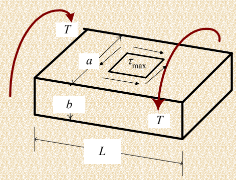
$$P = 2\pi f T = 2\pi\left(\frac{900}{60}\right)(69.0) = 6{,}503.1 \text{ in-kip/s} = 6.50\times10^6 \text{ in-lb/s}$$

$$P_b = \frac{6.50\times10^6}{6600} = 985 \text{ hp} \qquad \text{Change in } P = 100\left|\frac{985 - 889.3}{889.3}\right| = 11\%$$

Torsion of Noncircular Members

- Bar of Rectangular Cross Section

The maximum shearing stress and the angle of twist for a uniform bar of rectangular cross section, and subjected to pure torsion *T* are given by



$$\tau_{\max} = \frac{T}{k_1 a b^2} \qquad (9)$$

$$\phi = \frac{TL}{k_2 a b^3 G} \qquad (10)$$

The coefficients k_1 and k_2 can be obtained from Table 1.

Torsion of Noncircular Members

- Table 1. Coefficients for Rectangular Bars in Torsion

Beer and Johnston, 2002

a/b	k_1	k_2
1.0	0.208	0.1406
1.2	0.219	0.1661
1.5	0.231	0.1958
2.0	0.246	0.229
2.5	0.258	0.249
3.0	0.267	0.263
4.0	0.282	0.281
5.0	0.291	0.291
10.0	0.312	0.312
∞	0.333	0.333

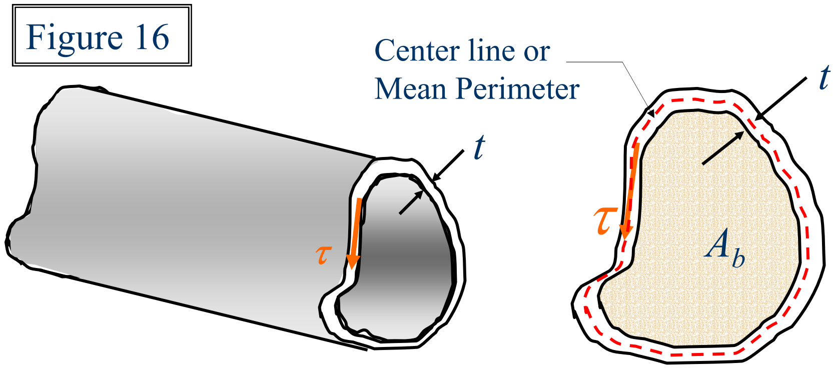
Torsion of Noncircular Members

- Thin-Walled Hollow Shafts
  - It was indicated earlier that the determination of the stresses in noncircular members generally requires the use of advanced mathematical methods.
  - In the case of thin-walled hollow noncircular shaft (Fig. 16), however, a good approximation of the distribution of stresses can be obtained.

Torsion of Noncircular Members

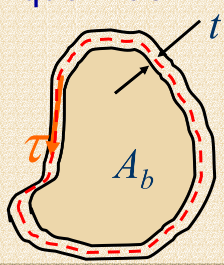
- Thin-Walled Hollow Shafts

Figure 16



Torsion of Noncircular Members

- Thin-Walled Hollow Shafts
  - The shearing stress τ at any given point of the wall may be expressed in terms of the torque T as



$$\tau = \frac{T}{2tA_b} \qquad (11)$$

A_b = area bounded by center line

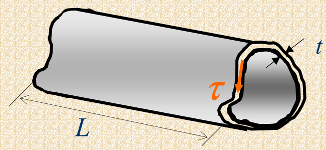
Torsion of Noncircular Members

- Thin-Walled Hollow Shafts
  - The shearing stress τ of Eq. 11 represents the average value of the shearing stress across the wall.
  - However, for elastic deformations the distribution of the stress across the wall may be assumed uniform, and Eq. 11 will give the actual value of the shearing stress at a given point of the wall.

Torsion of Noncircular Members

- Thin-Walled Hollow Shafts

  The angle of twist of a thin-walled shaft of length *L* and modulus of rigidity *G* is given by



$$\phi = \frac{TL}{4A_b^2 G}\oint \frac{ds}{t} \qquad (12)$$

  Where the integral is computed along the center line of the wall section.

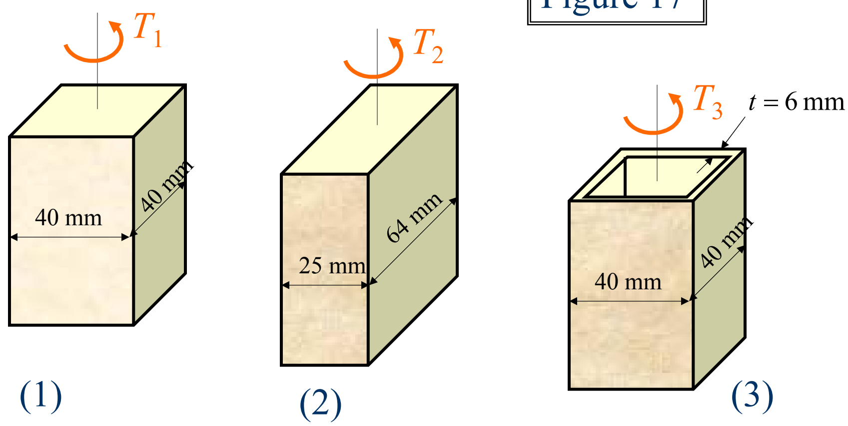
Torsion of Noncircular Members

- Example 7

  Using τ_{all} = 40 MPa, determine the largest torque which may be applied to each of the brass bars and to the brass tube shown. Note that the two solid bars have the same cross-sectional area, and that the square bar and square tube have the same outside dimensions

Torsion of Noncircular Members

Figure 17



Torsion of Noncircular Members

- Example 7 (cont'd)
  1. Bar with Square Cross Section:

     For a solid bar of rectangular cross section, the maximum shearing stress is given by Eq. 9:

$$\tau_{\max} = \frac{T}{k_1 a b^2}$$

where the coefficient k_1 is obtained from Table 1, therefore

$$a = b = 0.040 \text{ m} \qquad \frac{a}{b} = 1.00 \qquad k_1 = 0.208$$

Torsion of Noncircular Members

- Example 7 (cont'd)

  For $\tau_{max} = \tau_{all}$ = 40 MPa, we have

$$\tau_{max} = \frac{T_1}{k_1 ab^2} \quad 40 = \frac{T_1}{0.208(0.04)(0.04)^2} \Rightarrow \boxed{T_1 = 532 \text{ N} \cdot \text{m}}$$

2. Bar with Rectangular Cross Section:

$$a = 0.064 \text{ m} \qquad b = 0.025 \text{ m} \qquad \frac{a}{b} = \frac{0.064}{0.025} = 2.56$$

By interpolation, Table 1 gives: $k_1 = 0.259$

Torsion of Noncircular Members

- Example 7 (cont'd)

$$\tau_{max} = \frac{T_2}{k_1 ab^2} \quad 40 = \frac{T_2}{0.259(0.064)(0,025)^2} \Rightarrow \boxed{T_2 = 414 \text{ N} \cdot \text{m}}$$

3. Square Tube:

   For a tube of thickness t, the shearing stress is given by Eq. 11 as

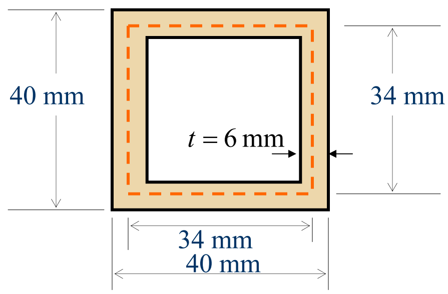
$$\tau = \frac{T}{2tA_b}$$

Torsion of Noncircular Members

- Example 7 (cont'd)

  where A_b is the area bounded by the center line of the cross section, therefore,

$$A_b = (0.034)(0.034) = 1.156 \times 10^{-3}\,\text{m}^2$$



Torsion of Noncircular Members

- Example 7 (cont'd)

  $\tau = \tau_{all} = 40$ MPa and $t = 0.006$ m.

  Substituting these value into Eq. 11 gives

$$\tau = \frac{T}{2tA_b}$$

$$40 = \frac{T_3}{2(0.006)(1.156 \times 10^{-3})}$$

$$\therefore T_3 = 555\ \text{N} \cdot \text{m}$$

Torsion of Noncircular Members

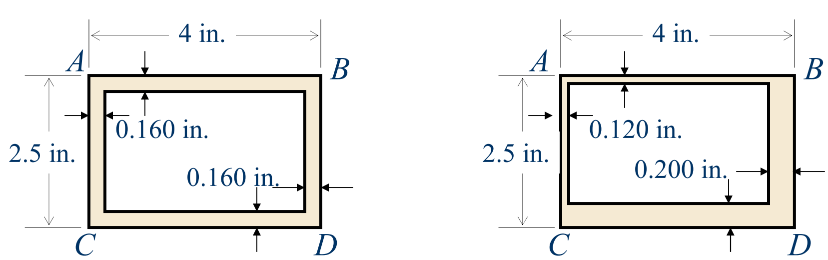
- Example 8

Structural aluminum tubing of 2.5 × 4-in. rectangular cross section was fabricated by extrusion. Determine the shearing stress in each of the four walls of a portion of such tubing when it is subjected to a torque of 24 kip·in., assuming (a) a uniform 0.160-in. wall thickness (Figure 18a), (b) that, as a result of defective fabrication, walls *AB*

Torsion of Noncircular Members

- Example 8 (cont'd)

and *AC* are 0.120-in thick, and walls *BD* and *CD* are 0.200-in thick (Fig. 18b)



(a) Figure 18 (b)

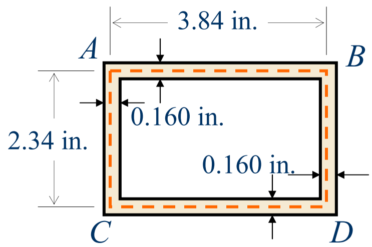
Torsion of Noncircular Members

- Example 8 (cont'd)

  (a) Tubing of Uniform Wall Thickness:

  The area bounded by the center line (Fig. 19) is given by

Figure 19



$$A_b = (3.84)(2.34) = 8.986\ \text{in}^2$$

Torsion of Noncircular Members

- Example 8 (cont'd)

  Since the thickness of each of the four walls is t = 0.160 in., we find from Eq. 11 that the shearing stress in each wall is

$$\tau = \frac{T}{2tA_b} = \frac{24}{2(0.160)(8.986)} = \boxed{8.35\ \text{ksi}}$$

  (b) Tubing with Variable Wall Thickness:

  Observing that the area A_b bounded by the center line is the same as in Part *a*, and substituting t = 0.120 in. and t = 0.200 in. into

Torsion of Noncircular Members

- Example 8 (cont'd)
  - Eq. 11, the following values for the shearing stresses are obtained:

$$\tau_{AB} = \tau_{AC} = \frac{24}{2(0.120)(8.986)} = \boxed{11.13 \text{ ksi}}$$

and

$$\tau_{BD} = \tau_{CD} = \frac{24}{2(0.200)(8.986)} = \boxed{6.68 \text{ ksi}}$$

  - Note that the stress in a given wall depends only upon its thickness *t*.