LECTURE

12

Chapter 5.1 – 5.2

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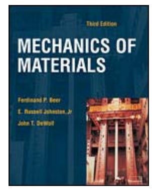
BEAMS: SHEAR AND MOMENT DIAGRAMS (FORMULA)

A. J. Clark School of Engineering •Department of Civil and Environmental Engineering



by
Dr. Ibrahim A. Assakkaf
SPRING 2003
ENES 220 – Mechanics of Materials
Department of Civil and Environmental Engineering
University of Maryland, College Park

Second Moments of Areas

- Moment of Inertia
  - There are many important topics in engineering practice that require evaluation of an integral of the second moment of *area* or *moment of inertia* of the type

$$\int x^2 \, dA \qquad (21)$$

Second Moments of Areas

- Moment of Inertia
  - The integral of Eq. 21 is referred to as the moment of inertia for an area.
  - Methods used to determine the area moment of inertia will be discussed briefly in this chapter.
  - Full treatment of this important topic can be found in almost every standard "statics" book.

Second Moments of Areas

- Moment of Inertia
  - Consider an area A located in the xy plane as shown in the figure.

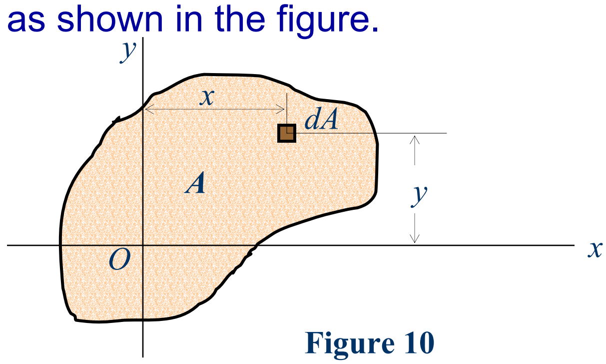


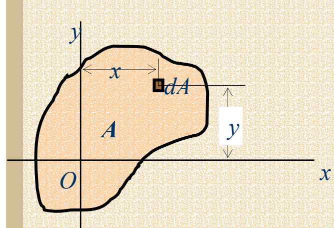
Figure 10

Second Moments of Areas

- Moment of Inertia
  - Also consider the element of area dA of coordinates x and y.
  - The second moment, or moment of inertia of the area A (see Fig. 10) with respect to the x axis, and the second moment, or moment of inertia with respect to the y axis are defined, respectively, as provided in the next viewgraph:

Second Moments of Areas

- Moment of Inertia



$$I_x = \int_A y^2 dA \quad \text{(22a)}$$

$$I_y = \int_A x^2 dA \quad \text{(22b)}$$

Where
I_x = moment of inertia with respect to x axis
I_y = moment of inertia with respect to y axis

Second Moments of Areas

- Moment of Inertia
  - The quantities I_x and I_y are referred to as *rectangular moments of inertia*, since they are computed from the rectangular coordinates of the element *dA*.
  - While each integral is basically a double integral, it is possible in many applications to select elements of area *dA* in the shape of thin horizontal or vertical strips.

Second Moments of Areas

- Polar Moment of Inertia
  - The second moment, or polar moment of inertia of an area with respect to an axis perpendicular to the plane of the area is denoted by the symbol *J*.
  - For example, the second moment of area *A* shown in Fig. 11 with respect to a *z*-axis that is perpendicular to the plane of the area at *O* of the *xy*-coordinate system is:

Second Moments of Areas

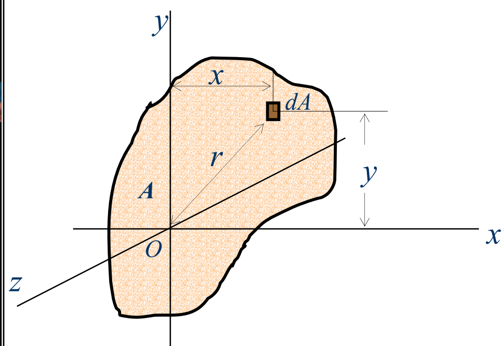


Figure 11

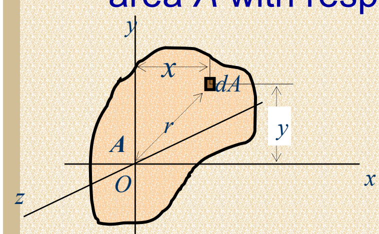
- Polar Moment of Inertia

$$J_z = \int_A r^2 dA$$
$$= \int_A \left(x^2 + y^2\right) dA$$
$$= \int_A x^2 dA + \int_A y^2 dA$$
$$= I_x + I_y \qquad (23)$$

Second Moments of Areas

- Polar Moment of Inertia

The second moment or polar moment of area *A* with respect to *z* axis is given by



$$J_z = I_x + I_y \qquad (24)$$

Second Moments of Areas

- Polar Moment of Inertia
  - The quantity J_z is known as the polar second moment of the area *A* and was used in Chapter 7 in the calculation of the stress in circular shafts transmitting torques.
  - For circular shaft of radius *r*,

$$J_z = \frac{\pi}{2} r^4 \qquad (25)$$

Second Moments of Areas

- Radius of Gyration of an Area
  - The radius of gyration of planar area has units of length and is a quantity often used for the design of columns in structural mechanics.
  - Provided the areas and moments of inertia are known, the radii of gyration are determined from the following formulas:

Second Moments of Areas

- Radii of Gyration of an Area

$$k_x = \sqrt{\frac{I_x}{A}} \qquad (26a)$$

$$k_y = \sqrt{\frac{I_y}{A}} \qquad (26b)$$

$$k_z = \sqrt{\frac{I_z}{A}} \qquad (26c)$$

Second Moments of Areas

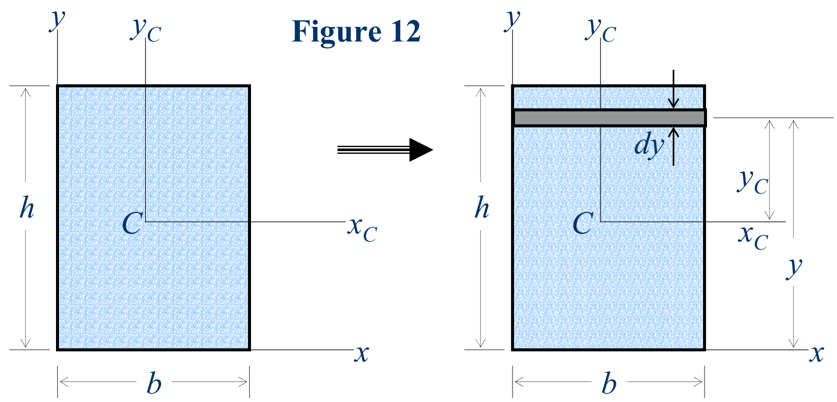
- Example 1

  Determine the moment of inertia for the rectangular area shown in Fig. 11 with respect to (a) the centroidal x_C axis, (b) the axis x passing through the base of the rectangle, and (c) the pole or z_C axis perpendicular to the x_C-y_C plane and passing through the centroid C.

Second Moments of Areas

- Example 1 (cont'd)

Figure 12



Second Moments of Areas

- Example 1 (cont'd)
  a) We select as an element of area a horizontal strip of length *b* and thickness dy_C. Therefore,

$$dI_{x_C} = y_C^2 dA = y_C^2 (b\, dy_C)$$

Integrating from $y_C = -h/2$ to $y_C = +h/2$, we obtain

$$I_{x_C} = \int_{-\frac{h}{2}}^{\frac{h}{2}} b\, y_C^2 dy_C = \frac{b}{3} y_C^3 \Big|_{-\frac{h}{2}}^{\frac{h}{2}} = \frac{b}{3}\left(\frac{h^3}{8} + \frac{h^3}{8}\right) = \boxed{\frac{1}{12} bh^3}$$

Second Moments of Areas

- Example 1 (cont'd)
  b) We select as an element of area a horizontal strip of length *b* and thickness *dy*. Therefore,

$$dI_x = y^2 dA = y^2 (b\,dy)$$

Integrating from *y* = 0 to y_C = *h*, we obtain

$$I_x = \int_0^h b\,y^2 dy = \frac{b}{3} y^3 \Big|_0^h = \boxed{\frac{b}{3} h^3}$$

Second Moments of Areas

- Example 1 (cont'd)
  c) We select as an element of area a vertical strip (Fig. 13) of length *h* and thickness dx_C. Therefore,

$$dI_{y_C} = x_C^2 dA = x_C^2 (h\,dx_C)$$

Integrating from x_C = -*b*/2 to y_C = +*b*/2, we obtain

$$I_{y_C} = \int_{-\frac{b}{2}}^{\frac{b}{2}} h\,x_C^2 dx_C = \frac{h}{3} y_C^3 \Big|_{-\frac{b}{2}}^{\frac{b}{2}} = \frac{h}{3}\left(\frac{b^3}{8} + \frac{b^3}{8}\right) = \frac{1}{12} hb^3$$

Second Moments of Areas

- Example 1 (cont'd)

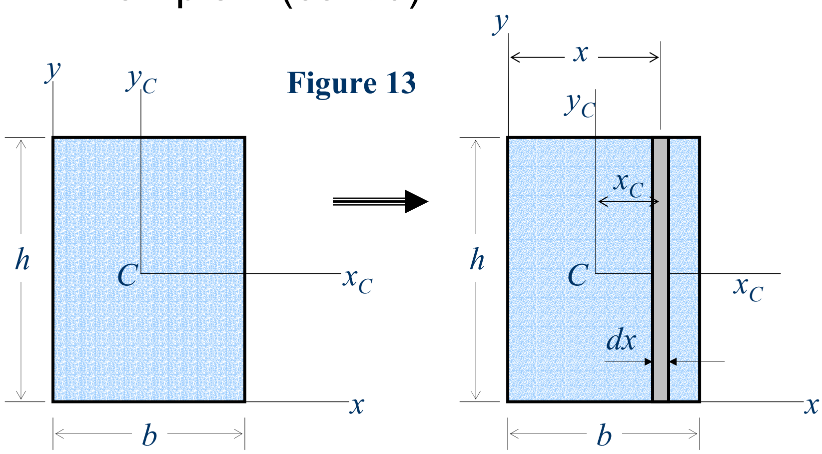


Figure 13

Second Moments of Areas

- Example 1 (cont'd)

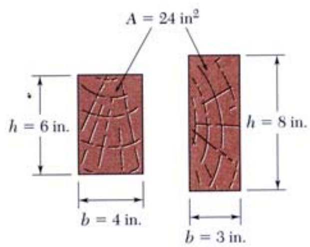
  Using Eq. 24, the polar moment of inertia about the z_C axis is computed as follows:

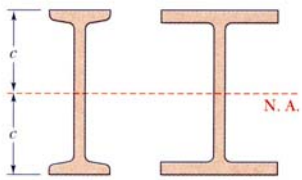
$$J_{z_C} = I_{x_C} + I_{y_C}$$

$$= \frac{bh^3}{12} + \frac{hb^3}{12}$$

$$= \frac{bh}{12}\left(h^2 + b^2\right) \quad \longleftarrow$$

Beam Section Properties





(a) S-beam (b) W-beam

- The maximum normal stress due to bending,

$$\sigma_m = \frac{Mc}{I} = \frac{M}{S}$$

I = section moment of inertia

$S = \frac{I}{c}$ = section modulus

A beam section with a larger section modulus will have a lower maximum stress

- Consider a rectangular beam cross section,

$$S = \frac{I}{c} = \frac{\frac{1}{12}bh^3}{h/2} = \frac{1}{6}bh^3 = \frac{1}{6}Ah$$

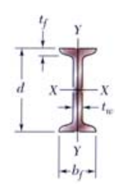
Between two beams with the same cross sectional area, the beam with the greater depth will be more effective in resisting bending.

- Structural steel beams are designed to have a large section modulus.

Properties of American Standard Shapes

Appendix C. Properties of Rolled-Steel Shapes
(SI Units)

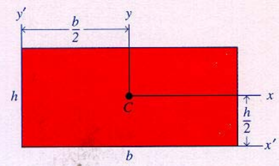
S Shapes
(American Standard Shapes)



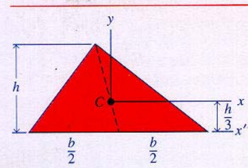
Designation†	Area A, mm²	Depth d, mm	Flange Width b_f, mm	Flange Thickness t_f, mm	Web Thickness t_w, mm	Axis X-X I_x 10^6 mm⁴	Axis X-X S_x 10^3 mm³	Axis X-X r_x mm	Axis Y-Y I_y 10^6 mm⁴	Axis Y-Y S_y 10^3 mm³	Axis Y-Y r_y mm
S610 × 180	22900	622	204	27.7	20.3	1320	4240	240	34.9	341	39.0
158	20100	622	200	27.7	15.7	1230	3950	247	32.5	321	39.9
149	19000	610	184	22.1	18.9	995	3260	229	20.2	215	32.3
134	17100	610	181	22.1	15.9	938	3080	234	19.0	206	33.0
119	15200	610	178	22.1	12.7	878	2880	240	17.9	198	34.0
S510 × 143	18200	516	183	23.4	20.3	700	2710	196	21.3	228	33.9
128	16400	516	179	23.4	16.8	658	2550	200	19.7	216	34.4
112	14200	508	162	20.2	16.1	530	2090	193	12.6	152	29.5
98.3	12500	508	159	20.2	12.8	495	1950	199	11.8	145	30.4
S460 × 104	13300	457	159	17.6	18.1	385	1685	170	10.4	127	27.5
81.4	10400	457	152	17.6	11.7	333	1460	179	8.83	113	28.8
S380 × 74	9500	381	143	15.6	14.0	201	1060	145	6.65	90.8	26.1
64	8150	381	140	15.8	10.4	185	971	151	6.15	85.7	27.1

Second Moments of Areas

- Commonly Used Second Moments of Plane Areas



$$I_x = \frac{bh^3}{12} \qquad I_{x'} = \frac{bh^3}{3} \qquad A = bh$$



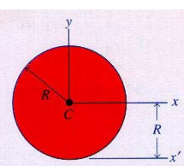
$$I_x = \frac{bh^3}{36} \qquad I_{x'} = \frac{bh^3}{12} \qquad A = \frac{1}{2}bh$$

Figure 14a

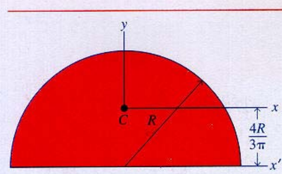
Second Moments of Areas

- Commonly Used Second Moments of Plane Areas

Figure 14b



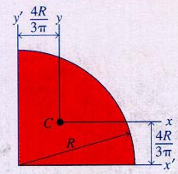
$$I_x = \frac{\pi R^4}{4} \qquad I_{x'} = \frac{5\pi R^4}{4} \qquad A = \pi R^2$$



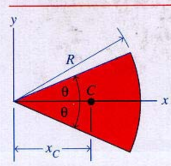
$$I_x = \frac{\pi R^4}{8} - \frac{8R^4}{9\pi} \qquad I_y = \frac{\pi R^4}{8} \qquad I_{x'} = \frac{\pi R^4}{8} \qquad A = \frac{1}{2}\pi R^2$$

Second Moments of Areas

- Commonly Used Second Moments of Plane Areas



$$I_x = \frac{\pi R^4}{16} - \frac{4R^4}{9\pi} \qquad I_{x'} = \frac{\pi R^4}{16} \qquad A = \frac{1}{4}\pi R^2$$



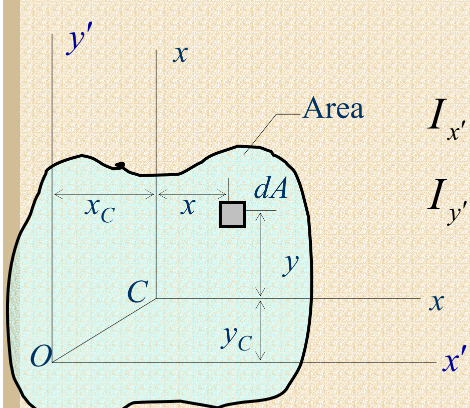
$$I_x = \frac{R^4}{4}\left(\theta - \frac{1}{2}\sin 2\theta\right) \qquad x_C = \frac{2}{3}\frac{R\sin\theta}{\theta}$$

$$I_y = \frac{R^4}{4}\left(\theta + \frac{1}{2}\sin 2\theta\right) \qquad A = \theta R^2$$

Figure 14c

Second Moments of Areas

- Parallel Axis Theorem



$$I_{x'} = I_{x_C} + y_C^2 A$$
$$I_{y'} = I_{y_C} + x_C^2 A \qquad (27)$$

Second Moments of Areas

- Example 2

  Repeat Part (b) of Example 1 using the parallel-axis theorem.

From Part (a),

$$I_{x_C} = \frac{bh^3}{12}$$

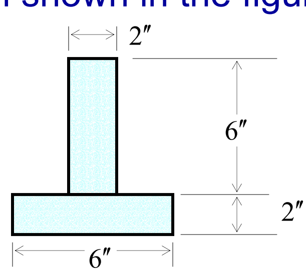
Using Eq. 27,

$$I_{x'} = I_{x_C} + y_c^2 A$$

$$= \frac{bh^3}{12} + \left(\frac{h}{2}\right)^2 (hb) = \frac{bh^3 + 3bh^3}{12} = \frac{bh^3}{3}$$

Examples: Elastic Flexure Formula

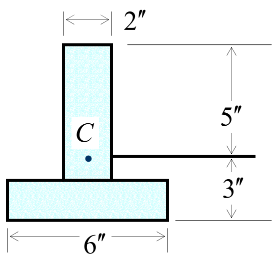
- Example 3
  - Determine the maximum flexural stress produced by a resisting moment M_r of +5000 ft·lb if the beam has the cross section shown in the figure.



Examples: Elastic Flexure Formula

- Example 3 (cont'd)

  First, we need to locate the neutral axis from the bottom edge:



$$y_C = \frac{(1)(2\times 6) + (2+3)(2\times 6)}{2\times 6 + 2\times 6} = \frac{72}{24} = 3''$$

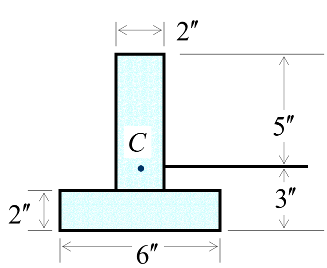
$$y_{\text{ten}} = 3'' \qquad y_{\text{com}} = 6 + 2 - 3 = 5'' = y_{\max}$$

$$\text{Max. Stress} = \frac{M_r y_{\max}}{I_x}$$

Examples: Elastic Flexure Formula

- Example 3 (cont'd)

  Find the moment of inertia I_x with respect to the *x* axis using parallel axis-theorem:



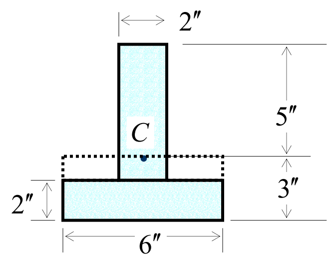
$$I_x = \frac{6(2)^3}{12} + (6\times 2)(2)^2 + \frac{2(6)^3}{12} + (2\times 6)(3-1)^2$$

$$= 4 + 48 + 36 + 48 = 136 \text{ in}^4$$

$$\text{Max. Stress (com)} = \frac{(5\times 12)(5)}{136} = \underline{2.21 \text{ ksi}}$$

Examples: Elastic Flexure Formula

- Example 3 (cont'd)
  - An alternative way for finding the moment of inertia I_x with respect to the x axis is as follows:



$$I_x = \frac{6(3)^3}{3} + \frac{2(5)^3}{3} - 2\left[\frac{2(1)^3}{3}\right] = 136$$

Examples: Elastic Flexure Formula

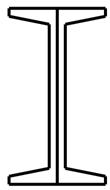
- Example 4

  A pair of channels fastened back-to-back will be used as a beam to resist a bending moment M_r of 60 kN · m. If the maximum flexural stress must not exceed 120 MPa, select the most economical channel section listed in Appendix B of the textbook.

Examples: Elastic Flexure Formula

- Example 4 (cont'd)

$$\sigma = \frac{M}{S}, \text{However, we have two channels, hence}$$



$$\sigma = \frac{M}{2S} \Rightarrow S = \frac{M}{2\sigma}$$

$$S = \frac{60 \times 10^3}{2\left(120 \times 10^6\right)} = 250 \times 10^{-6}\,\text{m}^3 = 250 \times 10^3\,\text{mm}^3$$

From Table B-6 of Textbook:

Select C254 × 30 channel

Example 4 (cont'd)

Select

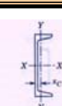


TABLE B-6 Standard Channels (SI Units)

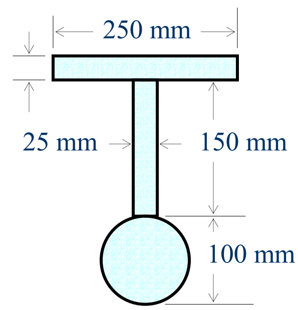
Designation*	Area (mm²)	Depth (mm)	FLANGE Width (mm)	FLANGE Thickness (mm)	Web Thickness (mm)	AXIS X–X I (10⁶ mm⁴)	AXIS X–X S (10³ mm³)	AXIS X–X r (mm)	AXIS Y–Y I (10⁶ mm⁴)	AXIS Y–Y S (10³ mm³)	AXIS Y–Y r (mm)	x_C (mm)
C457 × 86	11030	457.2	106.7	15.9	17.8	281	1230	160	7.41	87.2	25.9	21.9
× 77	9870	457.2	104.1	15.9	15.2	261	1140	163	6.83	83.1	26.4	21.8
× 68	8710	457.2	101.6	15.9	12.7	241	1055	167	6.29	79.0	26.9	22.0
× 64	8130	457.2	100.3	15.9	11.4	231	1010	169	5.99	76.9	27.2	22.3
C381 × 74	9485	381.0	94.4	16.5	18.2	168	882	133	4.58	61.9	22.0	20.3
× 60	7615	381.0	89.4	16.5	13.2	145	762	138	3.84	55.2	22.5	19.7
× 50	6425	381.0	86.4	16.5	10.2	131	688	143	3.38	51.0	23.0	20.0
C305 × 45	5690	304.8	80.5	12.7	13.0	67.4	442	109	2.14	33.8	19.4	17.1
× 37	4740	304.8	77.4	12.7	9.8	59.9	395	113	1.86	30.8	19.8	17.1
× 31	3930	304.8	74.7	12.7	7.2	53.7	352	117	1.61	28.3	20.3	17.7
C254 × 45	5690	254.0	77.0	11.1	17.1	42.9	339	86.9	1.64	27.0	17.0	16.5
× 37	4740	254.0	73.3	11.1	13.4	38.0	298	89.4	1.40	24.3	17.2	15.7
× 30	3795	254.0	69.6	11.1	9.6	32.8	259	93.0	1.17	21.6	17.6	15.4
× 23	2895	254.0	66.0	11.1	6.1	28.1	221	98.3	0.949	19.0	18.1	16.1
C229 × 30	3795	228.6	67.3	10.5	11.4	25.3	221	81.8	1.01	19.2	16.3	14.8
× 22	2845	228.6	63.1	10.5	7.2	21.2	185	86.4	0.803	16.6	16.8	14.9
× 20	2540	228.6	61.8	10.5	5.9	19.9	174	88.4	0.733	15.7	17.0	15.3
C203 × 28	3555	203.2	64.2	9.9	12.4	18.3	180	71.6	0.824	16.6	15.2	14.4
× 20	2605	203.2	59.5	9.9	7.7	15.0	148	75.9	0.637	14.0	15.6	14.0
× 17	2180	203.2	57.4	9.9	5.6	13.6	133	79.0	0.549	12.8	15.9	14.5
C178 × 22	2795	177.8	58.4	9.3	10.6	11.3	127	63.8	0.574	12.8	14.3	13.5
× 18	2320	177.8	55.7	9.3	8.0	10.1	114	66.0	0.487	11.5	14.5	13.3
× 15	1850	177.8	53.1	9.3	5.3	8.87	99.6	69.1	0.403	10.2	14.8	13.7
C152 × 19	2470	152.4	54.8	8.7	11.1	7.24	95.0	54.1	0.437	10.5	13.3	13.1
× 16	1995	152.4	51.7	8.7	8.0	6.33	82.9	56.4	0.360	9.24	13.4	12.7
× 12	1550	152.4	48.8	8.7	5.1	5.45	71.8	59.4	0.288	8.06	13.6	13.0
C127 × 13	1705	127.0	47.9	8.1	8.3	3.70	58.3	46.5	0.263	7.37	12.4	12.1
× 10	1270	127.0	44.5	8.1	4.8	3.12	49.2	49.5	0.199	6.19	12.5	12.3
C102 × 11	1375	101.6	43.7	7.5	8.2	1.91	37.5	37.3	0.180	5.62	11.4	11.7
× 8	1025	101.6	40.2	7.5	4.7	1.60	31.6	39.6	0.133	4.64	11.4	11.6
C76 × 9	1135	76.2	40.5	6.9	9.0	0.862	22.6	27.4	0.127	4.39	10.6	11.6
× 7	948	76.2	38.0	6.9	6.6	0.770	20.3	28.4	0.103	3.82	10.4	11.1
× 6	781	76.2	35.8	6.9	4.6	0.691	18.0	29.7	0.082	3.31	10.3	11.1

*C means channel, followed by the nominal depth in mm, then the mass in kg per meter of length.

Examples: Elastic Flexure Formula

- Example 5

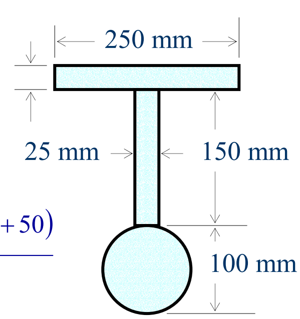
  Determine both the maximum flexural tensile and the maximum flexural compressive stresses produced by a resisting moment of 100 kN·m if the beam has the cross section shown in the figure.



Examples: Elastic Flexure Formula

- Example 5 (cont'd)

  Locate the neutral axis from the upper edge:



$$y_C = \frac{250\times 25(12.5) + 150\times 25(25+75) + \frac{\pi(100)^2}{4}(25+150+50)}{250\times 25 + 150\times 25 + \frac{\pi(100)^2}{4}}$$

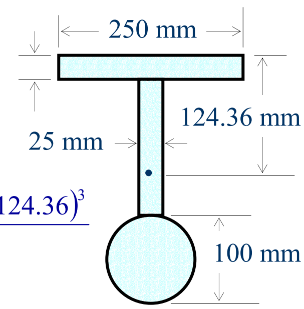
$$= \frac{2{,}220{,}270.87}{17{,}853.90}$$

$$= 124.36 \text{ mm}$$

Examples: Elastic Flexure Formula

- Example 5 (cont'd)

Calculate the moment of inertia with respect to the x axis:



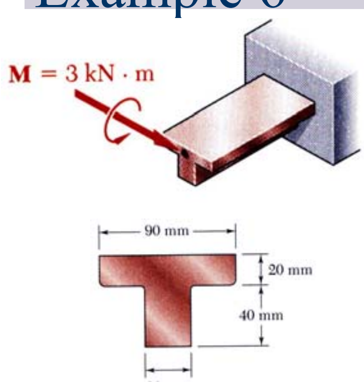
$$I_x = \frac{250(124.36)^3}{3} - \frac{(250-25)(124.36-25)^3}{3} + \frac{25(175-124.36)^3}{3} + \frac{\pi(100)^4}{64} + \frac{\pi(100)^2}{4}(225-124.36)^2$$

$$= 172.243\times10^6\,\text{mm}^4 = 172.243\times10^{-6}\,\text{m}^4$$

$$\sigma_{max}(\text{ten}) = \frac{M_r y}{I} = \frac{100\times10^3(275-124.36)\times10^{-3}}{172.243\times10^{-6}} = \underline{87.5\text{ MPa}}$$

$$\sigma_{max}(\text{com}) = \frac{100\times10^3(124.36)\times10^{-3}}{172.243\times10^{-6}} = \underline{72.2\text{ MPa}}$$

Example 6



A cast-iron machine part is acted upon by a 3 kN-m couple. Knowing E = 165 GPa and neglecting the effects of fillets, determine (a) the maximum tensile and compressive stresses, (b) the radius of curvature.

SOLUTION:

- Based on the cross section geometry, calculate the location of the section centroid and moment of inertia.

$$\bar{Y} = \frac{\sum \bar{y}A}{\sum A} \qquad I_{x'} = \sum\left(\bar{I} + A d^2\right)$$

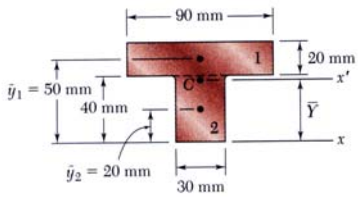
- Apply the elastic flexural formula to find the maximum tensile and compressive stresses.

$$\sigma_m = \frac{Mc}{I}$$

- Calculate the curvature

$$\frac{1}{\rho} = \frac{M}{EI}$$

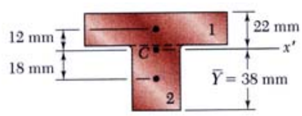
Example 6 (cont'd)



SOLUTION:

Based on the cross section geometry, calculate the location of the section centroid and moment of inertia.

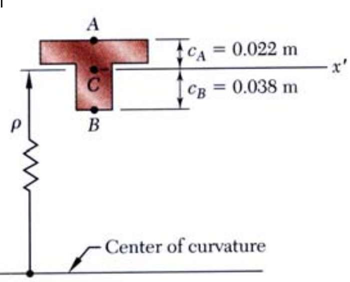
	Area, mm^2	$\bar{y}$, mm	$\bar{y}A$, mm^3
1	$20\times 90 = 1800$	50	90×10^3
2	$40\times 30 = 1200$	20	24×10^3
	$\sum A = 3000$		$\sum \bar{y}A = 114\times 10^3$



$$\bar{Y} = \frac{\sum \bar{y}A}{\sum A} = \frac{114\times 10^3}{3000} = 38\,\text{mm}$$

$$I_{x'} = \sum\left(\bar{I} + A d^2\right) = \sum\left(\tfrac{1}{12} bh^3 + A d^2\right)$$
$$= \left(\tfrac{1}{12} 90\times 20^3 + 1800\times 12^2\right) + \left(\tfrac{1}{12} 30\times 40^3 + 1200\times 18^2\right)$$
$$I = 868\times 10^3\,\text{mm} = 868\times 10^{-9}\,\text{m}^4$$

Example 6 (cont'd)



- Apply the elastic flexural formula to find the maximum tensile and compressive stresses.

$$\sigma_m = \frac{Mc}{I}$$

$$\sigma_A = \frac{M\,c_A}{I} = \frac{3\,\text{kN}\cdot\text{m}\times 0.022\,\text{m}}{868\times 10^{-9}\,\text{mm}^4} \qquad \boxed{\sigma_A = +76.0\,\text{MPa}}$$

$$\sigma_B = -\frac{M\,c_B}{I} = -\frac{3\,\text{kN}\cdot\text{m}\times 0.038\,\text{m}}{868\times 10^{-9}\,\text{mm}^4} \qquad \boxed{\sigma_B = -131.3\,\text{MPa}}$$

- Calculate the curvature

$$\frac{1}{\rho} = \frac{M}{EI}$$
$$= \frac{3\,\text{kN}\cdot\text{m}}{(165\,\text{GPa})\left(868\times 10^{-9}\,\text{m}^4\right)}$$

$$\boxed{\frac{1}{\rho} = 20.95\times 10^{-3}\,\text{m}^{-1} \qquad \rho = 47.7\,\text{m}}$$

Shear Forces and Bending Moments in Beams

- For any specified transverse cross section of a beam, the method for determining flexural stresses discussed previously is adequate if the objective is to determine the flexural stresses on that section.
- However, if the maximum flexural stress is required in a beam subjected to a

Shear Forces and Bending Moments in Beams

loading that produces a resisting moment that *varies* with position along the beam, it is necessary to have a method for determining the maximum resisting moment.

- Similarly, the maximum transverse shearing stress will occur at a section where the resisting shear V_r is maximum.

Shear Forces and Bending Moments in Beams

- Variation of Shear and Moment Forces
  - The variation of shear (*V*) and moment (*M*) forces as a function of the position *x* of an arbitrary point along the beam's axis can be obtained by using the method of section.
  - Here, it is necessary to locate the imaginary section at an arbitrary distance *x* from the end of the beam rather than at specified point.

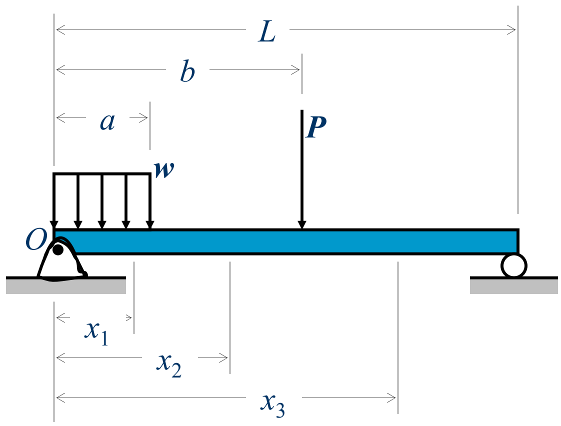
Shear Forces and Bending Moments in Beams

- Variation of Shear and Moment Forces
  - In general, the internal shear *V* and bending moment *M* variations will be discontinuous, or their slope will be discontinuous at points where a distributed load changes or where concentrated forces or couples are applied.

Shear Forces and Bending Moments in Beams

- Variation of Shear and Moment Forces

Figure 14



Shear Forces and Bending Moments in Beams

- Variation of Shear and Moment Forces
  - Shear and bending moment functions must be determined for *each segment* of the beam located between any two discontinuities of loading.
  - For example, sections located at x_1, x_2, and x_3 will have to be used to describe the variation of *V* and *M* throughout the length of the beam in Fig. 14.

Shear Forces and Bending Moments in Beams

- Variation of Shear and Moment Forces
  – These functions will be valid only within regions from

    O to *a* for x_1

    a to *b* for x_2, and

    b to *L* for x_3

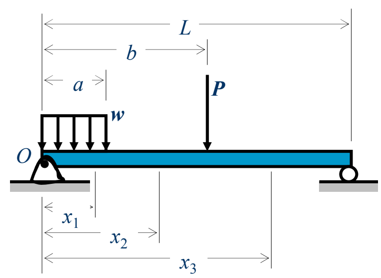


Figure 14

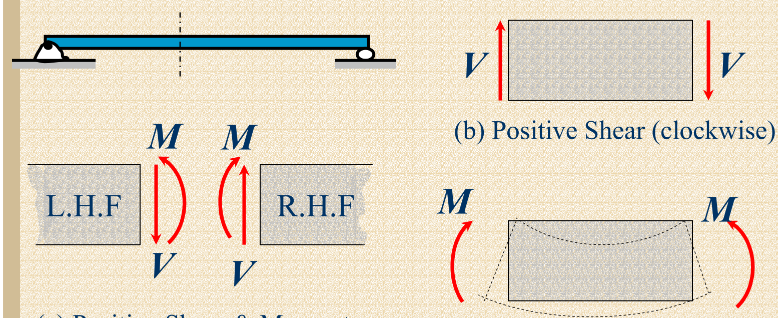
Shear Forces and Bending Moments in Beams

- Sign Convention
  – Before presenting a method for determining the shear and bending moment as functions of *x* and later plotting these functions (shear and moment diagrams), it is first necessary to establish a ***sign convention*** so define “positive” and “negative” shear force and bending moment acting in a beam.

Shear Forces and Bending Moments in Beams

- Sign Convention

Figure 15



(a) Positive Shear & Moment

(b) Positive Shear (clockwise)

(c) Positive Moment (concave upward)

Shear Forces and Bending Moments in Beams

- Sign Convention
  - With reference to Fig. 15a, on the left-hand face (L.H.F.) of the beam segment, the internal shear force ***V*** acts downward and the internal moment ***M*** acts counterclockwise.
  - In accordance with Newton's third law, an equal and opposite shear force and bending moment must act on the right-hand face (R.H.F.) of a segment.

Shear Forces and Bending Moments in Beams

- Sign Convention
  - Perhaps an easy way to remember this sign convention is to isolate a small beam segment and note that *positive shear* tends to rotate the segment clockwise (Fig. 15b), and a *positive moment* tends to bend the segment concave upward (Fig. 15c)

Shear Forces and Bending Moments in Beams

- Procedure for Analysis
  - The following procedure provides a method for constructing the shear and moment functions (formulas) for a beam.
    - Support Reactions
      - Determine all the reactive couples and forces acting on the beam and resolve the forces into components acting perpendicular and parallel to the beam's axis.
    - Shear and Moment Functions (Formulas)
      - Specify separate coordinates *x* having an origin at

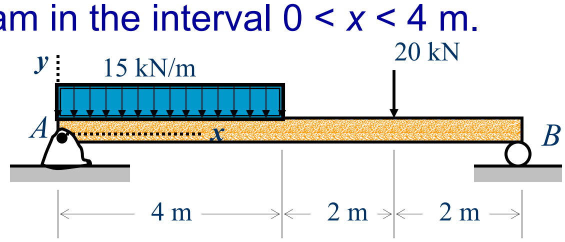
Shear Forces and Bending Moments in Beams

- Procedure for Analysis (cont'd)
  – The beam's left end and extending to regions of the beam between concentrated forces and/or couples and where there is no discontinuity of distributed loading.
  – Section the beam perpendicular to its axis at each distance *x* and from the free-body diagram of one of the segments, determine the unknowns ***V*** and ***M*** at the cut section as a function of *x*.
  – On the free-body diagram, ***V*** and ***M*** should be shown acting in their positive sense (see Fig 15).
  – ***V*** is obtained from $\sum F_y = 0$ and ***M*** is obtained by summing moments about point *S* located at the cut section, $\sum M_S = 0$

Shear Forces and Bending Moments in Beams

- Example 7

A beam is loaded and supported as shown in the figure. Using the coordinate axes shown, write equations for shear *V* and bending moment *M* for any section of the beam in the interval 0 < *x* < 4 m.



Shear Forces and Bending Moments in Beams

- Example 7 (cont'd)
  - A free-body diagram for the beam is shown Fig. 16. The reactions shown on the diagram are determined from equilibrium equations as follows:

$$+\circlearrowleft \sum M_B = 0;\ R_A(8) - (15\times 4)(6) - 20(2) = 0$$

$$\therefore R_A = 50 \text{ kN}$$

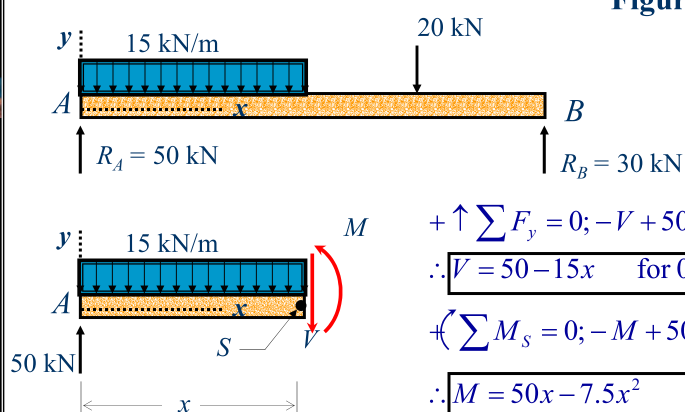
$$+\uparrow \sum F_y - 0;\ R_B + 50 - 15(4) - 20 = 0$$

$$\therefore R_B = 30 \text{ kN}$$

Shear Forces and Bending Moments in Beams

- Example 7 (cont'd)

Figure 16



$$+\uparrow \sum F_y = 0;\ -V + 50 - 15x = 0$$

$$\therefore \boxed{V = 50 - 15x \quad \text{for } 0 < x < 4}$$

$$+\circlearrowleft \sum M_S = 0;\ -M + 50x - 15(x)\frac{x}{2} = 0$$

$$\therefore \boxed{M = 50x - 7.5x^2 \quad \text{for } 0 < x < 4}$$