LECTURE

4

Chapter 2.9

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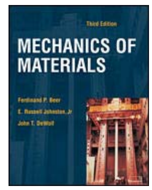
Third Edition

RODS: STATICALLY INDETERMINATE MEMBERS

A. J. Clark School of Engineering •Department of Civil and Environmental Engineering



by

Dr. Ibrahim A. Assakkaf

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ENES 220 – Mechanics of Materials

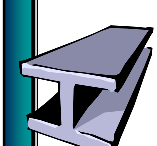
Department of Civil and Environmental Engineering

University of Maryland, College Park

Statically Indeterminate Structures

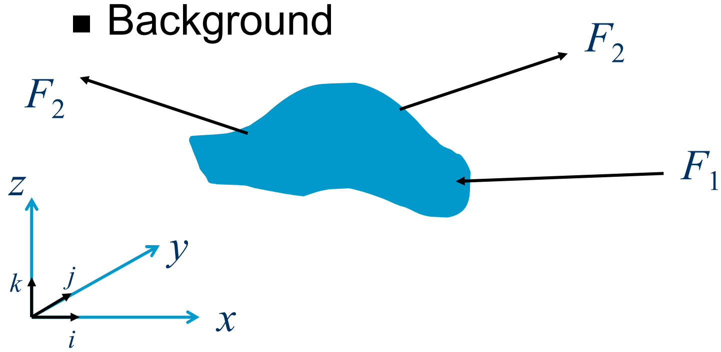
- Background
  - In all of the problems discussed so far, it was possible to determine the forces and stresses in the members by utilizing the equations of equilibrium, that is

$$\vec{R} = \sum F_x \mathrm{i} + \sum F_y \mathrm{j} + \sum F_z \mathrm{k} = \vec{0}$$
$$\vec{C} = \sum M_x \mathrm{i} + \sum M_y \mathrm{j} + \sum M_z \mathrm{k} = \vec{0} \qquad (1)$$



Statically Indeterminate Structures

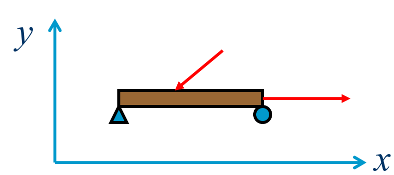
- Background



$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$
$$\sum M_x = 0 \qquad \sum M_y = 0 \qquad \sum M_z = 0 \qquad (2)$$

Statically Indeterminate Structures

- Background



$$\sum F_x = 0 \qquad \sum F_y = 0$$
$$\sum M_A = 0 \qquad (3)$$

Statically Indeterminate Structures

- Statically Determinate Member

  When equations of equilibrium are sufficient to determine the forces and stresses in a structural member, we say that the problem is *statically determinate*

Statically Indeterminate Structures

- Statically Indeterminate Member

  When the equilibrium equations alone are not sufficient to determine the loads or stresses, then such problems are referred to as *statically indeterminate* problems.

Statically Indeterminate Structures

- Determinacy
  - Equations 1, 2, and 3 provide both the necessary and sufficient conditions for equilibrium.
  - When all the forces in a structure can be determined strictly from these equations, the structure is referred to as *statically determinate*.

Statically Indeterminate Structures

- Determinacy (cont'd)
  - Structures having more unknown forces than available equilibrium equations are called statically *indeterminate*.
  - As a general rule, a structure can be identified as being either statically determinate or statically indeterminate by drawing free-body diagrams of all its members, or selective parts of its members.

Statically Indeterminate Structures

- Determinacy (cont'd)
  - When the free-body diagram is constructed, it will be possible to compare the total number of unknown reactive components with the total number of available equilibrium equations.
  - For simple structure, a "mental count" of the number of unknowns can also be made and compared with the number of Eqs.

Statically Indeterminate Structures

- Determinacy of Beams
  - For a coplanar (two-dimensional) structure, there are at most three equilibrium equations for each part, so that if there is a total of *n* parts and *r* reactions, we have

$$\boxed{\begin{array}{ll} r = 3n, & \Rightarrow \text{statically determinate} \\ r > 3n, & \Rightarrow \text{statically indeterminate} \end{array}} \qquad (4)$$

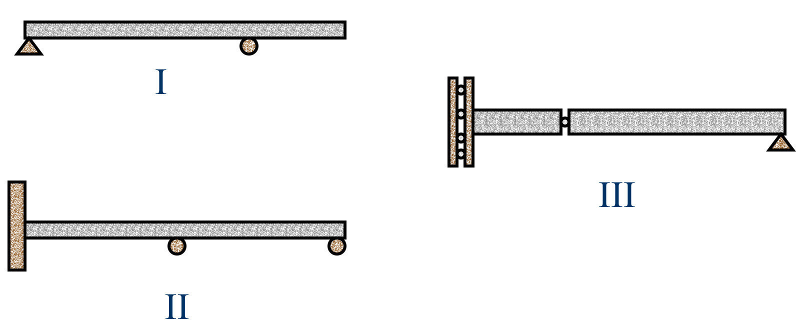
Statically Indeterminate Structures

- Example 1

  Classify each of the beams shown as statically determinate of statically indeterminate. If statically indeterminate, report the degrees of of determinacy. The beams are subjected to external loadings that are assumed to be known and can act anywhere on the beams.

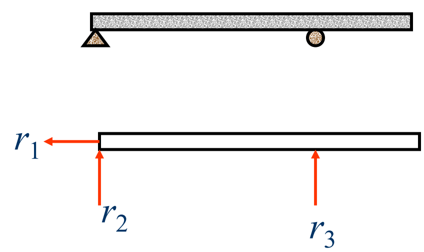
Statically Indeterminate Structures

- Example 1 (cont'd)



Statically Indeterminate Structures

- Example 1 (cont'd)
  - For part I:



Applying Eq. 4,

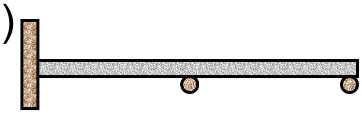
$r = 3, n = 1$, therefore,

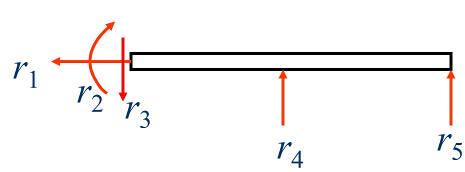
$r = 3n, \Rightarrow 3 = [3(1) = 3] \Rightarrow$ statically determinate

Statically Indeterminate Structures

- Example 1 (cont'd)

  For part II:





Applying Eq. 4,

$r = 5, n = 1$, therefore,

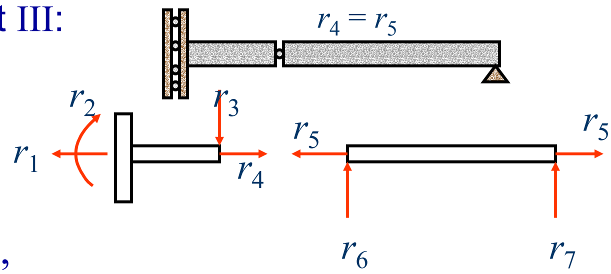
$r > 3n, \Rightarrow 5 > [3(1) > 3] \Rightarrow$ statically indeterminate to second degree

Statically Indeterminate Structures

- Example 1 (cont'd)

  For part III:

Note: $r_3 = r_6$ and $r_4 = r_5$



Applying Eq. 4,

$r = 6, n = 2$, therefore,

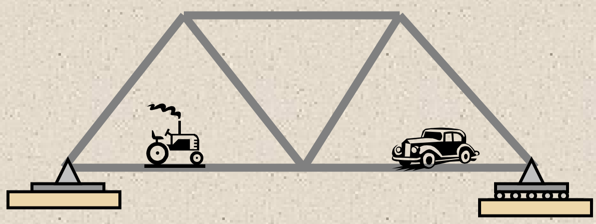
$r = 3n, \Rightarrow 6 = [3(2) = 6] \Rightarrow$ statically determinate

Statically Indeterminate Structures

- Determinacy of Trusses
  - For any problem in truss analysis, it should be noted that the total number of unknowns includes the forces in *b* number of bars of the truss and the total number of *r* of external support reactions.
  - Since the truss members are all straight axial force members lying in the same plane, the force system acting on each joint is coplanar and concurrent.

Statically Indeterminate Structures

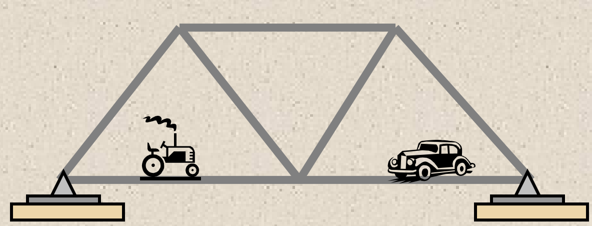
- Determinacy of Trusses



Statically Determinate Truss

Statically Indeterminate Structures

- Determinacy of Trusses



Statically Indeterminate Truss

Statically Indeterminate Structures

- Determinacy of Trusses

  Rotational or moment equilibrium is automatically satisfied at the joint (or pin) of a truss, and therefore, it is necessary to satisfy

$$\sum F_x = 0 \qquad \sum F_y = 0$$

to ensure transnational or force equilibrium

Statically Indeterminate Structures

- Determinacy of Trusses
  - For a coplanar (two-dimensional) truss, there are at most two equilibrium equations for each joint *j*, so that if there is a total of *b* members and *r* reactions, we have

$$\boxed{\begin{array}{ll} b + r = 2j, & \Rightarrow \text{statically determinate} \\ b + r > 2j, & \Rightarrow \text{statically indeterminate} \end{array}} \qquad (5)$$

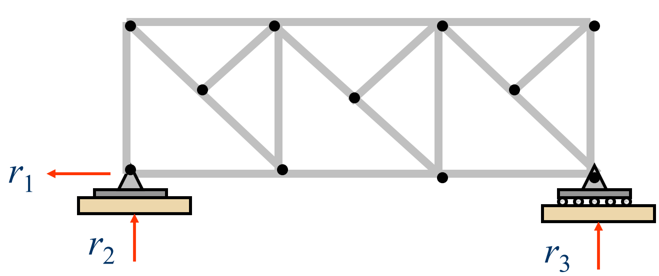
Statically Indeterminate Structures

- Example 2

  Classify each of the trusses shown as statically determinate or statically indeterminate. Also if the truss is statically indeterminate, what is the degree of indeterminacy

Statically Indeterminate Structures

- Example 2 (cont'd)



$$b = 19, r = 3, \text{ and } j = 11$$

$$b + r = 2j, \ (19+3) = (2)(11) = 22$$

$$\Rightarrow \text{statically determinate}$$

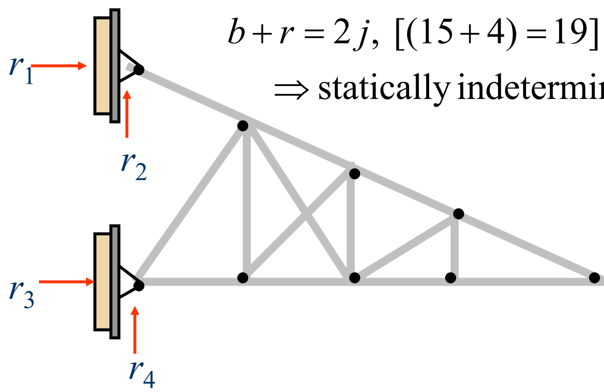
Statically Indeterminate Structures

- Example 2 (cont'd)

$$b = 15, r = 4, \text{ and } j = 9$$

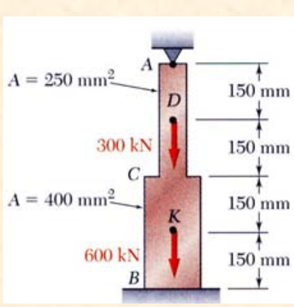
$$b + r = 2j, \; [(15+4) = 19] > [(2)(9) = 18]$$

$$\Rightarrow \text{statically indeterminate to first degree}$$



Statically Indeterminate Axially Loaded Members

- Static Indeterminacy

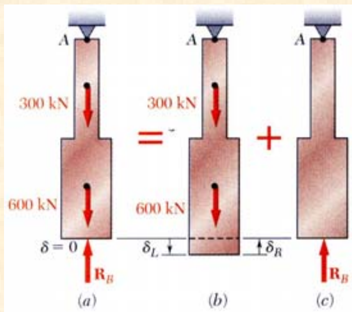


- Structures for which internal forces and reactions cannot be determined from statics alone are said to be *statically indeterminate*.

- A structure will be statically indeterminate whenever it is held by more supports than are required to maintain its equilibrium.

Statically Indeterminate Axially Loaded Members

- Static Indeterminacy



- Redundant reactions are replaced with unknown loads which along with the other loads must produce compatible deformations.

- Deformations due to actual loads and redundant reactions are determined separately and then added or *superposed*.

$$\delta = \delta_L + \delta_R = 0$$

Statically Indeterminate Axially Loaded Members

- *How to determine forces and stresses on axially loaded member that is statically indeterminate?*
  - In order to solve for the forces, and stresses in such member, it becomes necessary to supplement the equilibrium equations with additional relationships based on any conditions of restraint that may exist.

Statically Indeterminate Axially Loaded Members

- General Rules
  - Each statically indeterminate problem has its own peculiarities as to its method of solution.
  - But there are some general rules and ideas that are common to the solution of most types of problems.

Statically Indeterminate Axially Loaded Members

- General Rules
  - These general rules and guidelines are summarized as follows:
    1. Write the appropriate equations of equilibrium and examine them carefully to make sure whether or not the problem is statically determinate or indeterminate. Eqs. 4 and 5 can help in the case of coplanar problems.
    2. If the problem is statically indeterminate, examine the kinematic restraints to determine

Statically Indeterminate Axially Loaded Members

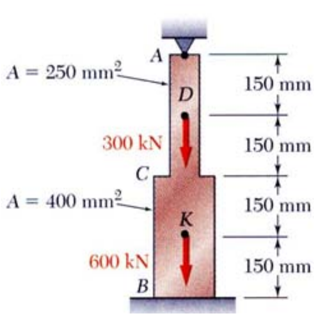
- General Rules

  the necessary conditions that must be satisfied by the deformation(s) of the member(s).

  3. Express the required deformations in terms of the loads or forces. When enough of these additional relationships have been obtained, they can be adjoined to the equilibrium equations and the problem can then be solved.

Statically Indeterminate Axially Loaded Members

- Example 3



Determine the reactions at A and B for the steel bar and loading shown, assuming a close fit at both supports before the loads are applied.

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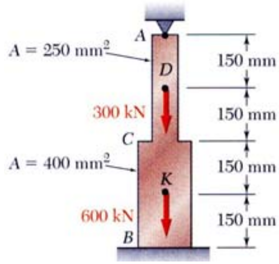
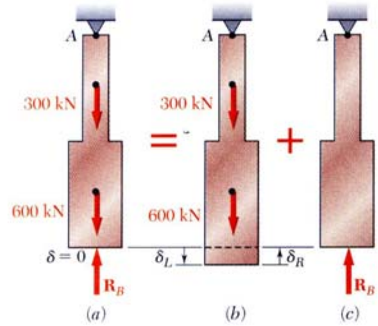
Statically Indeterminate Axially Loaded Members

- Example 3 (cont'd)





SOLUTION:

- Consider the reaction at *B* as redundant, release the bar from that support, and solve for the displacement at *B* due to the applied loads.

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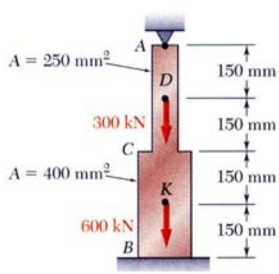
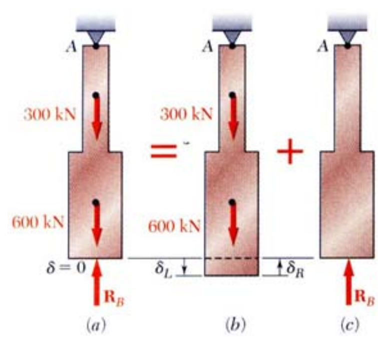
Statically Indeterminate Axially Loaded Members

- Example 3 (cont'd)

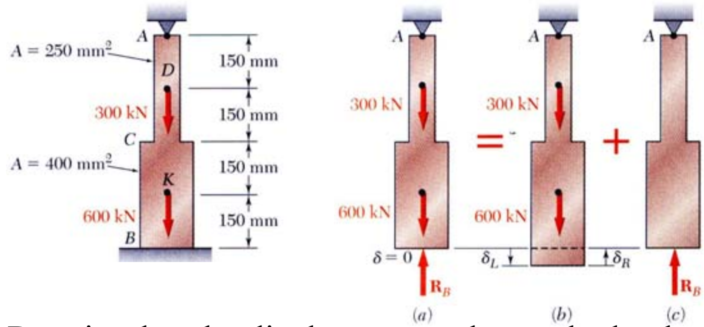




- Solve for the displacement at *B* due to the redundant reaction at *B*.

Statically Indeterminate Axially Loaded Members

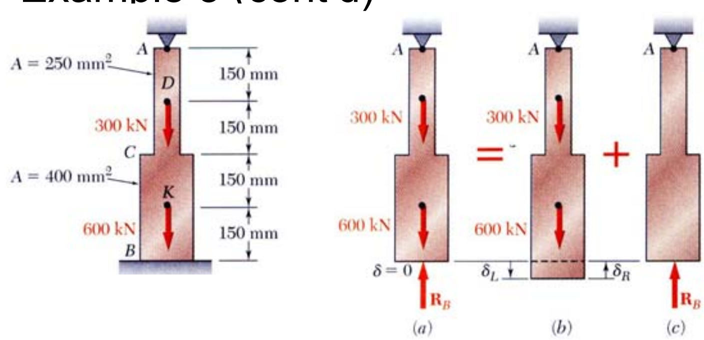
- Example 3 (cont'd)



- Require that the displacements due to the loads and due to the redundant reaction be compatible, i.e., require that their sum be zero.

Statically Indeterminate Axially Loaded Members

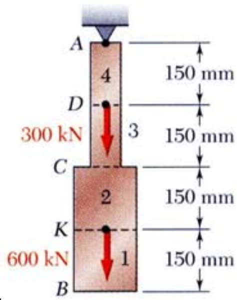
- Example 3 (cont'd)



- Solve for the reaction at *A* due to applied loads and the reaction found at *B*.

Statically Indeterminate Axially Loaded Members

- Example 3 (cont'd)



- Solve for the displacement at B due to the applied loads with the redundant constraint released,

$$P_1 = 0 \quad P_2 = P_3 = 600\times10^3\,\mathrm{N} \quad P_4 = 900\times10^3\,\mathrm{N}$$

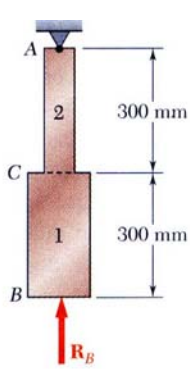
$$A_1 = A_2 = 400\times10^{-6}\,\mathrm{m}^2 \quad A_3 = A_4 = 250\times10^{-6}\,\mathrm{m}^2$$

$$L_1 = L_2 = L_3 = L_4 = 0.150\,\mathrm{m}$$

$$\delta_L = \sum_i \frac{P_i L_i}{A_i E_i} = \frac{1.125\times10^9}{E}$$

Statically Indeterminate Axially Loaded Members

- Example 3 (cont'd)



- Solve for the displacement at B due to the redundant constraint,

$$P_1 = P_2 = -R_B$$

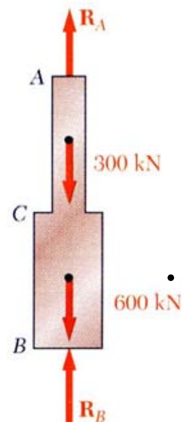
$$A_1 = 400\times10^{-6}\,\mathrm{m}^2 \quad A_2 = 250\times10^{-6}\,\mathrm{m}^2$$

$$L_1 = L_2 = 0.300\,\mathrm{m}$$

$$\delta_R = \sum_i \frac{P_i L_i}{A_i E_i} = -\frac{\left(1.95\times10^3\right)R_B}{E}$$

Statically Indeterminate Axially Loaded Members

- Example 3 (cont'd)



- Require that the displacements due to the loads and due to the redundant reaction be compatible,

$$\delta = \delta_L + \delta_R = 0$$

$$\delta = \frac{1.125\times10^9}{E} - \frac{\left(1.95\times10^3\right)R_B}{E} = 0$$

$$R_B = 577\times10^3\,\text{N} = 577\,\text{kN}$$

- Find the reaction at A due to the loads and the reaction at B

$$\sum F_y = 0 = R_A - 300\,\text{kN} - 600\,\text{kN} + 577\,\text{kN}$$

$$R_A = 323\,\text{kN}$$

$$\boxed{\begin{aligned} R_A &= 323\,\text{kN} \\ R_B &= 577\,\text{kN} \end{aligned}}$$

Statically Indeterminate Axially Loaded Members

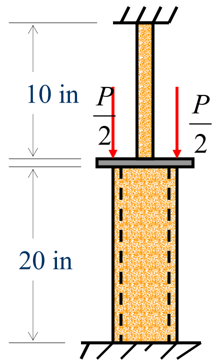
- Example 4

A rigid plate C is used to transfer a 20-kip load P to a steel (Es = 30,000 ksi) rod A and aluminum alloy (Ea = 10,000 ksi) pipe B, as shown. The supports at the top of the rod and bottom of the pipe are rigid and there are no stresses in the rod or pipe before the load P applied. The cross-sectional areas of rod A and pipe B are

Statically Indeterminate Axially Loaded Members

- Example 4 (cont'd)

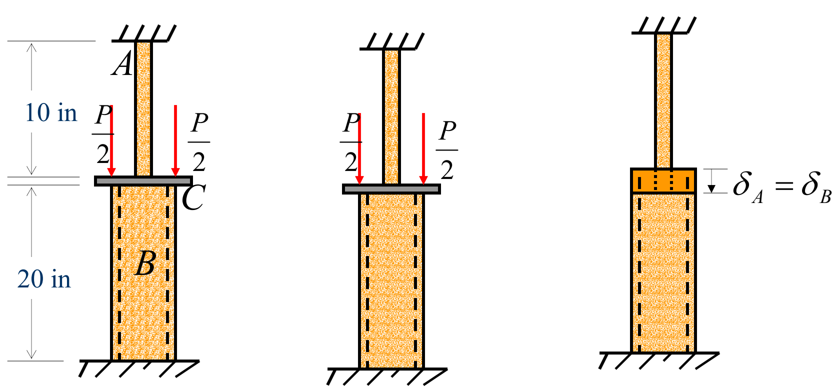
0.8 in2 and 3.0 in2, respectively. Determine



(a) the axial stresses in rod A and pipe B,
(b) the displacement of plate C, and
(c) the reactions.

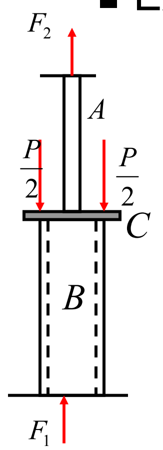
Statically Indeterminate Axially Loaded Members

- Example 4 (cont'd)



Statically Indeterminate Axially Loaded Members

- Example 4 (cont'd)



FBD

Equilibrium equation:

$$+\uparrow \sum F_y = 0; F_1 + F_2 - \frac{P}{2} - \frac{P}{2} = 0$$

$$F_1 + F_2 - 20 = 0 \qquad (6)$$

Internal Forces in *A* and *B* are

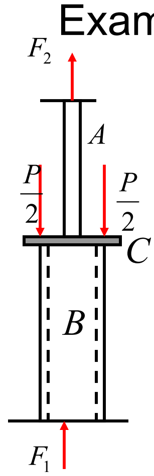
F_1 and F_2

But

$$F_1 = \sigma_A A_a \quad \text{and} \quad F_2 = \sigma_B A_B$$

Statically Indeterminate Axially Loaded Members

Example 4 (cont'd)



Therefore, Eq. 6 can be written as

$$\sigma_A A_A + \sigma_B A_B - 20 = 0, \text{ or}$$

$$0.8\sigma_A + 3\sigma_B = 20 \qquad (7)$$

Deformation equation yields:

$$\delta_A = \delta_B = \frac{\sigma_A L_A}{E_A} = \frac{\sigma_B L_B}{E_B} = \frac{\sigma_A (10)}{30{,}000} = \frac{\sigma_B (20)}{10{,}000}$$

From which,

$$\sigma_A = 6\sigma_B \qquad (8)$$

Statically Indeterminate Axially Loaded Members

- Example 4 (cont'd)

Solving Eqs. 7 and 8 simultaneously, gives

$$\sigma_A = 15.38 \text{ ksi} \quad \text{and} \quad \sigma_B = 2.56 \text{ ksi}$$

The displacement of *C*:

$$\delta_C = \delta_A = \delta_B = \frac{\sigma_A(10)}{30{,}000} = \frac{15.38(10)}{30{,}000} = 0.00513 \text{ in}$$

The reactions are determined as follows:

$$F_1 = \sigma_B A_B = 2.56(3) = 7.68 \text{ kips}$$

$$F_2 = \sigma_A A_A = 15.38(0.8) = 12.3 \text{ kips}$$