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LECTURE

5

Chapter
2.10
2.18

RODS: THERMAL STRESS AND STRESS CONCENTRATION

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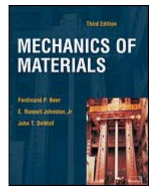

by
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ENES 220 – Mechanics of Materials

Department of Civil and Environmental Engineering
University of Maryland, College Park

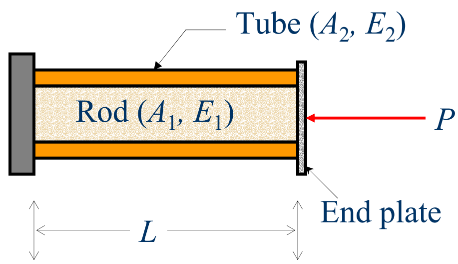
Statically Indeterminate Axially Loaded Members

- Example 5

  A rod of length L, cross-sectional area A1, and modulus of elasticity E1, has been placed inside a tube of the same length L, but of cross-sectional area A2 and modulus of elasticity E2. What is the deformation of the rod and tube when a force P is exerted on a rigid end plate as shown? What are the internal forces in the rod and the tube?

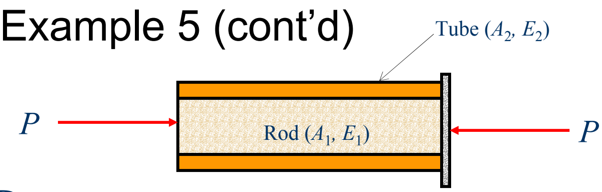
Statically Indeterminate Axially Loaded Members

- Example 5 (cont'd)



Statically Indeterminate Axially Loaded Members

- Example 5 (cont'd)



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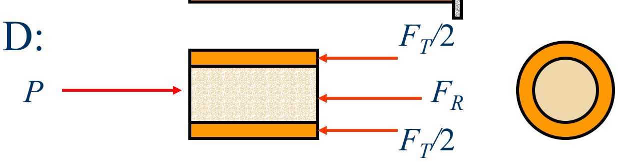


$$\rightarrow + \sum F_x = 0; \; P - \frac{F_T}{2} - \frac{F_T}{2} - F_R = 0$$

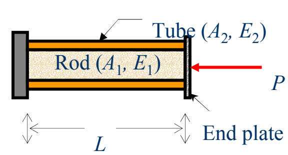
$$F_R + F_T = P \qquad (9)$$

Statically Indeterminate Axially Loaded Members

- Example 5 (cont'd)
  - Clearly on equation is not sufficient to determine the two unknown internal forces FR and FT. The problem is statically indeterminate.
  - However, the geometry of the problem shows that the deformations δR and δT of the rod and tube must be equal, that is

Statically Indeterminate Axially Loaded Members

- Example 5 (cont'd)



$$\delta_R = \delta_T$$

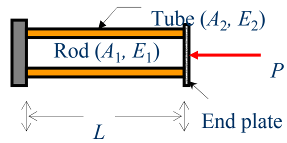
$$\frac{F_R L}{A_1 E_1} = \frac{F_T L}{A_2 E_2}$$

$$\frac{F_R}{A_1 E_1} = \frac{F_T}{A_2 E_2}$$

$$F_T = \frac{F_R A_2 E_2}{A_1 E_1} \qquad (10)$$

Statically Indeterminate Axially Loaded Members

- Example 5 (cont'd)
  - Substituting Eq. 10 into Eq. 9, therefore



$$F_R + F_T = P$$

$$F_R + \frac{F_R A_2 E_2}{A_1 E_1} = P$$

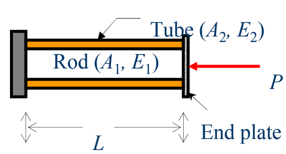
$$F_R\left(1 + \frac{A_2 E_2}{A_1 E_1}\right) = P \Rightarrow F_R\left(\frac{A_1 E_1 + A_2 E_2}{A_1 E_1}\right) = P$$

Statically Indeterminate Axially Loaded Members

- Example 5 (cont'd)

  Substituting Eq. 10 into Eq. 9, therefore

  Or



$$F_R = \frac{P A_1 E_1}{A_1 E_1 + A_2 E_2} \qquad \textbf{Ans.}$$

and from Eq. 9,

$$F_T = P - F_R = P - \frac{P A_1 E_1}{A_1 E_1 + A_2 E_2} = \frac{P A_2 E_2}{A_1 E_1 + A_2 E_2} \qquad \textbf{Ans.}$$

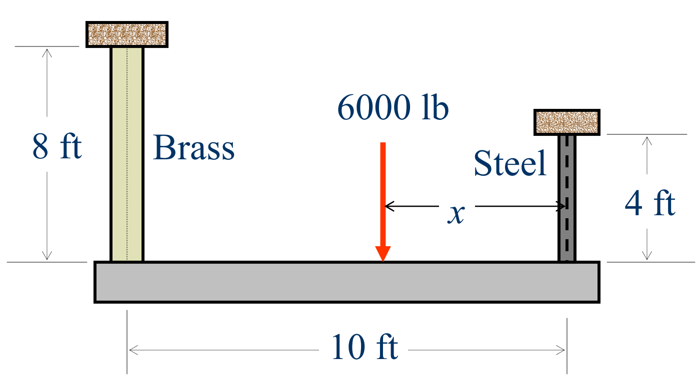
Statically Indeterminate Axially Loaded Members

- Example 6

  A very stiff bar of negligible weight is suspended horizontally by two vertical rods as shown. One of the rods is of steel, and is ½-in in diameter and 4 ft long; the other is of brass and is 7/8-in in diameter and 8 ft long. If a vertical load of 6000 lb is applied to the bar, where must be placed in order that the bar will remain horizontal? Take $E_s = 30\times10^6$ psi and $E_b = 14\times10^6$.

Statically Indeterminate Axially Loaded Members

- Example 6 (cont'd)
  - Also find the stresses in the brass and steel rods.



Statically Indeterminate Axially Loaded Members

- Example 6 (cont'd)

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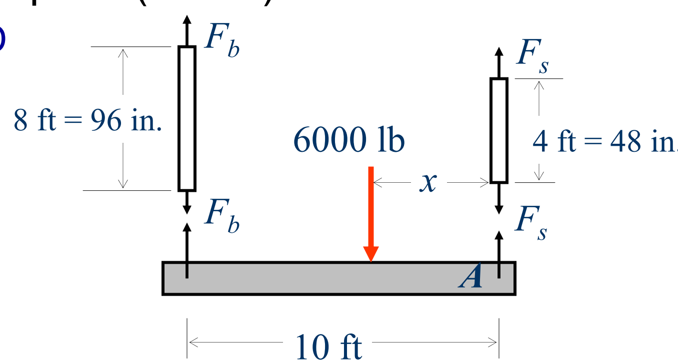
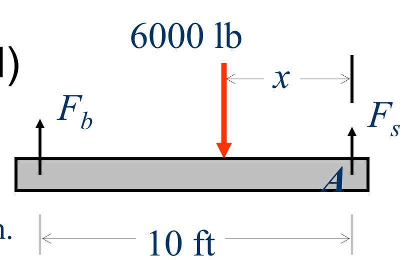


Statically Indeterminate Axially Loaded Members

- Example 6 (cont'd)



Two independent equations of static equilibrium may be written for the free-body diagram. The possible equations are

$$\uparrow + \sum F_y = 0;\ F_s + F_b - 6000 = 0 \qquad (11)$$

$$\circlearrowleft + \sum M_A = 0;\ F_b(10) - 6000(x) = 0 \qquad (12)$$

Statically Indeterminate Axially Loaded Members

- Example 6 (cont'd)
  - Since no more independent equations of equilibrium can be written and there are three unknown quantities, the structure is statically indeterminate.
  - One additional independent equation is needed. The problem requires that the bar remain ***horizontal***. Therefore, the rods must undergo equal elongations, that is

$$\delta_s = \delta_b \qquad (13)$$

Statically Indeterminate Axially Loaded Members

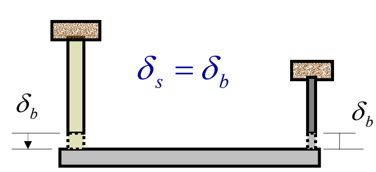
- Example 6 (cont'd)

$\delta_b = \delta_s$, but in general $\delta = \dfrac{\sigma L}{E}$

$$\frac{\sigma_b L}{E_b} = \frac{\sigma_s L}{E_s}$$

$$\frac{\sigma_b (96)}{14\times 10^6} = \frac{\sigma_s (48)}{30\times 10^6}$$

$$6.857143\times 10^{-6}\sigma_b - 1.6\times 10^{-6}\sigma_s = 0 \qquad (14)$$



But $\sigma_b = \dfrac{F_b}{A_b}$ and $\sigma_s = \dfrac{F_s}{A_s}$, therefore $\qquad$ (15)

Statically Indeterminate Axially Loaded Members

- Example 6 (cont'd)

Substituting Eq.15 into Eq.14, thus

$$6.857143\times10^{-6}\frac{F_b}{A_b}-1.6\times10^{-6}\frac{F_s}{A_s}=0 \qquad (16)$$

The areas of brass and steel bar are

$$A_b=\frac{\pi(7/8)^2}{4}=0.60132\text{ in}^2 \qquad (17)$$

$$A_s=\frac{\pi(1/2)^2}{4}=0.19635\text{ in}^2 \qquad (18)$$

Statically Indeterminate Axially Loaded Members

- Example 6 (cont'd)

Substituting Eqs. 17 and 18 into Eq. 16, gives

$$11.40348\times10^{-6}F_b-8.148714\times10^{-6}F_s=0 \qquad (19)$$

Recalling Eqs. 11 and 12,

$$F_b+Fs-6000=0$$

$$10F_b-6000x=0$$

Statically Indeterminate Axially Loaded Members

- Example 6 (cont'd)
  - The solution of the following system of simultaneous equations, gives F_b, F_s, and x:

$$\begin{aligned} F_b + F_s &= 6000 \\ 10F_b - 6000x &= 0 \end{aligned} \qquad (20)$$

$$11.40348\times10^{-6}F_b - 8.148714\times10^{-6}F_s = 0$$

Statically Indeterminate Axially Loaded Members

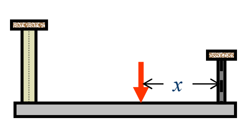
- Example 6 (cont'd)

From the system of Equation 20:

F_b = 2500 lb

F_s = 3500 lb

x = 4.167 ft



Hence, if the bar is to remain horizontal, the 6000-lb load should placed 4.167 ft from the steel rod as shown.

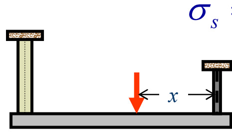
Statically Indeterminate Axially Loaded Members

- Example 6 (cont'd)

  The stresses in brass and steel rods can be calculated from Eq.15 as follows:

$$\sigma_b = \frac{F_b}{A_b} = \frac{2500}{0.60132} = 4{,}157.5 \text{ psi} = 4.16 \text{ ksi}$$

$$\sigma_s = \frac{F_s}{A_s} = \frac{3500}{0.19635} = 17{,}825.3 \text{ psi} = 17.83 \text{ ksi}$$



Statically Indeterminate Axially Loaded Members

- Thermal Stress
  - Temperature Strain
    - Most materials when unstrained expand when heated and contract when cooled.
    - The thermal strain due to one degree (10) change in temperature is given by α and is known is the coefficient of thermal expansion.
    - The thermal strain due a temperature change of ΔT degrees is given by

$$\varepsilon_T = \alpha \Delta T \qquad (21)$$

Statically Indeterminate Axially Loaded Members

- Thermal Stress
  - Total Strain
    - The sum of the normal strain caused by the loads and the thermal strain is called the total strain, and it is given by

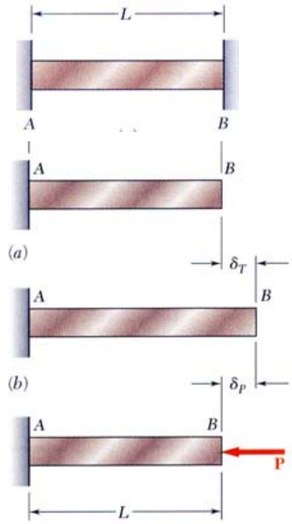
$$\varepsilon_{\text{total}} = \varepsilon_{\sigma} + \varepsilon_{T} = \frac{\sigma}{E} + \alpha\,\Delta T \qquad (22)$$

Statically Indeterminate Axially Loaded Members

- Thermal Stress
  - Definition

    "***Thermal stress is the stress that is induced in a structural member due a temperature change while the member is restrained (free movement restricted or prevented***)"

Thermal Stresses



- A temperature change results in a change in length or *thermal strain*. There is no stress associated with the thermal strain unless the elongation is restrained by the supports.
- Treat the additional support as redundant and apply the principle of superposition.

$$\delta_T = \alpha(\Delta T)L \qquad \delta_P = \frac{PL}{AE}$$

$\alpha =$ thermal expansion coef.

- The thermal deformation and the deformation from the redundant support must be compatible.

$$\delta = \delta_T + \delta_P = 0$$

$$\alpha(\Delta T)L + \frac{PL}{AE} = 0$$

$$\delta = \delta_T + \delta_P = 0$$

$$P = -AE\alpha(\Delta T)$$

$$\sigma = \frac{P}{A} = -E\alpha(\Delta T)$$

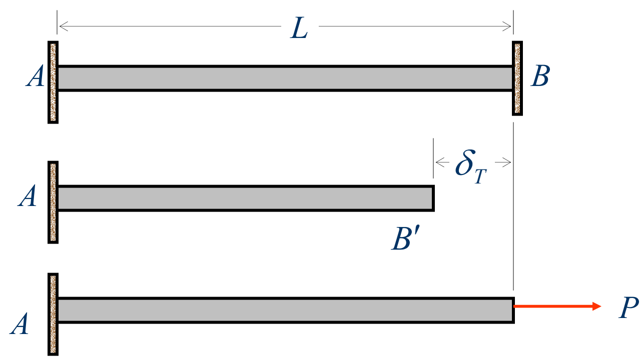
Statically Indeterminate Axially Loaded Members

- Thermal Stress
  - Example
    - The bar *AB* is securely fastened to rigid supports at the ends and is subjected to a temperature change.
    - Since the ends of the bar are fixed, the total deformation of the bar must be zero

$$\delta_{\text{total}} = \delta_T + \delta_\sigma = \varepsilon_T L + \varepsilon_\sigma L = \alpha \Delta T L + \frac{\sigma}{E} L \qquad (23)$$

Statically Indeterminate Axially Loaded Members

- Thermal Stress
  - Example (cont'd)



Statically Indeterminate Axially Loaded Members

- Thermal Stress
  - Example (cont'd)
    - If the temperature of the bar increases (ΔT positive), then the induced stress must be negative, and the wall must push on the ends of the rod.
    - If the temperature of the bar decreases (ΔT negative), then the induced stress must be positive, and the wall must pull on the ends of the rod.

Statically Indeterminate Axially Loaded Members

- Thermal Stress
  - Example (cont'd)
    - This means that if end *B* were not attached to the wall and the temperature drops, then end *B* would move to B', a distance

$$|\delta_T| = |\varepsilon_T L| = |\alpha \Delta T L| \qquad (24)$$

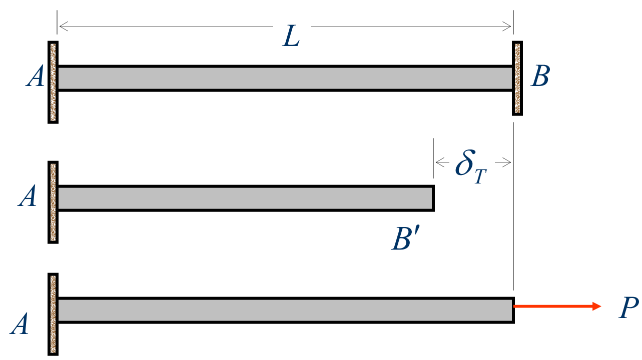
as shown in the figure

Statically Indeterminate Axially Loaded Members

- Thermal Stress
  - Example (cont'd)
    - Therefore, for total deformation of the bar to be zero, the wall at *B* must apply a force $P = \sigma A$ of sufficient magnitude to move end *B* through a distance $\delta_P = \varepsilon_\sigma L = (\sigma/E) L$ so that the length of the bar is a gain *L*, which is the distance between the walls.

Statically Indeterminate Axially Loaded Members

- Thermal Stress
  - Example (cont'd)



Statically Indeterminate Axially Loaded Members

- Thermal Stress
  - Since the walls do not move, then

$$|\delta_T| = |\delta_P|$$

Or

$$\delta_P - |\delta_T| = \delta_P + \delta_T = 0$$

and hence, the total deformation of the bar is zero.

Statically Indeterminate Axially Loaded Members

- Example 7

A 10-m section of steel (E = 200 Gpa and $\alpha = 11.9\times10^{-6}$ /^{0}C) rail has a cross-sectional area of 7500 mm^2. Both ends of the rail are tight against adjacent rails that can be assumed to be rigid. The rail is supported against lateral moment. For an increase in temperature of 50^{0}C, determine

a) The normal stress in the rail.
b) The internal force on the cross section.

Statically Indeterminate Axially Loaded Members

- Example 7 (cont'd)

(a) The change in length can be calculated as follows:

$$\delta = \varepsilon_T L = \alpha L \Delta T = 11.9\times10^{-6}(10)(50) = 5.95 \text{ mm}$$

The stress required to resist a change in length of 5.95 mm is computed as

$$\sigma = \frac{E\delta}{L} = \frac{200\times10^{9}(5.95\times10^{-3})}{10} = 119 \text{ MPa}$$

Statically Indeterminate Axially Loaded Members

- Example 7 (cont'd)

  (b) The internal force on a cross section of the rail is computed as follows:

$$F = \sigma A = 119 \times 10^6 (7500 \times 10^{-6}) = 892.5 \times 10^3 \text{ N}$$
$$= 893 \text{ kN}$$

Statically Indeterminate Axially Loaded Members

- General Notes on Thermal Stress and Thermal Deformation
  – The results that was discussed earlier apply only in the case of a _homogenous_ rod (or bar) of _uniform cross section_.
  – Any other problem involving a restrained member undergoing a change in temperature must be analyzed on its own merits.

Statically Indeterminate Axially Loaded Members

- General Notes on Thermal Stress and Thermal Deformation
  – However, the same general approach may be used, that is, we may consider separately the deformation due to temperature change and the deformation due to the redundant reaction and superimpose the solutions obtained.

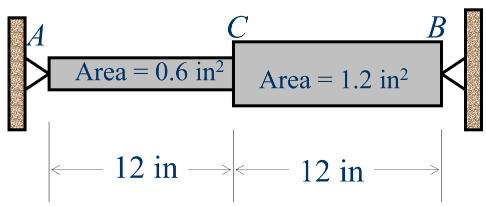
Statically Indeterminate Axially Loaded Members

- Example 8

  Determine the values of the stress in portion *AC* and *CB* of the steel bar shown when a the temperature of the bar is – 50^0F, knowing that a close fit exists at both of the rigid supports when the temperature is +70^0F. Use $E = 29\times10^6$ psi and $\alpha = 6.5 \times10^{-6}/^0$F for steel.

Statically Indeterminate Axially Loaded Members

- Example 8 (cont'd)



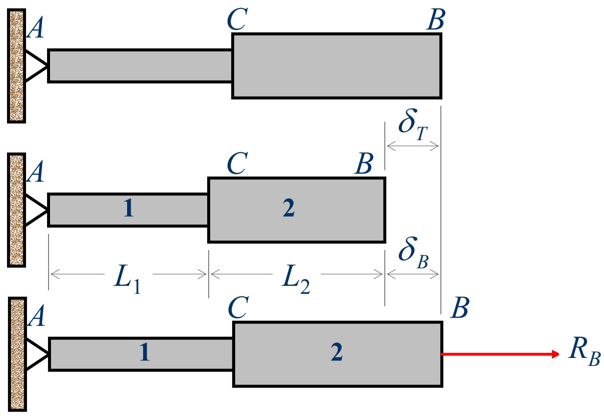
Statically Indeterminate Axially Loaded Members

- Example 8 (cont'd)
  - The reaction at the supports needs to be determined.
  - Sine the problem is statically indeterminate, the bar can be detached from its support at *B*, and let it undergo the temperature change

$$\Delta T = (-50) - (75) = -125^0\,\mathrm{F}$$

Statically Indeterminate Axially Loaded Members

- Example 8 (cont'd)



Statically Indeterminate Axially Loaded Members

- Example 8 (cont'd)

  The total deformation that correspond to this temperature change becomes

$$\delta_T = \alpha \Delta T L = 6.5\times10^{-6}(-125)(24) = 0.0195\ \text{in}$$

  The input data is

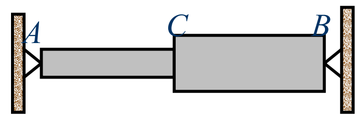
$$L_1 = L_2 = 12\ \text{in}$$

$$A_1 = 0.6\ \text{in}^2 \qquad A_2 = 1.2\ \text{in}^2$$

$$F_1 = F_2 = R_B \qquad E = 29\times10^6\ \text{psi}$$

Statically Indeterminate Axially Loaded Members

- Example 8 (cont'd)



$$\delta_B = \frac{F_1 L_1}{A_1 E} + \frac{F_2 L_2}{A_2 E}$$

$$= \frac{R_B}{29\times 10^6}\left[\frac{12}{0.6} + \frac{12}{1.2}\right] = 1.0345\times 10^{-6} R_B$$

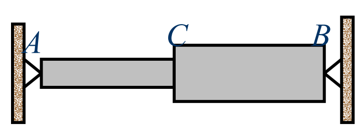
Expressing that the total deformation of the bar must be zero, hence

$$\delta = \delta_T + \delta_R = 0; -0.0195 + 1.0345\times 10^{-6} R_B = 0$$

$$\text{from which}\quad R_B = 18.85\times 10^3 \text{ lb} = 18.85 \text{ kips}$$

Statically Indeterminate Axially Loaded Members

- Example 8 (cont'd)



Therefore,

$F_1 = F_2 = 18.85$ kips, and

$$\sigma_1(\text{stress in } AC) = \frac{F_1}{A_1} = \frac{18.85}{0.6} = 31.42 \text{ ksi}$$

$$\sigma_2(\text{stress in } CB) = \frac{F_2}{A_2} = \frac{18.85}{1.2} = 15.71 \text{ ksi}$$

Stress Concentrations

- The stresses near the points of application of concentrated loads can values much larger than the average value of the stress in a member.
- When a structural member contains a discontinuity, such as a hole or a sudden change in cross section, high localized stresses can also occur near the discontinuity (Figs 1 and 2).

Stress Concentrations

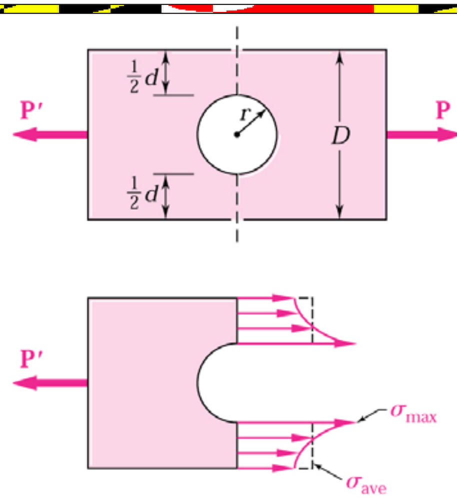


Fig. 1. Stress distribution near circular hole in flat bar under axial loading

Stress Concentrations

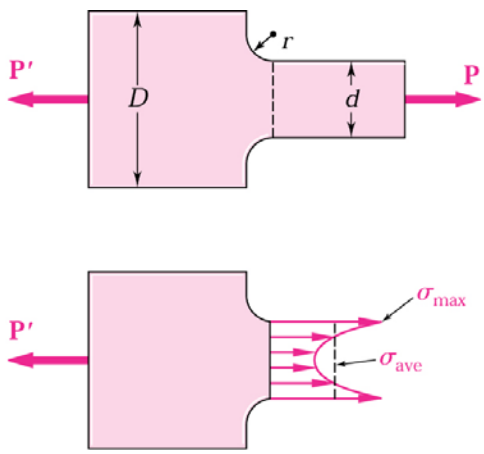
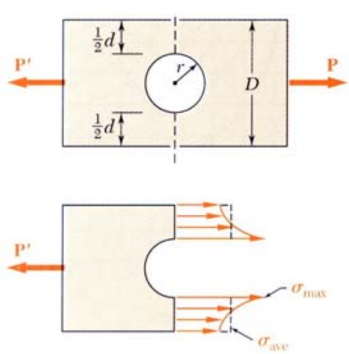


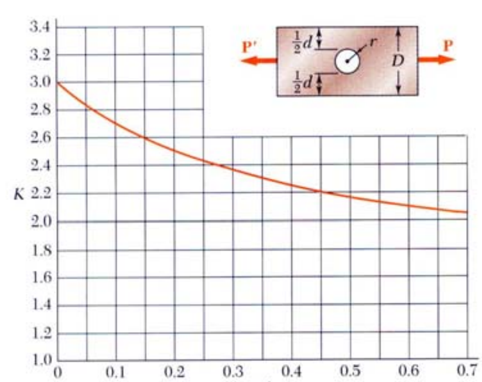
Fig. 2. Stress distribution near fillets in flat bar under axial loading

Stress Concentrations

- Hole

Fig. 3





(a) Flat bars with holes

Discontinuities of cross section may result in high localized or *concentrated* stresses.

$$K = \frac{\sigma_{max}}{\sigma_{ave}}$$

Stress Concentrations

- Fillet

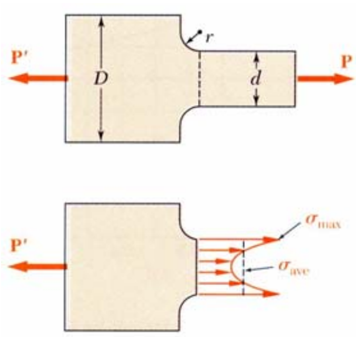
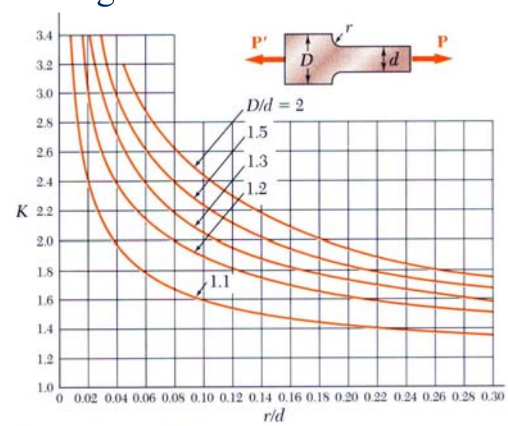


Fig. 4



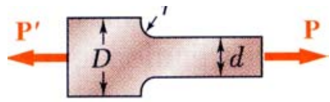
(*b*) Flat bars with fillets

Stress Concentrations

- To determine the maximum stress occurring near discontinuity in a given member subjected to a given axial load *P*, it is only required that the average stress $\sigma_{ave} = P/A$ be computed in the critical section, and the result be multiplied by the appropriate value of the stress-concentration factor *K*.
- It is to be noted that this procedure is valid as long as $\sigma_{max} \leq \sigma_y$

Stress Concentrations

- Example 9



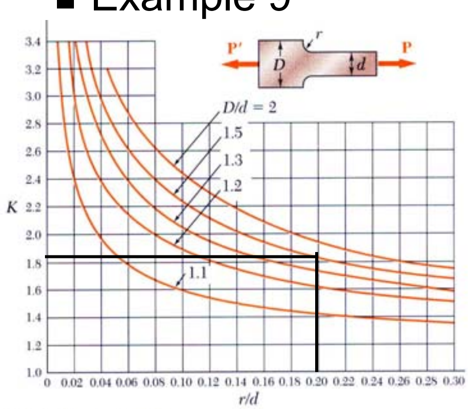
Determine the largest axial load P that can be safely supported by a flat steel bar consisting of two portions, both 10 mm thick, and respectively 40 and 60 mm wide, connected by fillets of radius r = 8 mm. Assume an allowable normal stress of 165 MPa.

SOLUTION:

- Determine the geometric ratios and find the stress concentration factor from Fig. 4.
- Find the allowable average normal stress using the material allowable normal stress and the stress concentration factor.
- Apply the definition of normal stress to find the allowable load.

Stress Concentrations

- Example 9



(b) Flat bars with fillets

- Determine the geometric ratios and find the stress concentration factor from Fig. 4.

$$\frac{D}{d} = \frac{60\,\text{mm}}{40\,\text{mm}} = 1.50 \qquad \frac{r}{d} = \frac{8\,\text{mm}}{40\,\text{mm}} = 0.20$$

$$K = 1.82$$

- Find the allowable average normal stress using the material allowable normal stress and the stress concentration factor.

$$\sigma_{ave} = \frac{\sigma_{max}}{K} = \frac{165\,\text{MPa}}{1.82} = 90.7\,\text{MPa}$$

- Apply the definition of normal stress to find the allowable load.

$$P = A\sigma_{ave} = (40\,\text{mm})(10\,\text{mm})(90.7\,\text{MPa})$$

$$= 36.3 \times 10^3\,\text{N}$$

$$\boxed{P = 36.3\,\text{kN}}$$