LECTURE

15

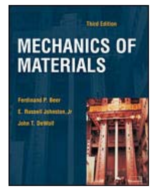
Chapter 6.6 – 6.7

The McGraw·Hill Companies

Third Edition

BEAMS: SHEAR FLOW, THIN WALLED MEMBERS

A. J. Clark School of Engineering •Department of Civil and Environmental Engineering



by

Dr. Ibrahim A. Assakkaf

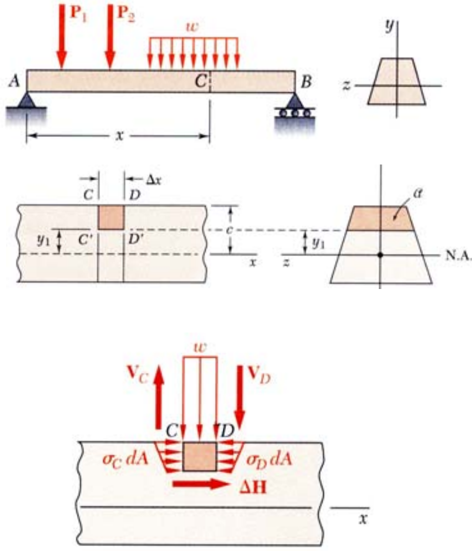
SPRING 2003

ENES 220 – Mechanics of Materials

Department of Civil and Environmental Engineering

University of Maryland, College Park

Shear on the Horizontal Face of a Beam Element



- Consider prismatic beam
- For equilibrium of beam element

$$\sum F_x = 0 = \Delta H + \int_A (\sigma_D - \sigma_D)\, dA$$

$$\Delta H = \frac{M_D - M_C}{I} \int_A y\, dA$$

- Note,

$$Q = \int_A y\, dA$$

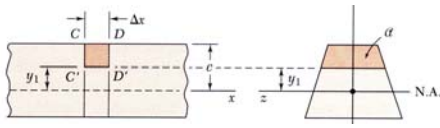
$$M_D - M_C = \frac{dM}{dx}\Delta x = V\,\Delta x$$

- Substituting,

$$\Delta H = \frac{VQ}{I}\Delta x$$

$$q = \frac{\Delta H}{\Delta x} = \frac{VQ}{I} = \underline{shear\ \ flow}$$

Shear on the Horizontal Face of a Beam Element



- Shear flow,

$$q = \frac{\Delta H}{\Delta x} = \frac{VQ}{I} = shear\ \ flow$$

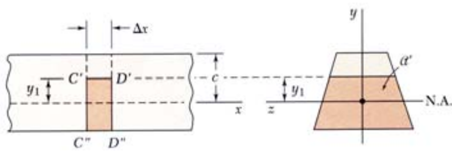
- where

$$Q = \int_A y\,dA$$

$= $ first moment of area above y_1

$$I = \int_{A+A'} y^2 dA$$

$= $ second moment of full cross section



- Same result found for lower area

$$q' = \frac{\Delta H'}{\Delta x} = \frac{VQ'}{I} = -q'$$

$$Q + Q' = 0$$

$= $ first moment with respect to neutral axis

$$\Delta H' = -\Delta H$$

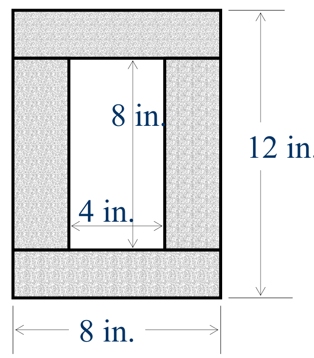
Shearing Stress in Beams

- Example 16

  The transverse shear *V* at a certain section of a timber beam is 600 lb. If the beam has the cross section shown in the figure, determine (a) the vertical shearing stress 3 in. below the top of the beam, and (b) the maximum vertical stress on the cross section.

Shearing Stress in Beams

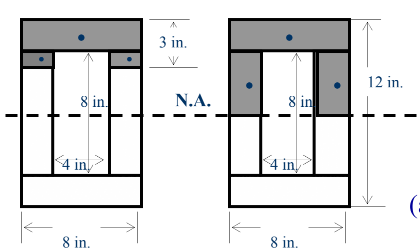
- Example 16 (cont'd)



Shearing Stress in Beams

- Example 16 (cont'd)

  From symmetry, the neutral axis is located 6 in. from either the top or bottom edge.



$$I = \frac{8(12)^3}{12} - \frac{4(8)^3}{12} = 981.3\ \text{in}^4$$

$$Q_{3''} = 8(2)(5) + 2[1(2)(3.5)] = 94.0\ \text{in}^3$$

$$Q_{NA} = 8(2)(5) + 2[2(2)(4)] = 112.0\ \text{in}^3$$

$$\text{(a) } \tau_{Q_{3''}} = \frac{VQ_{3''}}{It} = \frac{6000(94)}{981.3(4)} = \underline{143.7\ \text{psi}}$$

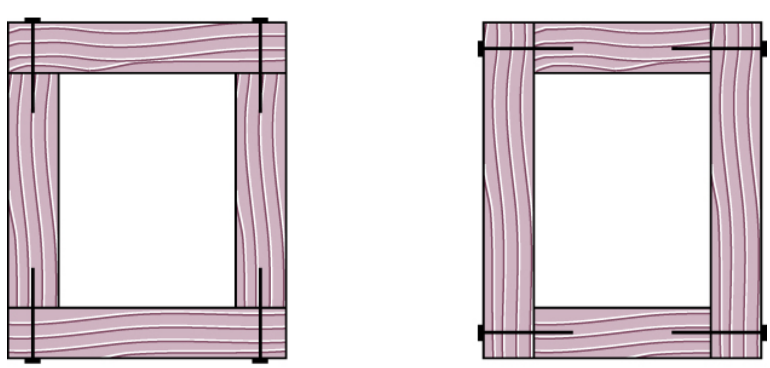
$$\text{(b) } \tau_{\max} = \frac{VQ_{\max}}{It} = \frac{6000(112)}{981.3(4)} = \underline{171.2\ \text{psi}}$$

Longitudinal Shear on a Beam Element of Arbitrary Shape

- Consider a box beam obtained by nailing together four planks as shown in Fig. 1.
- The shear per unit length (Shear flow) q on a horizontal surfaces along which the planks are joined is given by

$$q = \frac{VQ}{I} = shear\ \ flow \qquad (1)$$

Longitudinal Shear on a Beam Element of Arbitrary Shape



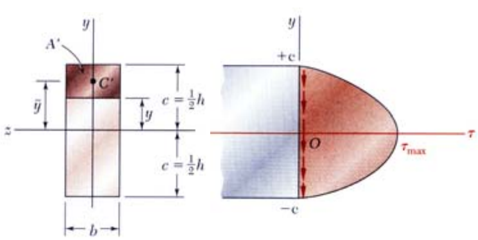
(a) (b)

Figure 1

Longitudinal Shear on a Beam Element of Arbitrary Shape

- But could q be determined if the planks had been joined along *vertical surfaces*, as shown in Fig. 1b?
- Previously, we had examined the distribution of the vertical components τ_{xy} of the stresses on a transverse section of a W-beam or an S-beam as shown in the following viewgraph.

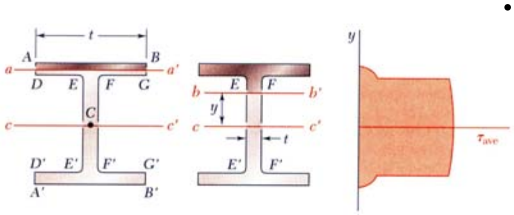
Shearing Stresses τ_{xy} in Common Types of Beams



- For a narrow rectangular beam,

$$\tau_{xy} = \frac{VQ}{Ib} = \frac{3}{2}\frac{V}{A}\left(1 - \frac{y^2}{c^2}\right)$$

$$\tau_{max} = \frac{3}{2}\frac{V}{A}$$



- For American Standard (S-beam) and wide-flange (W-beam) beams

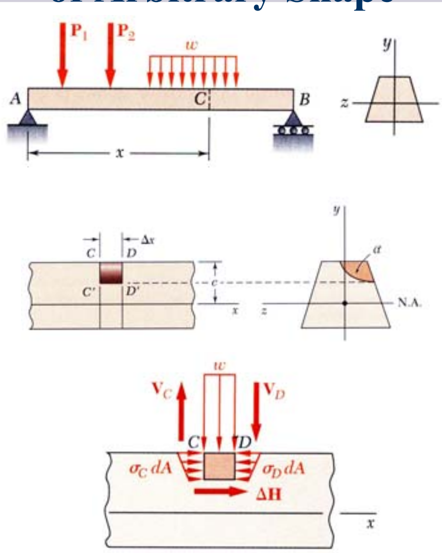
$$\tau_{ave} = \frac{VQ}{It}$$

$$\tau_{max} = \frac{V}{A_{web}}$$

Longitudinal Shear on A Beam Element of Arbitrary Shape

- But what about the *horizontal* component τ_{xz} of the stresses in the flanges?
- To answer these questions, the procedure developed earlier must be extended for the determination of the shear per unit length q so that it will apply to the cases just described.

Longitudinal Shear on a Beam Element of Arbitrary Shape



- We have examined the distribution of the vertical components τ_{xy} on a transverse section of a beam. We now wish to consider the horizontal components τ_{xz} of the stresses.
- Consider prismatic beam with an element defined by the curved surface CDD'C'.

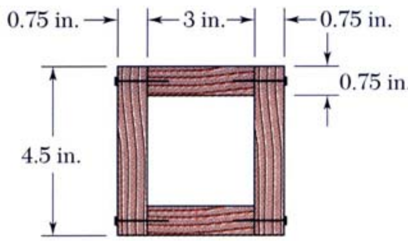
$$\sum F_x = 0 = \Delta H + \int_a (\sigma_D - \sigma_C) dA$$

- Except for the differences in integration areas, this is the same result obtained before which led to

$$\Delta H = \frac{VQ}{I}\Delta x \qquad q = \frac{\Delta H}{\Delta x} = \frac{VQ}{I}$$

Shearing Stress in Beams

- Example 17

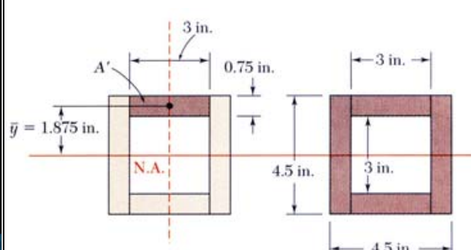


A square box beam is constructed from four planks as shown. Knowing that the spacing between nails is 1.75 in. and the beam is subjected to a vertical shear of magnitude $V = 600$ lb, determine the shearing force in each nail.

SOLUTION:

- Determine the shear force per unit length along each edge of the upper plank.
- Based on the spacing between nails, determine the shear force in each nail.

Example 17 (cont'd)



For the upper plank,

$$Q = A'y = (0.75\text{in.})(3\text{in.})(1.875\text{in.})$$
$$= 4.22\,\text{in}^3$$

For the overall beam cross-section,

$$I = \tfrac{1}{12}(4.5\,\text{in})^3 - \tfrac{1}{12}(3\,\text{in})^3$$
$$= 27.42\,\text{in}^4$$

SOLUTION:

- Determine the shear force per unit length along each edge of the upper plank.

$$q = \frac{VQ}{I} = \frac{(600\,\text{lb})(4.22\,\text{in}^3)}{27.42\,\text{in}^4} = 92.3\frac{\text{lb}}{\text{in}}$$

$$f = \frac{q}{2} = 46.15\frac{\text{lb}}{\text{in}}$$
$$= \text{edge force per unit length}$$

- Based on the spacing between nails, determine the shear force in each nail.

$$F = f\,\ell = \left(46.15\frac{\text{lb}}{\text{in}}\right)(1.75\,\text{in})$$

$$\boxed{F = 80.8\,\text{lb}}$$

Shearing Stress in Thin-Walled Members

- It was noted earlier that Eq. 1 can be used to determine the shear flow in an arbitrary shape of a beam cross section.
- This equation will be used in this section to calculate both the shear flow and the average shearing stress in thin-walled members such as flanges of wide-flange beams (Fig. 2) and box beams or the walls of structural tubes.

Shearing Stress in Thin-Walled Members

Wide-Flange Beams

Figure 2

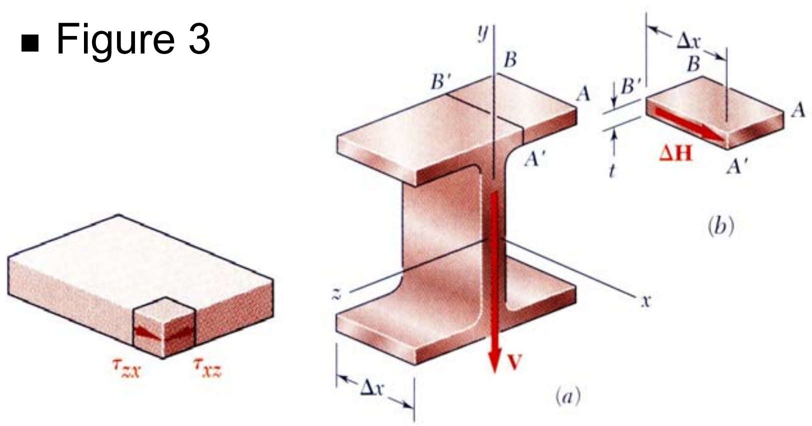
Shearing Stress in Thin-Walled Members

- Consider a segment of a wide-flange beam subjected to the vertical shear *V*.
- The longitudinal shear force on the element is

$$\Delta H = \frac{VQ}{I}\Delta x \qquad (2)$$

Shearing Stress in Thin-Walled Members

- Figure 3



Shearing Stress in Thin-Walled Members

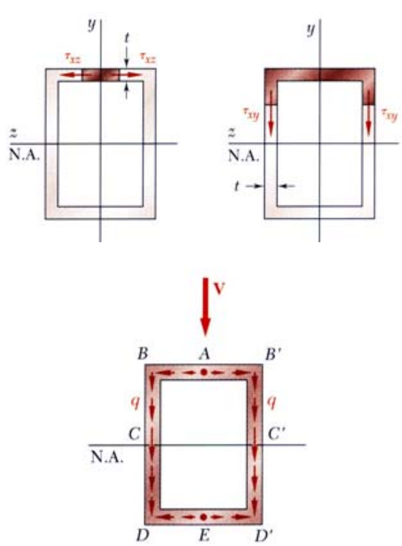
- The corresponding shear stress is

$$\tau_{zx} = \tau_{xz} \approx \frac{\Delta H}{t\,\Delta x} = \frac{VQ}{It} \qquad (3)$$

- Previously found a similar expression for the shearing stress in the web

$$\tau_{xy} = \frac{VQ}{It} \qquad (4)$$

Shearing Stress in Thin-Walled Members



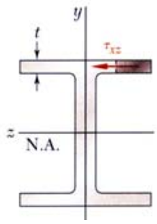
- The variation of shear flow across the section depends only on the variation of the first moment.

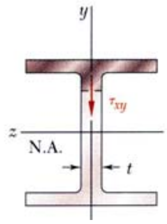
$$q = \tau t = \frac{VQ}{I}$$

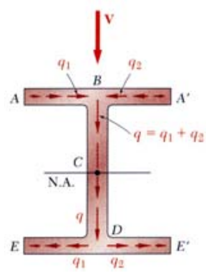
- For a box beam, q grows smoothly from zero at A to a maximum at C and C' and then decreases back to zero at E.

- The sense of q in the horizontal portions of the section may be deduced from the sense in the vertical portions or the sense of the shear V.

Shearing Stress in Thin-Walled Members





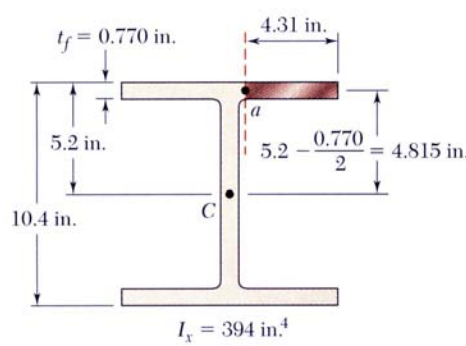


- For a wide-flange beam, the shear flow increases symmetrically from zero at *A* and *A'*, reaches a maximum at *C* and the decreases to zero at *E* and *E'*.
- The continuity of the variation in *q* and the merging of *q* from section branches suggests an analogy to fluid flow.

Shearing Stress in Thin-Walled Members

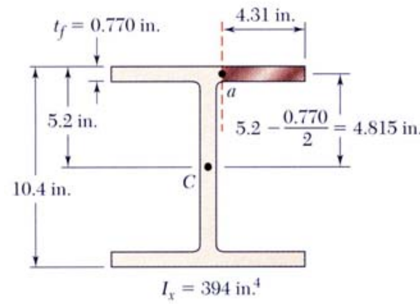
- Example 18

  Knowing that the vertical shear is 50 kips in a W10 × 68 rolled-steel beam, determine the horizontal shearing stress in the top flange at the point a.



Shearing Stress in Thin-Walled Members

- Example 18 (cont'd)



SOLUTION:

- For the shaded area,

$$Q = (4.31\,\text{in})(0.770\,\text{in})(4.815\,\text{in})$$
$$= 15.98\,\text{in}^3$$

- The shear stress at a,

$$\tau = \frac{VQ}{It} = \frac{(50\,\text{kips})(15.98\,\text{in}^3)}{(394\,\text{in}^4)(0.770\,\text{in})}$$

$$\boxed{\tau = 2.63\,\text{ksi}}$$