LECTURE

13

Chapter 5.3

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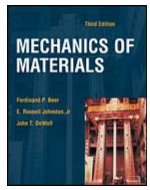
Third Edition

BEAMS: SHEAR AND MOMENT DIAGRAMS (GRAPHICAL)

A. J. Clark School of Engineering •Department of Civil and Environmental Engineering



by

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ENES 220 – Mechanics of Materials

Department of Civil and Environmental Engineering

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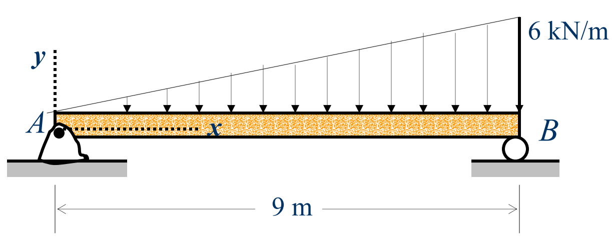
Shear Forces and Bending Moments in Beams

- Example 8

The beam is loaded and supported as shown in the figure. Write equations for the shear *V* and bending moment *M* for any section of the beam in the interval *AB.*



Shear Forces and Bending Moments in Beams

- Example 8 (cont'd)

A free-body diagram for the beam is shown Fig. 17. The reactions shown on the diagram are determined from equilibrium equations as follows:

$$\circlearrowleft + \sum M_B = 0;\ R_A(9) - \left(\frac{6\times 9}{2}\right)\left(9\times\frac{1}{3}\right) = 0$$

$$\therefore R_A = 9\ \text{kN}$$

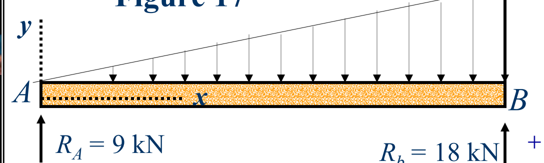
$$+\uparrow \sum F_y - 0;\ R_B + 9 - \frac{6\times 9}{2} = 0$$

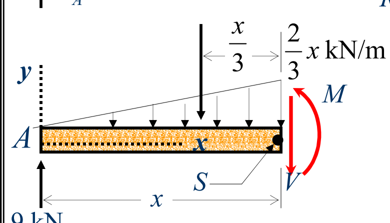
$$\therefore R_B = 18\ \text{kN}$$

Shear Forces and Bending Moments in Beams

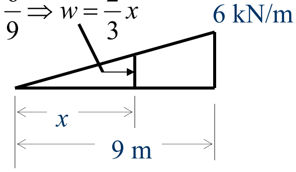
- Example 8 (cont'd)

Figure 17





$$\frac{w}{x} = \frac{6}{9} \Rightarrow w = \frac{2}{3}x$$



$$+\uparrow \sum F_y = 0;\ -V + 9 - \left(\frac{2}{3}x\right)(x)\frac{1}{2} = 0$$

$$\boxed{\therefore V = 9 - \frac{x^2}{3} \quad \text{for } 0 < x < 9}$$

$$\circlearrowleft + \sum M_S = 0;\ -M + 9x - \frac{2}{3}x\left(\frac{x}{2}\right)\left(\frac{x}{3}\right) = 0$$

$$\boxed{\therefore M = 9x - \frac{x^3}{9} \quad \text{for } 0 < x < 9}$$

Shear Forces and Bending Moments in Beams

- Example 9

  A timber beam is loaded as shown in Fig. 18a. The beam has the cross section shown in Fig.18.b. On a transverse cross section 1 ft from the left end, determine

  a) The flexural stress at point *A* of the cross section

  b) The flexural stress at point *B* of the cross section.

Shear Forces and Bending Moments in Beams

- Example 9 (cont'd)

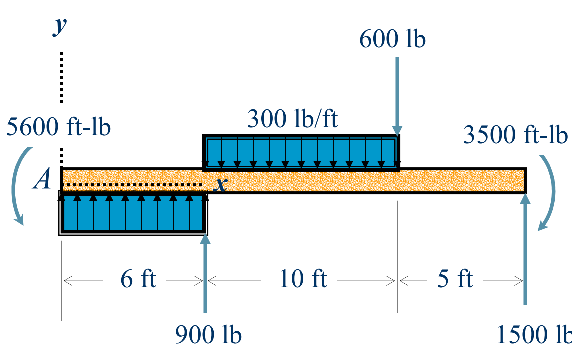


Figure 18a

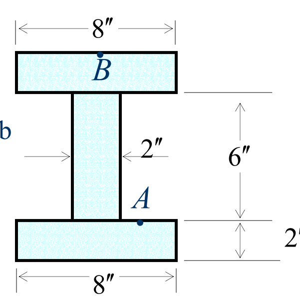
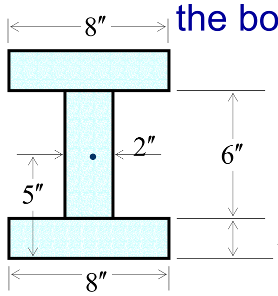


Figure 18b

Shear Forces and Bending Moments in Beams

- Example 9 (cont'd)

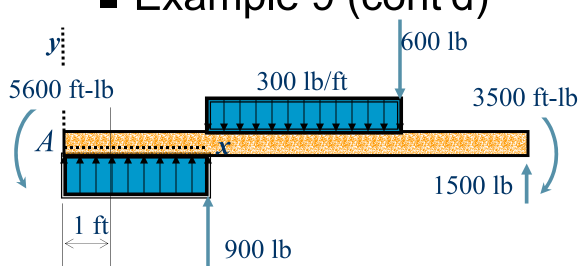
First, we have to determine the moment of inertia I_x. From symmetry, the neutral axis is located at a distance y = 5 in. either from the bottom or the upper edge. Therefore,

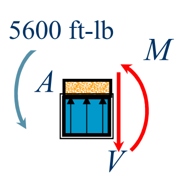


$$I_x = 2\left[\frac{8(5)^3}{3} - \frac{6(3)^3}{3}\right] = 558.7\ \text{in}^4$$

Shear Forces and Bending Moments in Beams

- Example 9 (cont'd)





$$+\uparrow \sum F_y = 0; -V + 1(200) = 0$$

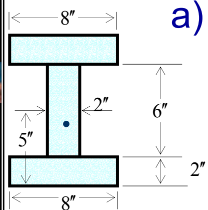
$$\therefore V = 200\ \text{lb} \quad \text{for } 0 < x < 6$$

$$+ \sum M_S = 0; -M - 5600 + (1)(200)(0.5) = 0$$

$$\therefore M = -5600 + 100 = -5500\ \text{ft-lb} \quad \text{at } x = 1\ \text{ft}$$

Shear Forces and Bending Moments in Beams

- Example 9 (cont'd)



a) Stress at *A*

$$\sigma_A = -\frac{M_r y}{I_x} = -\frac{(-5500\times 12)(-3)}{558.7} = -354.4\text{ psi}$$

$$= \underline{354.4\text{ psi Compression (C)}}$$

b) Stress at *B*

$$\sigma_B = -\frac{M_r y}{I_x} = -\frac{(-5500\times 12)(5)}{558.7} = +591\text{ psi}$$

$$= \underline{591\text{ psi Tension (T)}}$$

Shear Forces and Bending Moments in Beams

- Load, Shear Force, and Bending Moment Relationships
  - In cases where a beam is subjected to several concentrated forces, couples, and distributed loads, the equilibrium approach discussed previously can be tedious because it would then require several cuts and several free-body diagrams.
  - In this section, a simpler method for constructing shear and moment diagrams are discussed.

Shear Forces and Bending Moments in Beams

- Load, Shear Force, and Bending Moment Relationships

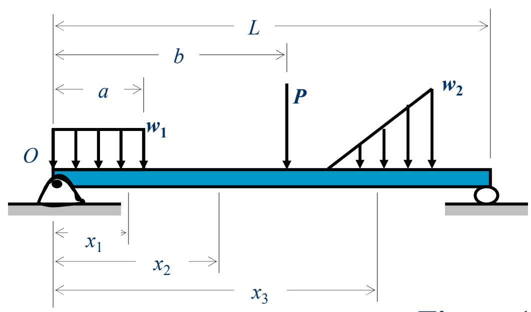


Figure 19

Shear Forces and Bending Moments in Beams

- Load, Shear Force, and Bending Moment Relationships
  - The beam shown in the Figure 20 is subjected to an arbitrary distributed loading $w = w(x)$ and a series of concentrated forces and couple moments.
  - We will consider the distributed load w to positive when the loading acts upward as shown in Figure 20.

Shear Forces and Bending Moments in Beams

- Load, Shear Force, and Bending Moment Relationships

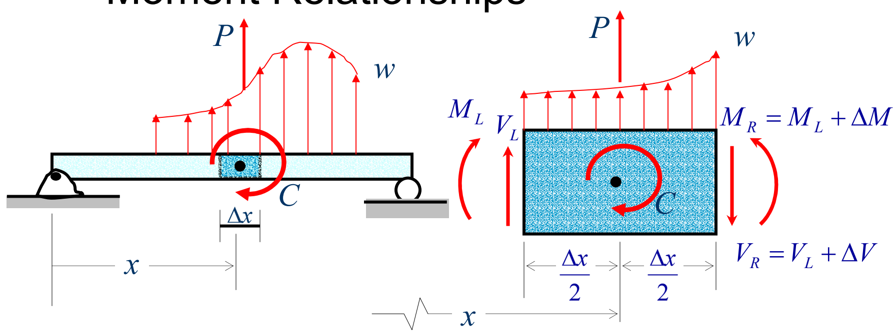


Figure 20a Figure 20b

Shear Forces and Bending Moments in Beams

- Load, Shear Force, and Bending Moment Relationships
  - In reference to Fig. 20a at some location *x*, the beam is acted upon by a distributed load *w*(*x*), a concentrated load force *P*, and a concentrated couple *C*.
  - A free-body diagram segment of the beam centered at the location *x* is shown in Figure 20b.

Shear Forces and Bending Moments in Beams

- Load, Shear Force, and Bending Moment Relationships

The element must be in equilibrium, therefore,

$$+\uparrow \sum F_y = 0; V_L + w_{\text{avg}}\Delta x + P - \left(V_L + \Delta V\right) = 0$$

From which

$$\Delta V = P + w_{\text{avg}}\Delta x \qquad (28)$$

Shear Forces and Bending Moments in Beams

- Load, Shear Force, and Bending Moment Relationships

In Eq. 28, if the concentrated force *P* and distributed force *w* are both zero in some region of the beam, then

$$\Delta V = 0 \qquad \text{or} \qquad V_L = V_R \qquad (29)$$

This implies that the shear force is constant in any segment of the beam where there are no loads.

Shear Forces and Bending Moments in Beams

- Load, Shear Force, and Bending Moment Relationships

If the concentrated load *P* is not zero, then in the limit as $\Delta x \to 0$,

$$\Delta V = P \qquad \text{or} \qquad V_R = V_L + P \qquad (30)$$

That is, across any concentrated load *P*, the shear force graph (shear force versus *x*) jumps by the amount of the concentrated load.

Shear Forces and Bending Moments in Beams

- Load, Shear Force, and Bending Moment Relationships

Furthermore, moving from left to right along the beam, the shear force graph jumps in the direction of the concentrated load.

If the concentrated load is zero, then in the limit as $\Delta x \to 0$, we have

$$\Delta V = w_{\text{avg}} \Delta x \to 0 \qquad (31)$$

Shear Forces and Bending Moments in Beams

- Load, Shear Force, and Bending Moment Relationships

And the shear force is a continuous function at *x*. Dividing through by Δx in Eq. 31, gives

$$\lim_{\Delta x \to 0} \frac{\Delta V}{\Delta X} = \frac{dV}{dx} = w \qquad (32)$$

That is, ***the slope of the shear force graph at any section x in the beam is equal to the intensity of loading at that section of the beam***.

Shear Forces and Bending Moments in Beams

- Load, Shear Force, and Bending Moment Relationships

Moving from left to right along the beam, if the distributed force is upward, the slope of the shear force graph ($dV/dx = w$) is positive and the shear force graph is increasing (moving upward).

If the distributed force is zero, then the slope of the shear graph ($dV/dx = 0$) and the shear force is constant.

Shear Forces and Bending Moments in Beams

- Load, Shear Force, and Bending Moment Relationships

  In any region of the beam in which Eq. 32 is valid (any region in which there are no concentrated loads), the equation can be integrated between definite limits to obtain

$$\Delta V = V_2 - V_1 = \int_{V_1}^{V_2} dV = \int_{x_1}^{x_2} w\,dx$$

Shear Forces and Bending Moments in Beams

- Load and Shear Force Relationships

$$\frac{dV}{dx} = w(x) \qquad (33)$$

Slope of Shear Diagram = Distributed Load Intensity

Shear Forces and Bending Moments in Beams

- Load and Shear Force Relationships

$$\Delta V_{2-1} = V_2 - V_1 = \int_{V_1}^{V_2} dV = \int_{x_1}^{x_2} w(x)\, dx \qquad (34)$$

Change in Shear = Area under Loading Curve between x_1 and x_2

Shear Forces and Bending Moments in Beams

- Load, Shear Force, and Bending Moment Relationships

Similarly, applying moment equilibrium to the free-body diagram of Fig. 20b we obtain

$$+\sum M_c = 0; -\left(M_L + \Delta M\right) - M_L - V_L \frac{\Delta x}{2} - \left(V_L + \Delta V\right)\frac{\Delta x}{2} + a\left(w_{\text{avg}} \Delta x\right) = 0$$

From which

$$\Delta M = C + V_L \Delta x + \Delta V \frac{\Delta x}{2} - a\left(w_{\text{avg}} \Delta x\right) \qquad (35)$$

Shear Forces and Bending Moments in Beams

- Load, Shear Force, and Bending Moment Relationships

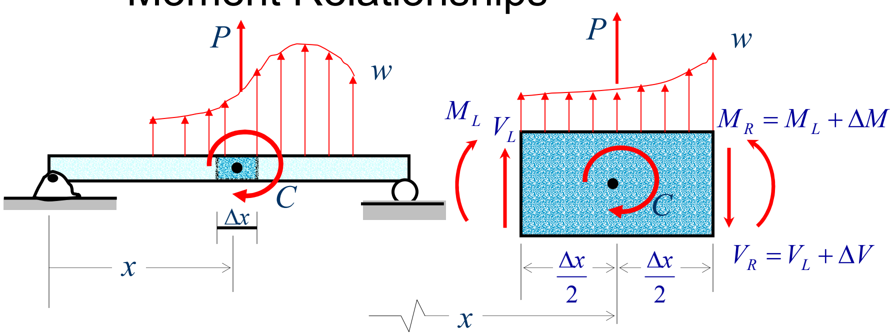


Figure 20a **Figure 20b**

Shear Forces and Bending Moments in Beams

- Load, Shear Force, and Bending Moment Relationships

  In which $-\Delta x/2 < a < \Delta x/2$ and in the limit as $\Delta x \to 0$, $a \to 0$ and $w_{avg} \to w$.

  Three important relationships are clear from Eq. 35. First, if the concentrated couple is not zero, then in the limit as $\Delta x \to 0$,

$$\Delta M = C \qquad \text{or} \qquad M_R = M_L + C \quad (36)$$

Shear Forces and Bending Moments in Beams

- Load, Shear Force, and Bending Moment Relationships

  That is, across any concentrated couple *C*, the bending moment graph (bending moment versus *x*) jumps by the amount of the concentrated couple.

  Furthermore, moving from left to right along the beam, the bending moment graph jumps upward for a clockwise concentrated couple and jumps downward for a concentrated counterclockwise *C*.

Shear Forces and Bending Moments in Beams

- Load, Shear Force, and Bending Moment Relationships

  Second, if the concentrated couple *C* and concentrated force *P* are both zero, then in the limit as $\Delta x \rightarrow 0$, we have

$$\Delta M = V_L \Delta x + \Delta V \frac{\Delta x}{2} - a\left(w_{\text{avg}} \Delta x\right) \rightarrow 0 \qquad (37)$$

  and dividing by Δx, gives

$$\lim_{\Delta x \rightarrow 0} \frac{\Delta M}{\Delta x} = \frac{dM}{dx} = V \qquad (38)$$

Shear Forces and Bending Moments in Beams

- Load, Shear Force, and Bending Moment Relationships

  That is, ***the slope of the bending moment graph at any location x in the beam is equal to the value of the shear force at that section of the beam***.

  Moving from left to right along the beam, if *V* is positive, then *dM/dx* = *V* is positive, and the bending moment graph is increasing

Shear Forces and Bending Moments in Beams

- Load, Shear Force, and Bending Moment Relationships
  - In the region of the beam I which Eq. 38 is valid (any region in which there are no concentrated loads or couples), the equation can be integrated between definite limits to get

$$\Delta M = M_2 - M_1 = \int_{M_1}^{M_2} dM = \int_{x_1}^{x_2} V\,dx \qquad (39)$$

Shear Forces and Bending Moments in Beams

- Shear Force and Bending Moment Relationships

$$\frac{dM}{dx} = V \qquad (40)$$

Slope of Moment Diagram = Shear

Shear Forces and Bending Moments in Beams

- Shear Force and Bending Moment Relationships

$$\Delta M_{2-1} = M_2 - M_1 = \int_{M_1}^{M_2} dM = \int_{x_1}^{x_2} V\, dx \qquad (41)$$

Change in Moment = Area under Shear diagram between x_1 and x_2

Shear Forces and Bending Moments in Beams

- Shear and Moment Diagrams
  - Procedure for Analysis
    - Support Reactions
      - Determine the support reactions and resolve the forces acting on the beam into components which are perpendicular and parallel to the beam's axis
    - Shear Diagram
      - Establish the *V* and *x* axes and plot the values of the shear at two ends of the beam. Since $dV/dx = w$, the slope of the shear diagram at any point is equal to the intensity of the distributed loading at the point.

Shear Forces and Bending Moments in Beams

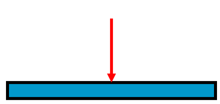
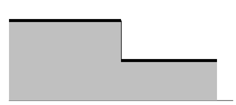
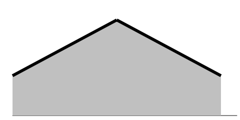
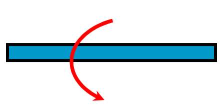
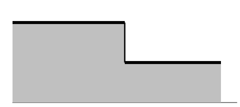
- Shear and Moment Diagrams
      - If a numerical value of the shear is to be determined at the point, one can find this value either by using the methods of establishing equations (formulas)for each section under study or by using Eq. 34, *which states that the change in the shear force is equal to the area under the distributed loading diagram*. Since $w(x)$ is integrated to obtain V, if $w(x)$ is a curve of degree n, then $V(x)$ will be a curve of degree $n + 1$. For example, if $w(x)$ is uniform, $V(x)$ will be linear.
    - Moment Diagram
      - Establish the *M* and *x* axes and plot the values of the moment at the ends of the beam.

Shear Forces and Bending Moments in Beams

- Shear and Moment Diagrams
  - Since *dM/dx* = *V*, the slope of the moment diagram at any point is equal to the intensity of the shear at the point. In particular, note that at the point where the shear is zero, that is *dM/dx* = 0, and therefore this may be a point of maximum or minimum moment. If the numerical value of the moment is to be determined at a point, one can find this value either by using the method of establishing equations (formulas) for each section under study or by using Eq. 41, *which states that the change in the moment is equal to the area under the shear diagram.* Since *w*(*x*) is integrated to obtain *V*, if *w*(*x*) is a curve of degree *n*, then *V*(*x*) will be a curve of degree *n* + 1. For example, if *w*(*x*) is uniform, *V*(*x*) will be linear.

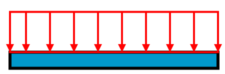
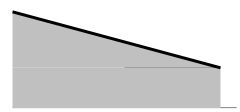
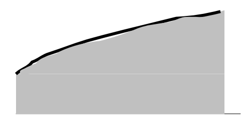
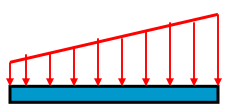
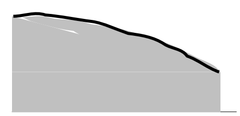
Shear Forces and Bending Moments in Beams

- Shear and Moment Diagrams

Loading	Shear Diagram, $\frac{dV}{dx} = w$	Moment Diagram, $\frac{dM}{dx} = V$
		
		

Shear Forces and Bending Moments in Beams

- Shear and Moment Diagrams

Loading	Shear Diagram, $\frac{dV}{dx} = w$	Moment Diagram, $\frac{dM}{dx} = V$
		
		

Shear and Moment Diagrams

- Example 10

  Draw the shear and bending moment diagrams for the beam shown in Figure 21a.

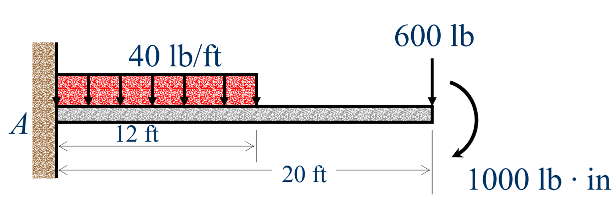


Figure 21a

Shear and Moment Diagrams

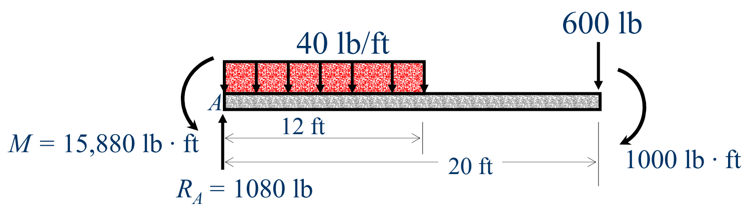
- Example 10 (cont'd)
  - Support Reactions
    - The reactions at the fixed support can be calculated as follows:

Figure 21b

$$+\uparrow \sum F_y = 0;\ R_A - 40(12) - 600 = 0 \rightarrow R_A = 1080 \text{ lb}$$

$$+ \ \sum M_A = 0;\ -M + 40(12)(6) + 600(20) + 1000 = 0$$

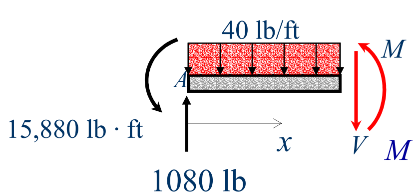
$$\therefore M = 15{,}880 \text{ lb} \cdot \text{in}$$



Shear and Moment Diagrams

- Example 10 (cont'd)
  - Shear Diagram
    - Using the established sign convention, the shear at the ends of the beam is plotted first. For example, when $x = 0$, $V = 1080$; and when $x = 20$, $V = 600$

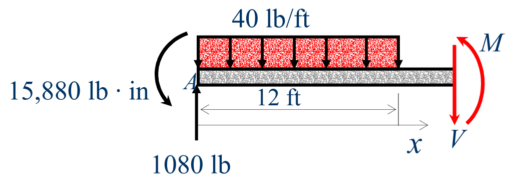
$$V(x) = 1080 - 40x \qquad \text{for } 0 < x < 12$$



$$M = 1080x - 40\frac{x^2}{2} - 15{,}880 \quad \text{for } 0 < x < 12$$

Shear and Moment Diagrams

- Example 10 (cont'd)

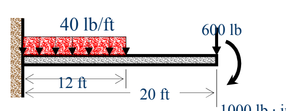


$$V(x) = 600 \qquad \text{for } 12 < x < 20$$

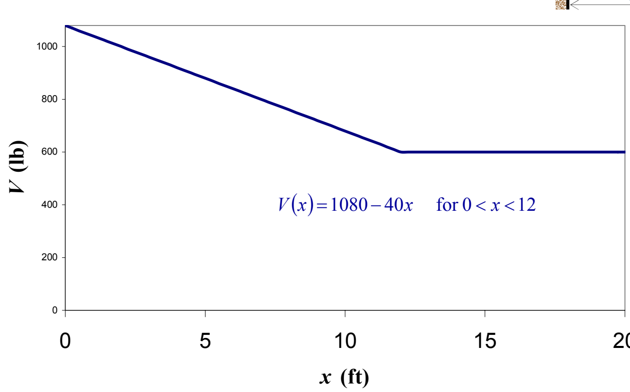
$$M = -15{,}880 - 40(12)(x - 6) + 1080x \qquad \text{for } 12 < x < 20$$

Shear and Moment Diagrams

- Example 10 (cont'd)

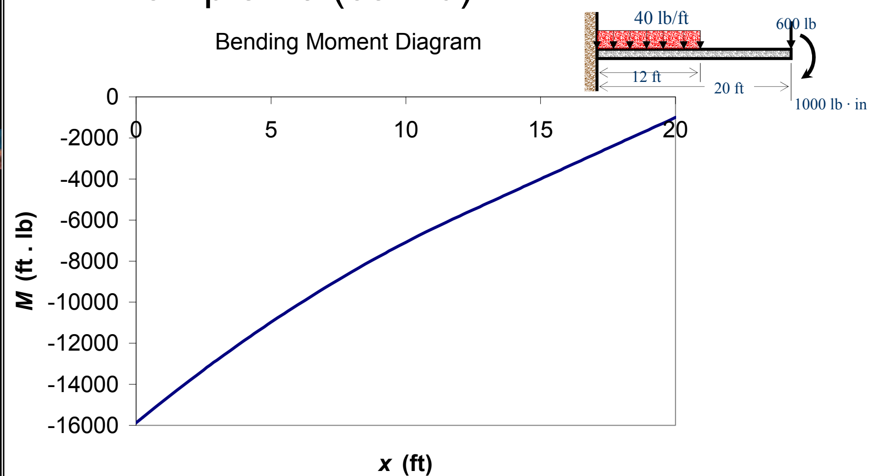


Shear Digram



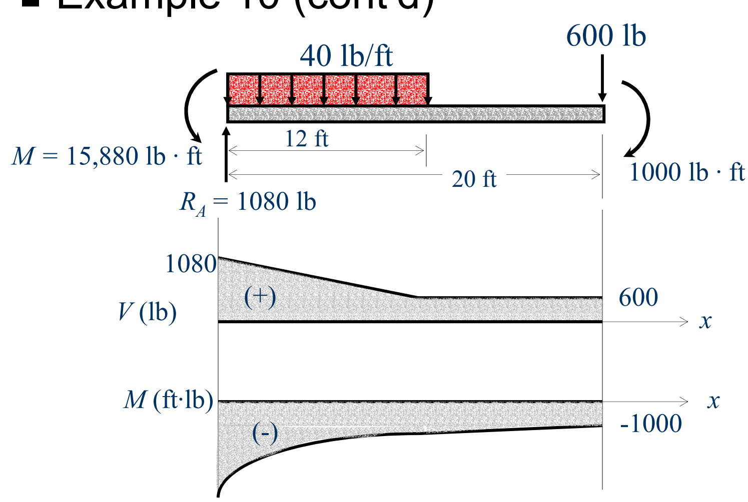
Shear and Moment Diagrams

- Example 10 (cont'd)



Shear and Moment Diagrams

- Example 10 (cont'd)



Shear and Moment Diagrams

- Example 11

  Draw complete shear and bending moment diagrams for the beam shown in Fig. 22

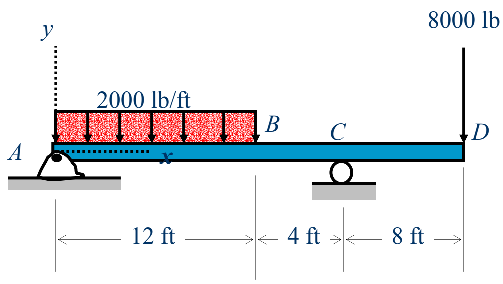


Figure 22a

Shear and Moment Diagrams

- Example 11 (cont'd)
  - The support reactions were computed from equilibrium as shown in Fig. 22.b.

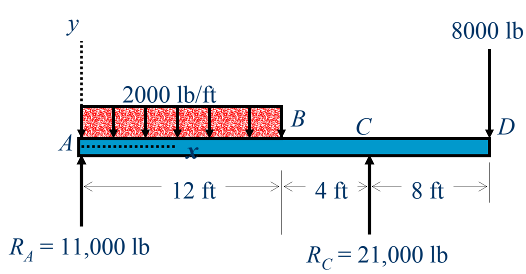
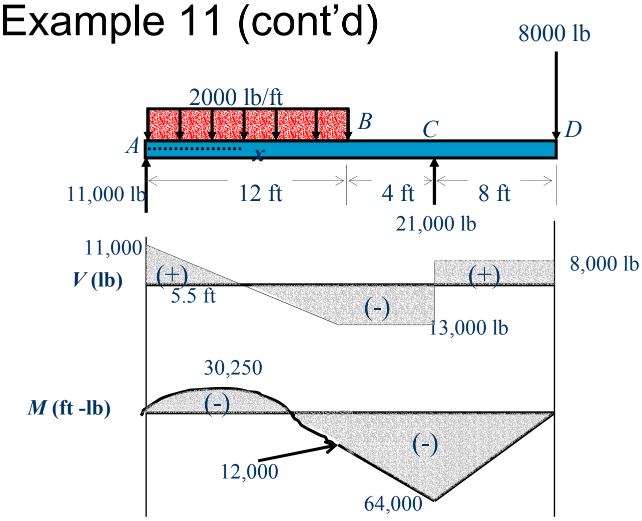


Figure 22a

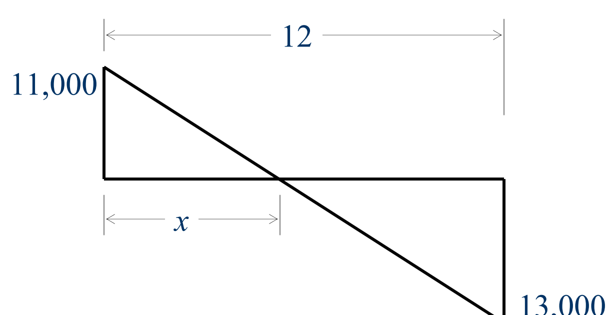
Shear and Moment Diagrams

- Example 11 (cont'd)



Shear and Moment Diagrams

- Example 11 (cont'd)



$$\frac{x}{11{,}000} = \frac{12 - x}{13{,}000} \Rightarrow x = \frac{12}{2.18} = 5.5$$