LECTURE

29

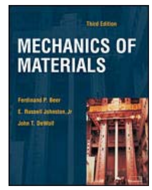
Chapter H2

The McGraw-Hill Companies

Third Edition

ADVANCED TOPICS IN MECHANICS-II

A. J. Clark School of Engineering •Department of Civil and Environmental Engineering



by

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SPRING 2003

ENES 220 – Mechanics of Materials

Department of Civil and Environmental Engineering

University of Maryland, College Park

ADVANCED TOPICS IN MECHANICS-II

1. Buckling: Eccentric Loading
   - References:
     - Beer and Johnston, 1992. "Mechanics of Materials," McGraw-Hill, Inc.
     - Byars and Snyder, 1975. "Engineering Mechanics of Deformable Bodies," Thomas Y. Crowell Company Inc.

ADVANCED TOPICS IN MECHANICS-I

2. Torsion of Noncircular Members and Thin-Walled Hollow Shafts
   - References
     - Beer and Johnston, 1992. "Mechanics of Materials," McGraw-Hill, Inc.
     - Byars and Snyder, 1975. "Engineering Mechanics of Deformable Bodies," Thomas Y. Crowell Company Inc.

ADVANCED TOPICS IN MECHANICS-I

3. Introduction to Plastic Moment
   - References:
     - Salmon, C. G. and Johnson, J. E., 1990. "*Steel Structures – Design and Behavior*," Chapter 10, HarperCollins Publishers Inc.
     - McCormac, J. C., 1989. "*Structural Steel Design*," Ch. 8,9, Harper & Row, Publishers, Inc.

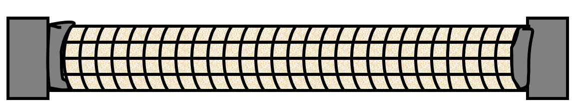
Torsion of Noncircular Members

- Introduction
  - The analysis of a noncircular torsion structural member is far more complicated than a circular shaft.
  - The major difficulty basically lies in determining the shear-strain distribution.
  - In these noncircular members, the discussion presented previously for circular shafts is not applicable.

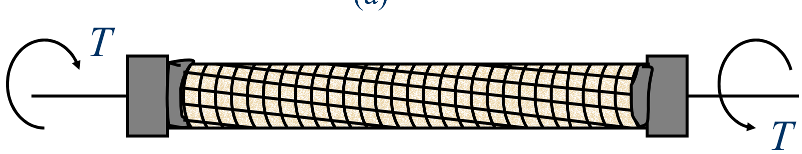
Torsion of Noncircular Members

- Introduction
  - Deformation of Circular Shaft Subjected to Torque *T*

Figure 11



(a)

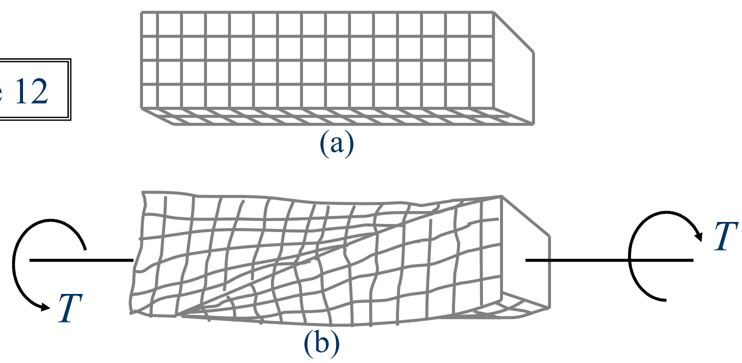


(b)

Torsion of Noncircular Members

- Introduction
  - Deformation of a Bar of Square Cross Section Subjected to Torque *T*

Figure 12



Torsion of Noncircular Members

- Introduction
  - In circular or cylindrical member, it was concluded that plane transverse sections remain plane and the shear strain varies linearly from the geometric center.
  - A simple experiment indicates that these conclusions are not true for a torsion member having a rectangular cross section, as shown in Figure 12.

Torsion of Noncircular Members

- Introduction
  - For circular elastic shaft, the equations that define respectively the distribution of stain and stress are as follows:

$$\gamma = \frac{\rho}{c}\gamma_{\max} \qquad (24)$$

$$\tau = \frac{\rho}{c}\tau_{\max} \qquad (25)$$

Torsion of Noncircular Members

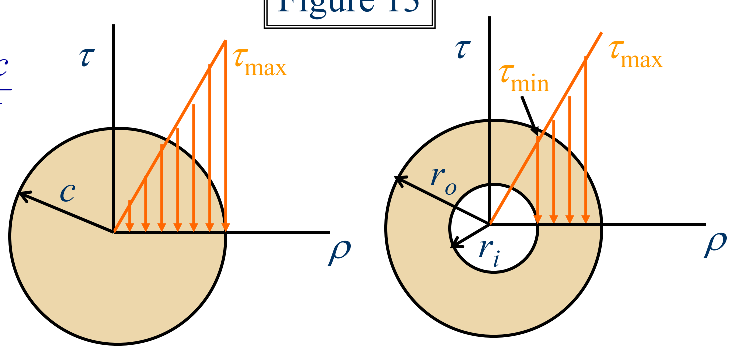
- Introduction
  - Distribution of Shearing Stress within the Circular Cross Section

Figure 13

$$\tau_c = \frac{Tc}{J}$$

$$\tau_{\max} = \frac{Tc}{J}$$

$$\tau_\rho = \frac{T\rho}{J}$$



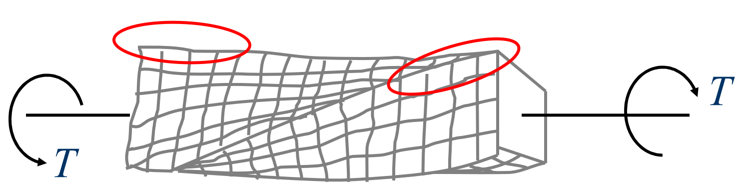
Torsion of Noncircular Members

- Introduction
  - Eqs. 24 and 25 that define respectively the distribution of stain and stress in an elastic shaft cannot be used for noncircular members.
  - For example, it would be wrong to assume that the shearing stress in the cross section of a square bar varies linearly with the distance from the axis of the bar and

Torsion of Noncircular Members

- Introduction

  is therefore largest at the corners of the cross section.
  - The shearing stress is actually zero at the corners of a square cross section



Torsion of Noncircular Members

- Determination of Shearing Stress
  - Several rigorous methods have been derived to determine the shear strain distribution in noncircular torsion members.
  - Probably the foremost of these in the *membrane analogy*.
  - However, the mathematics required to pursue this derivation is beyond the level of this introductory course in mechanics.

Torsion of Noncircular Members

- Determination of Shearing Stress
  - The solutions of many problems for solid noncircular torsion members can be found in several advanced books such as
    - Seely, F. B. and Smith, J. O., 1952. "Advanced Mechanics of Materials," 2nd edition, Wiley.
    - Timoshenko, S. P. and Goodier, J. N., 1970. "Theory of Elasticity," 3rd edition, McGraw-Hill, New York, section. 109.

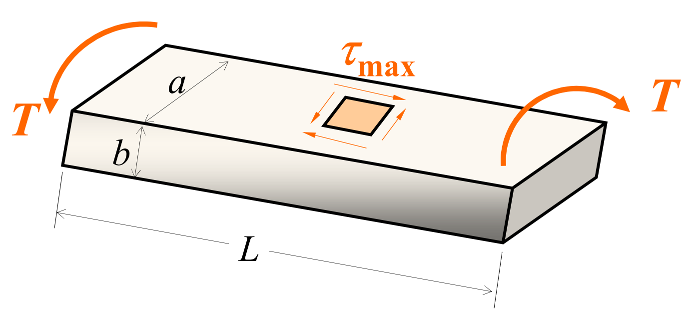
Torsion of Noncircular Members

- Bar of Rectangular Cross Section
  - Results obtained from the mathematical theory of elasticity (Timoshenko and Goodier 1970) for straight bars with a uniform rectangular cross section are presented herein for real applications.
  - Denoting by *L* the length of the bar, by *a* and *b*, respectively, the wider and narrower side of its cross section, and by *T* the

Torsion of Noncircular Members

- Bar of Rectangular Cross Section

Figure 14



Torsion of Noncircular Members

- Bar of Rectangular Cross Section

  magnitude of the torques applied to the bar (Fig. 14), the maximum shearing stress occurs along the center line of the wider face of the bar and is equal to

$$\tau_{\max} = \frac{T}{k_1 a b^2} \qquad (26)$$

Torsion of Noncircular Members

- Bar of Rectangular Cross Section
  - The angle of twist ϕ, on the other hand, may be expressed as

$$\phi = \frac{TL}{k_2 a b^3 G} \qquad (27)$$

  - The coefficients k_1 and k_2 depend only upon the ratio *a/b* and are given in the following table (Table 2) for various values for that ratio.

Torsion of Noncircular Members

- Table 2. Coefficients for Rectangular Bars in Torsion

Beer and Johnston, 1992

a/b	k_1	k_2
1.0	0.208	0.1406
1.2	0.219	0.1661
1.5	0.231	0.1958
2.0	0.246	0.229
2.5	0.258	0.249
3.0	0.267	0.263
4.0	0.282	0.281
5.0	0.291	0.291
10.0	0.312	0.312
∞	0.333	0.333

Torsion of Noncircular Members

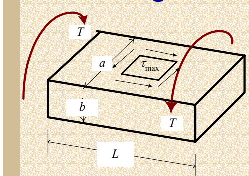
- Bar of Rectangular Cross Section
  - Note that Eqs. 26 and 27 are valid only within the elastic range.
  - It is clear from Table 1 tat for $a/b \geq 5$, the coefficients k_1 and k_2 are equal. It can be shown that for such values of *a/b*, we have

$$k_1 = k_2 = \frac{1}{3}\left(1 - \frac{0.630}{(a/b)}\right) \quad \text{for } a/b \geq 5 \text{ only} \qquad (28)$$

Torsion of Noncircular Members

- Bar of Rectangular Cross Section

The maximum shearing stress and the angle of twist for a uniform bar of rectangular cross section, and subjected to pure torsion *T* are given by



$$\tau_{\max} = \frac{T}{k_1 a b^2} \qquad (26)$$

$$\phi = \frac{TL}{k_2 a b^3 G} \qquad (27)$$

The coefficients k_1 and k_2 can be obtained from Table 2.

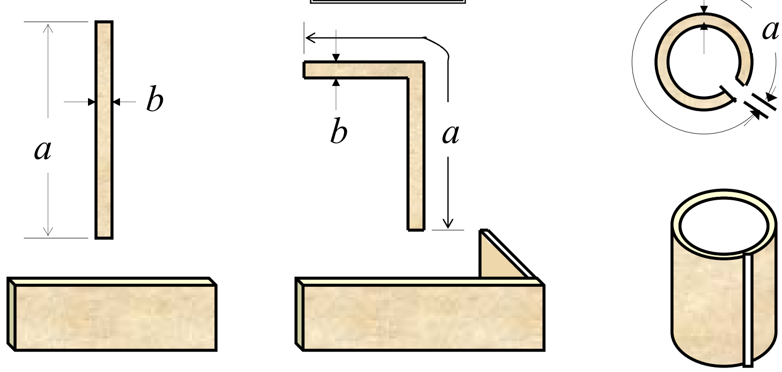
Torsion of Noncircular Members

- Thin-Walled Member of Uniform Thickness and Arbitrary Shape
  - Let us consider several thin-walled members with the cross sections shown in Fig. 15, which are subjected to the same torque.
  - Based on the *membrane analogy* (Timoshenko and Goodier 1970), for a thin-walled member of uniform thickness and

Torsion of Noncircular Members

- Thin-Walled Member of Uniform Thickness and Arbitrary Shape

Figure 15



Torsion of Noncircular Members

- Thin-Walled Member of Uniform Thickness and Arbitrary Shape

  arbitrary shape, the maximum shearing stress is the same as for a rectangular bar with a very large value of *a/b* and can be determined from Eq. 26 with k_1 = 0.333.

  – Also, it can be shown that the angle of twist ϕ can be determined from Eq. 27 with k_2 = 0.333.

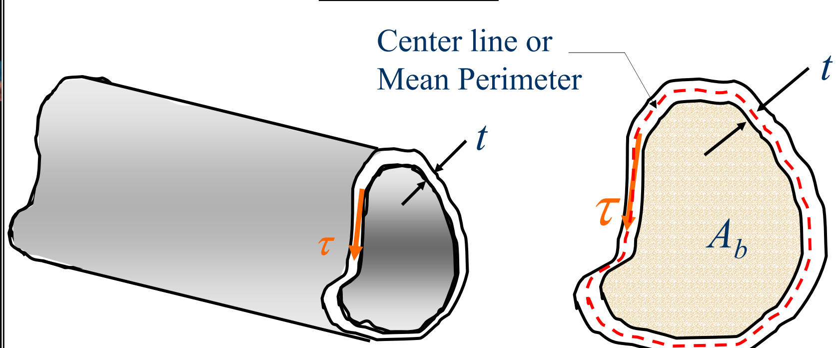
Torsion of Noncircular Members

- Thin-Walled Hollow Shafts
  - It was indicated earlier that the determination of the stresses in noncircular members generally requires the use of advanced mathematical methods.
  - In the case of thin-walled hollow noncircular shaft (Fig. 16), however, a good approximation of the distribution of stresses can be obtained.

Torsion of Noncircular Members

- Thin-Walled Hollow Shafts

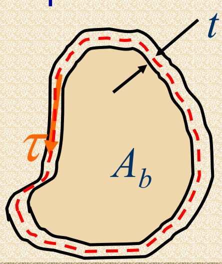
Figure 16



Torsion of Noncircular Members

- Thin-Walled Hollow Shafts

  The shearing stress τ at any given point of the wall may be expressed in terms of the torque T as



$$\tau = \frac{T}{2tA_b} \qquad (29)$$

A_b = area bounded by center line

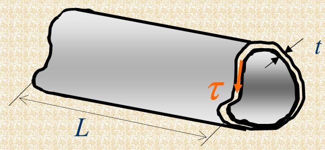
Torsion of Noncircular Members

- Thin-Walled Hollow Shafts
  - The shearing stress τ of Eq. 29 represents the average value of the shearing stress across the wall.
  - However, for elastic deformations the distribution of the stress across the wall may be assumed uniform, and Eq. 29 will give the actual value of the shearing stress at a given point of the wall.

Torsion of Noncircular Members

- Thin-Walled Hollow Shafts

The angle of twist of a thin-walled shaft of length L and modulus of rigidity G is given by



$$\phi = \frac{TL}{4A_b^2 G} \oint \frac{ds}{t} \qquad (30)$$

Where the integral is computed along the center line of the wall section.

Torsion of Noncircular Members

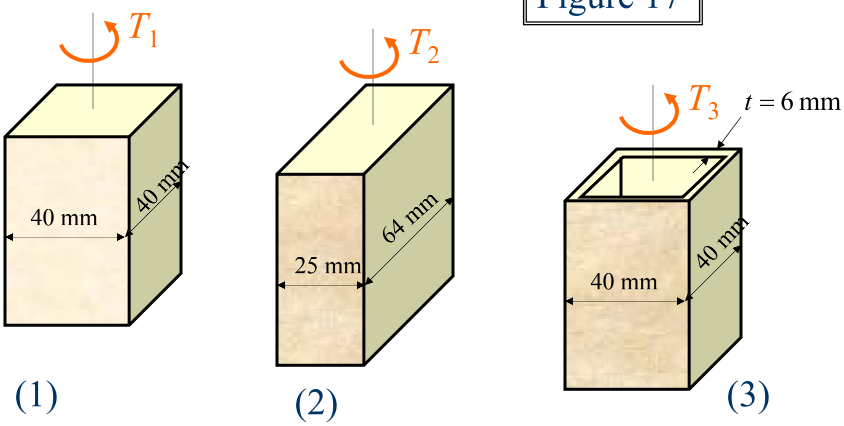
- Example 3

Using τ_{all} = 40 MPa, determine the largest torque which may be applied to each of the brass bars and to the brass tube shown. Note that the two solid bars have the same cross-sectional area, and that the square bar and square tube have the same outside dimensions.

Torsion of Noncircular Members

- Example 3 (cont'd)

Figure 17



Torsion of Noncircular Members

- Example 3 (cont'd)
  1. Bar with Square Cross Section:

     For a solid bar of rectangular cross section, the maximum shearing stress is given by Eq. 26:

$$\tau_{\max} = \frac{T}{k_1 ab^2}$$

where the coefficient k_1 is obtained from Table 2, therefore

$$a = b = 0.040 \text{ m} \qquad \frac{a}{b} = 1.00 \qquad k_1 = 0.208$$

Torsion of Noncircular Members

- Example 3 (cont'd)

  For τ_{max} = τ_{all} = 40 MPa, we have

$$\tau_{max} = \frac{T_1}{k_1 ab^2} \qquad 40 = \frac{T_1}{0.208(0.04)(0.04)^2} \Rightarrow \boxed{T_1 = 532 \text{ N}\cdot\text{m}}$$

2. Bar with Rectangular Cross Section:

$$a = 0.064 \text{ m} \qquad b = 0.025 \text{ m} \qquad \frac{a}{b} = \frac{0.064}{0.025} = 2.56$$

By interpolation, Table 2 gives : $k_1 = 0.259$

Torsion of Noncircular Members

- Example 3 (cont'd)

$$\tau_{max} = \frac{T_2}{k_1 ab^2} \qquad 40 = \frac{T_2}{0.259(0.064)(0,025)^2} \Rightarrow \boxed{T_2 = 414 \text{ N}\cdot\text{m}}$$

3. Square Tube:

   For a tube of thickness *t*, the shearing stress is given by Eq. 29 as

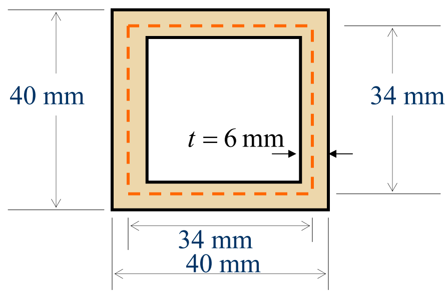
$$\tau = \frac{T}{2tA_b}$$

Torsion of Noncircular Members

- Example 3 (cont'd)

where A_b is the area bounded by the center line of the cross section, therefore,

$$A_b = (0.034)(0.034) = 1.156 \times 10^{-3}\ \text{m}^2$$



Torsion of Noncircular Members

- Example 3 (cont'd)

$\tau = \tau_{all} = 40$ MPa and $t = 0.006$ m.

Substituting these value into Eq. 27 gives

$$\tau = \frac{T}{2tA_b}$$

$$40 = \frac{T_3}{2(0.006)(1.156 \times 10^{-3})}$$

$$\therefore T_3 = \boxed{555\ \text{N} \cdot \text{m}}$$

Torsion of Noncircular Members

- Example 4

  Structural aluminum tubing of 2.5 × 4-in. rectangular cross section was fabricated by extrusion. Determine the shearing stress in each of the four walls of a portion of such tubing when it is subjected to a torque of 24 kip·in., assuming (a) a uniform 0.160-in. wall thickness (Figure 18a), (b) that, as a result of defective fabrication, walls *AB*

Torsion of Noncircular Members

- Example 4 (cont'd)

  and *AC* are 0.120-in thick, and walls *BD* and *CD* are 0.200-in thick (Fig. 18b)

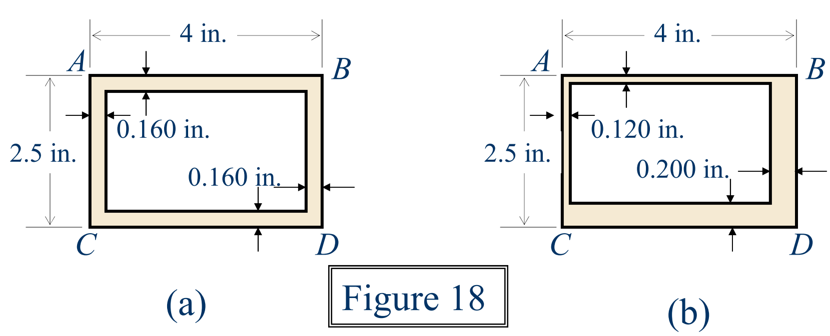
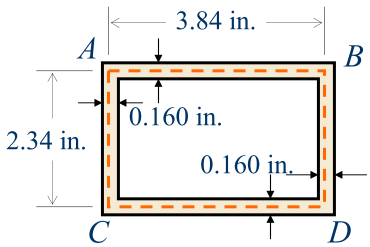


Figure 18

Torsion of Noncircular Members

- Example 4 (cont'd)
  - (a) Tubing of Uniform Wall Thickness:

    The area bounded by the center line (Fig. 19) is given by



$$A_b = (3.84)(2.34) = 8.986\ \text{in}^2$$

Figure 19

Torsion of Noncircular Members

- Example 4 (cont'd)

  Since the thickness of each of the four walls is t = 0.160 in., we find from Eq. 29 that the shearing stress in each wall is

$$\tau = \frac{T}{2tA_b} = \frac{24}{2(0.160)(8.986)} = \boxed{8.35\ \text{ksi}}$$

  - (b) Tubing with Variable Wall Thickness:

    Observing that the area A_b bounded by the center line is the same as in Part a, and substituting t = 0.120 in. and t = 0.200 in. into

Torsion of Noncircular Members

- Example 4 (cont'd)
  - Eq. 29, the following values for the shearing stresses are obtained:

$$\tau_{AB} = \tau_{AC} = \frac{24}{2(0.120)(8.986)} = \boxed{11.13\ \text{ksi}}$$

and

$$\tau_{BD} = \tau_{CD} = \frac{24}{2(0.200)(8.986)} = \boxed{6.68\ \text{ksi}}$$

  - Note that the stress in a given wall depends only upon its thickness *t*.

Introduction to Plastic Moment

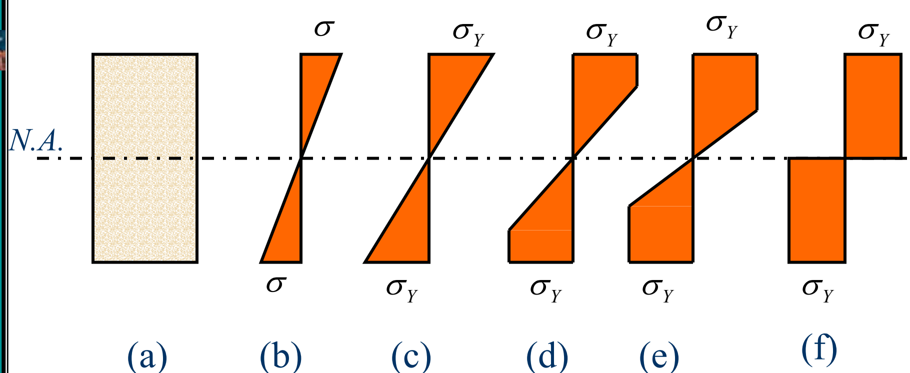
- Background
  - Stresses in Beams
    - For introduction to bending stress the rectangular beam and stress diagrams of Fig. 20 are considered.
    - If the beam is subjected to some bending moment that stress at any point may be computed with the usual *flexure formula*:

$$\sigma = \frac{Mc}{I} \qquad (31)$$

Introduction to Plastic Moment

- Background

Figure 20



Introduction to Plastic Moment

- Background
  - It is important to remember that the expression given by Eq. 31 is *only* applicable when the maximum computed stress in the beam is below the *elastic limit*.
  - The formula of Eq. 31 is based on the assumption that the stress is proportional to the strain, and a plane section before bending remains plane after bending.

Introduction to Plastic Moment

- Background
  - The value of I/c is a constant for a particular section and is known as the *section modulus* S.
  - The flexure formula may then be written as follows:

$$\sigma = \frac{M}{S} \qquad (32)$$

Introduction to Plastic Moment

- Plastic Moment
  - In reference to Fig. 20:
    - Stress varies linearly from the neutral axis to extreme fibers, as shown in Fig. 20b
    - When the moment increases, there will also be a linear relationship between the moment and the stress until the stress reaches the yield stress σ_Y, as shown in Fig. 20c.
    - In Fig. 20d, when the moment increases beyond the yield moment, the outermost fibers

Introduction to Plastic Moment

- Plastic Moment
  - In reference to Fig. 20 (cont'd):

    that had previously stressed to their yield point will continue to have the same but will yield.

    - The process will continue with more and more parts of the beam cross section stressed to the yield point as shown by the stress diagrams of parts (d) and (e) of Fig. 20., until finally a full plastic distribution is approached as shown in Fig. 20f.

Introduction to Plastic Moment

- Plastic Hinge
  - When the stress has reached this stage, a *plastic hinge* is said to have formed because no additional moment can be resisted at the section.
  - Any additional moment applied at the section will cause the beam to rotate with little increase in stress.

Introduction to Plastic Moment

- Plastic Moment
  - Definition

    “*The plastic moment can be defined as the moment that will produce full plasticity in a member cross section and create a plastic hinge*”.

Introduction to Plastic Moment

- Shape Factor
  - Definition

    “The shape factor of a member cross section can be defined as the ratio of the plastic moment M_p to yield moment M_y”.
  - The shape factor equals 1.50 for rectangular cross sections and varies from about 1.10 to 1.20 for standard rolled-beam sections

Introduction to Plastic Moment

- Plastic Modulus
  - Definitions

"The plastic modulus *Z* is defined as the ratio of the plastic moment M_p to the yield stress σ_Y."

"It can also be defined as the first moment of area about the neutral axis when the areas above and below the neutral axis are equal."

Introduction to Plastic Moment

- Shape Factor

The shape factor *Z* can be computed from the following expressions:

$$\text{Shape Factor} = \frac{M_P}{M_y} \qquad (33)$$

Or from

$$\text{Shape Factor} = \frac{Z}{S} \qquad (34)$$

Introduction to Plastic Moment

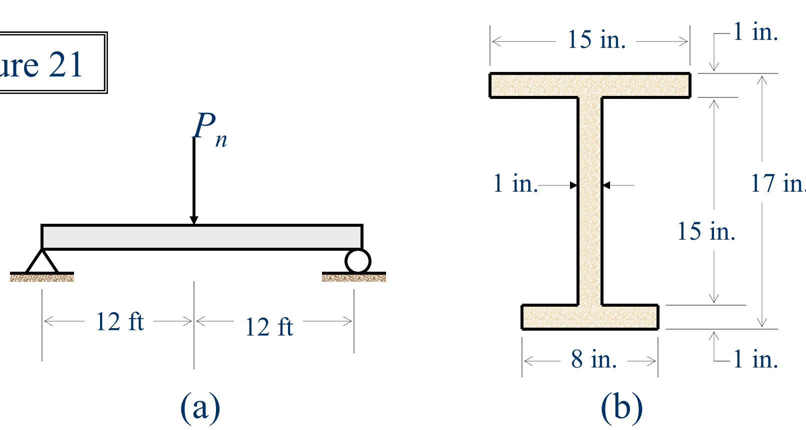
- Example 5

  Determine the yield moment M_y, the plastic or nominal moment M_p (M_n), and the plastic modulus Z for the simply supported beam having the cross section shown in Fig. 21b. Also calculate the shape factor and nominal load P_n acting transversely through the midspan of the beam. Assume that σ_Y = 50 ksi.

Introduction to Plastic Moment

- Example 5 (cont'd)

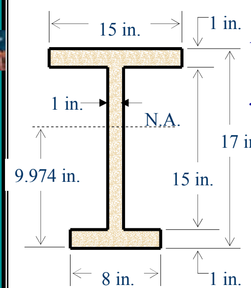
Figure 21



Introduction to Plastic Moment

- Example 5 (cont'd)

Elastic Calculations:



$$A = 15(1) + 15(1) + 8(1) = 38 \text{ in}^2$$

$$y_C = \frac{15(1)(16.5) + 15(1)(8.5) + 8(1)(0.5)}{38} = 9.974 \text{ in from lower base}$$

$$I_x = \frac{8(9.974)^3}{3} - \frac{7(8.974)^3}{3} + \frac{15(7.026)^3}{3} - \frac{14(6.026)^3}{3}$$

$$= 1{,}672.64 \text{ in}^4$$

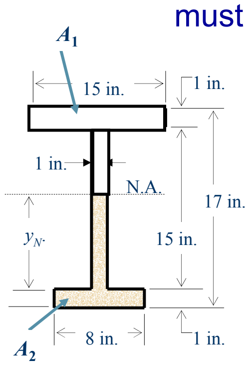
$$S = \frac{I}{c} = \frac{1{,}672.64}{9.974} = 167.7 \text{ in}^3 \quad M_y = \sigma_Y S = \frac{50(167.7)}{12} = \boxed{698.75 \text{ ft - kip}}$$

Introduction to Plastic Moment

- Example 5 (cont'd)

Plastic Calculations:

- The areas above and below the neutral axis must be equal for plastic analysis



$$A_1 = A_2$$

$$15(1) + (15 - y_N)(1) = 8(1) + y_N(1)$$

$$15 + 15 - y_N = 8 + y_N$$

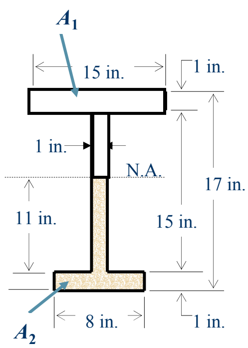
$$2y_N = 15 + 15 - 8 = 22$$

$$y_N = 11 \text{ in}$$

Introduction to Plastic Moment

- Example 5 (cont'd)

  Plastic Calculations (cont'd):



$$Z = 8(1)(11.5) + 11(1)(5.5) + 15(1)(4.5) + 4(1)(2) = 228\ \text{in}^3$$

$$M_p = M_n = \sigma_Y Z = \frac{50(228)}{12} = \boxed{950\ \text{ft - kip}}$$

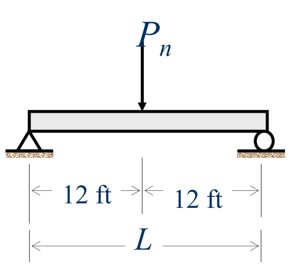
$$\text{Shape Factor} = \frac{M_n}{M_y} = \frac{950}{698.75} = \boxed{1.36}$$

Note, the shape factor can also be calculated from

$$\text{Shape Factor} = \frac{Z}{S} = \frac{228}{167.7} = 1.36$$

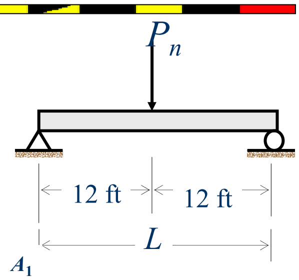
Introduction to Plastic Moment

- Example 5 (cont'd)
  - In order to find the nominal load P_n, we need to find an expression that gives the maximum moment on the beam. This maximum moment occurs at midspan of the simply supported beam, and is given by



$$M_P = M_{L/2} = \frac{P_n L}{4} \qquad (35)$$

Introduction to Plastic Moment



- Example 5 (cont'd)

$$M_P = M_{L/2} = \frac{P_n L}{4}$$

$$950 = \frac{P_n (24)}{4}$$

Therefore,

$$P_n = \frac{4(950)}{24} = \boxed{158.3 \text{ kips}}$$