Shear on the Horizontal Face of a Beam Element

- Consider prismatic beam
- For equilibrium of beam element
  \[ \sum F_x = 0 = \Delta H + \int (\sigma_D - \sigma) \, dA \]
  \[ \Delta H = \frac{M_D - M_C}{l} \int y \, dA \]
- Note,
  \[ Q = \int y \, dA \]
  \[ M_D - M_C = \frac{dM}{dx} \Delta x = V \Delta x \]
- Substituting,
  \[ \Delta H = \frac{VQ}{l} \Delta x \]
  \[ q = \frac{\Delta H}{\Delta x} = \frac{VQ}{I} = \text{shear flow} \]
**Shear on the Horizontal Face of a Beam Element**

- Shear flow,
  \[ q = \frac{\Delta H}{\Delta x} = \frac{VQ}{I} \] = shear flow

- where
  \[ Q = \int y \, dA \] = first moment of area above \( y_1 \)
  \[ I = \int y^2 \, dA \] = second moment of full cross section

- Same result found for lower area
  \[ q' = \frac{\Delta H'}{\Delta x} = \frac{VQ'}{I'} = -q' \]
  \[ Q + Q' = 0 \] = first moment with respect to neutral axis
  \[ \Delta H' = -\Delta H \]

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**Shearing Stress in Beams**

- **Example 16**
  The transverse shear \( V \) at a certain section of a timber beam is 600 lb. If the beam has the cross section shown in the figure, determine (a) the vertical shearing stress 3 in. below the top of the beam, and (b) the maximum vertical stress on the cross section.
Shearing Stress in Beams

- Example 16 (cont’d)

From symmetry, the neutral axis is located 6 in. from either the top or bottom edge.

\[ I = \frac{8(12)^3}{12} - \frac{4(8)^3}{12} = 981.3 \text{ in}^4 \]

\[ Q_v = 8(2)(5) + 2[2(2)(3.5)] = 94.0 \text{ in}^3 \]

\[ Q_{NAd} = 8(2)(5) + 2[2(2)(4)] = 112.0 \text{ in}^3 \]

(a) \[ \tau_{Qv} = \frac{VQ_v}{It} = \frac{6000(94)}{981.3(4)} = 143.7 \text{ psi} \]

(b) \[ \tau_{\text{max}} = \frac{VQ_{\text{max}}}{It} = \frac{6000(112)}{981.3(4)} = 171.2 \text{ psi} \]
Longitudinal Shear on a Beam Element of Arbitrary Shape

- Consider a box beam obtained by nailing together four planks as shown in Fig. 1.
- The shear per unit length (Shear flow) $q$ on a horizontal surfaces along which the planks are joined is given by

$$q = \frac{VQ}{I} = \text{shear flow}$$  \hspace{1cm} (1)
Longitudinal Shear on a Beam
Element of Arbitrary Shape

- But could $q$ be determined if the planks had been joined along vertical surfaces, as shown in Fig. 1b?

- Previously, we had examined the distribution of the vertical components $\tau_{xy}$ of the stresses on a transverse section of a W-beam or an S-beam as shown in the following viewgraph.

Shearing Stresses $\tau_{xy}$ in Common Types of Beams

- For a narrow rectangular beam,
  
  \[ \tau_{xy} = \frac{VQ}{lb} = \frac{3V}{2A} \left( 1 - \frac{y^2}{c^2} \right) \]
  
  \[ \tau_{\text{max}} = \frac{3V}{2A} \]

- For American Standard (S-beam) and wide-flange (W-beam) beams
  
  \[ \tau_{\text{ave}} = \frac{VQ}{It} \]
  
  \[ \tau_{\text{max}} = \frac{V}{A_{\text{web}}} \]
Longitudinal Shear on A Beam Element of Arbitrary Shape

- But what about the horizontal component $\tau_{xz}$ of the stresses in the flanges?

- To answer these questions, the procedure developed earlier must be extended for the determination of the shear per unit length $q$ so that it will apply to the cases just described.

Longitudinal Shear on a Beam Element of Arbitrary Shape

- We have examined the distribution of the vertical components $\tau_{yy}$ on a transverse section of a beam. We now wish to consider the horizontal components $\tau_{xz}$ of the stresses.

- Consider prismatic beam with an element defined by the curved surface CDD'C'.

$$\sum F_x = 0 = \Delta H + \int_{a}^{b} (\sigma_D - \sigma_C) dA$$

- Except for the differences in integration areas, this is the same result obtained before which led to

$$\Delta H = \frac{V Q}{I} \Delta x \quad q = \frac{\Delta H}{\Delta x} = \frac{V Q}{I}$$
**Shearing Stress in Beams**

- **Example 17**

A square box beam is constructed from four planks as shown. Knowing that the spacing between nails is 1.75 in. and the beam is subjected to a vertical shear of magnitude \( V = 600 \text{ lb} \), determine the shearing force in each nail.

**SOLUTION:**

- Determine the shear force per unit length along each edge of the upper plank.

- Based on the spacing between nails, determine the shear force in each nail.

**Example 17 (cont’d)**

For the upper plank,

\[
Q = A' y = (0.75\text{ in.})(3\text{ in.})(1.875\text{ in.}) = 4.22\text{ in}^3
\]

For the overall beam cross-section,

\[
I = \frac{1}{12}(4.5\text{ in.})^3 - \frac{1}{12}(3\text{ in.})^3 = 27.42\text{ in}^4
\]

**SOLUTION:**

- Determine the shear force per unit length along each edge of the upper plank.

\[
q = \frac{VQ}{T} = \left(\frac{600\text{ lb}}{27.42\text{ in}^4}\right) = 21.93\text{ lb/in}
\]

\[
f = \frac{q}{2} = \frac{21.93\text{ lb/in}}{2} = 10.965\text{ lb/in}
\]

- Based on the spacing between nails, determine the shear force in each nail.

\[
F = f t = \left(10.965\text{ lb/in}\right)(1.75\text{ in.}) = 19.32\text{ lb}
\]

\[
F = 80.8\text{ lb}
\]
Shearing Stress in Thin-Walled Members

- It was noted earlier that Eq. 1 can be used to determine the shear flow in an arbitrary shape of a beam cross section.
- This equation will be used in this section to calculate both the shear flow and the average shearing stress in thin-walled members such as flanges of wide-flange beams (Fig. 2) and box beams or the walls of structural tubes.
Shearing Stress in Thin-Walled Members

- Consider a segment of a wide-flange beam subjected to the vertical shear $V$.
- The longitudinal shear force on the element is

$$\Delta H = \frac{VQ}{I} \Delta x \quad (2)$$

Shearing Stress in Thin-Walled Members

- Figure 3
Shearing Stress in Thin-Walled Members

- The corresponding shear stress is

\[ \tau_{zx} = \tau_{xz} \approx \frac{\Delta H}{t \Delta x} = \frac{VQ}{It} \]  \hspace{1cm} (3)

- Previously found a similar expression for the shearing stress in the web

\[ \tau_{xy} = \frac{VQ}{It} \] \hspace{1cm} (4)

Shearing Stress in Thin-Walled Members

- The variation of shear flow across the section depends only on the variation of the first moment.

\[ q = \frac{VQ}{T} \]

- For a box beam, \( q \) grows smoothly from zero at \( A \) to a maximum at \( C \) and \( C' \) and then decreases back to zero at \( E \).

- The sense of \( q \) in the horizontal portions of the section may be deduced from the sense in the vertical portions or the sense of the shear \( V \).
Shearing Stress in Thin-Walled Members

- For a wide-flange beam, the shear flow increases symmetrically from zero at \( A \) and \( A' \), reaches a maximum at \( C \) and the decreases to zero at \( E \) and \( E' \).

- The continuity of the variation in \( q \) and the merging of \( q \) from section branches suggests an analogy to fluid flow.

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Example 18

Knowing that the vertical shear is 50 kips in a W10 × 68 rolled-steel beam, determine the horizontal shearing stress in the top flange at the point \( a \).
Shearing Stress in Thin-Walled Members

- **Example 18 (cont'd)**

**SOLUTION:**

- For the shaded area,
  \[ Q = (4.31\text{in})(0.770\text{in})(4.815\text{in}) \]
  \[ = 15.98\text{in}^3 \]

- The shear stress at \( a \),
  \[ \tau = \frac{VQ}{It} = \frac{(50\text{kip})(15.98\text{in}^3)}{(394\text{in}^4)(0.770\text{in})} \]
  \[ \tau = 2.65\text{kpsi} \]