LECTURE

10

Chapters 1, 2, and 3

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REVIEW FOR EXAM #1

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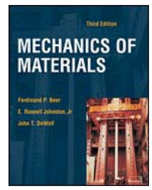


by

Dr. Ibrahim A. Assakkaf

SPRING 2003

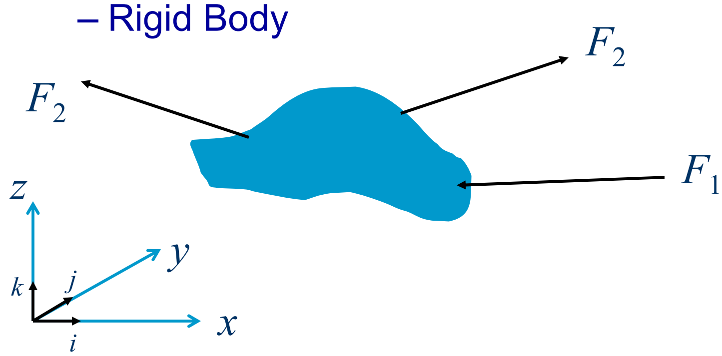
ENES 220 – Mechanics of Materials

Department of Civil and Environmental Engineering

University of Maryland, College Park

Review: Statics

- Equations of Equilibrium
  - Rigid Body



Review: Statics

- Equations of Equilibrium
  – For a rigid body to be in equilibrium, both the resultant force ***R*** and a resultant moments (couples) ***C*** must vanish.
  – These two conditions can be expressed mathematically in vector form as

$$\vec{R} = \sum F_x \mathrm{i} + \sum F_y \mathrm{j} + \sum F_z \mathrm{k} = \vec{0}$$
$$\vec{C} = \sum M_x \mathrm{i} + \sum M_y \mathrm{j} + \sum M_z \mathrm{k} = \vec{0}$$

Review: Statics

- Equations of Equilibrium
  – The two conditions can also be expressed in scalar form as

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$
$$\sum M_x = 0 \qquad \sum M_y = 0 \qquad \sum M_z = 0$$

Review: Statics

- Equilibrium in Two Dimensions
  - The term “two dimensional” is used to describe problems in which the forces under consideration are contained in a plane (say the *xy*-plane)

Review: Statics

- Equilibrium in Two Dimensions
  - For two-dimensional problems, since a force in the *xy*-plane has no *z*-component and produces no moments about the *x*- or *y*-axes, hence

$$\vec{R} = \sum F_x \mathrm{i} + \sum F_y \mathrm{j} = \vec{0}$$
$$\vec{C} = \sum M_z \mathrm{k} = \vec{0}$$

Review: Statics

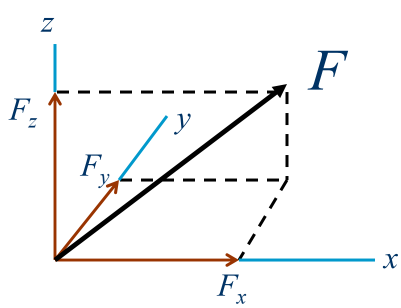
- Equilibrium in Two Dimensions
  – In scalar form, these conditions can be expressed as

$$\sum F_x = 0 \qquad \sum F_y = 0$$

$$\sum M_A = 0$$

Review: Statics

- Cartesian Vector Representation of A Force

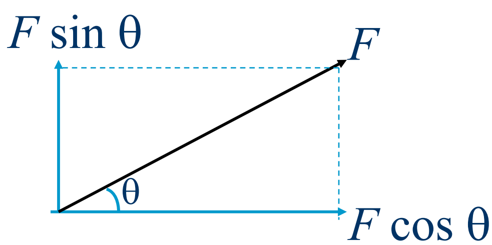


$$F = F_x\mathrm{i} + F_y\mathrm{j} + F_z\mathrm{k}$$

Review: Statics

- Cartesian Vector Representation of A Force in Two Dimensions

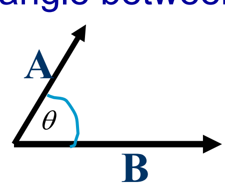
$$\vec{F} = F_x\mathrm{i} + F_y\mathrm{j}$$
$$= F\cos\theta\mathrm{i} + F\sin\theta\mathrm{j}$$



Review: Vector Operations

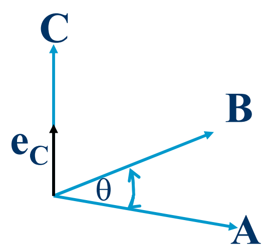
- Dot or Scalar Product
  - The dot or scalar product or two intersecting vectors is defined as the product of the magnitudes of the vectors and the cosine of the angle between them.

$$\mathbf{A} \bullet \mathbf{B} = AB\cos\theta$$



Review: Vector Operations

- Cross or Vector Product



$$\boxed{\mathbf{C} = \mathbf{A} \times \mathbf{B} = (AB \sin \theta)\, \mathbf{e}_C}$$

$$\mathbf{A} \times \mathbf{B} = -\, \mathbf{B} \times \mathbf{A}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \mathrm{i} & \mathrm{j} & \mathrm{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \vec{C}$$

Review: Vector Operations

- Example 3
  - If **A** = -3.75**i** – 2.50**j** + 1.50**k** and
    B = 32**i** + 44**j** + 64 **k**
    determine the magnitude and direction of the vector **C** = **A** × **B**

$$\vec{C} = \vec{A} \times \vec{B} = \begin{vmatrix} \mathrm{i} & \mathrm{j} & \mathrm{k} \\ -3.75 & -2.5 & 1.5 \\ 32 & 44 & 64 \end{vmatrix}$$

Review: Vector Operations

- Example 3 (cont'd)

$$\vec{C} = \vec{A} \times \vec{B} = \begin{vmatrix} \mathrm{i} & \mathrm{j} & \mathrm{k} \\ -3.75 & -2.5 & 1.5 \\ 32 & 44 & 64 \end{vmatrix}$$

$$= [-2.5(64) - 1.5(44)]\mathrm{i} - [-3.75(64) - 1.5(32)]\mathrm{j} + [-3.75(44) - (-2.5)32]$$

$$= -226\mathrm{i} + 288\mathrm{j} - 85\mathrm{k}$$

Review: Vector Operations

- Example 3 (cont'd)

$$C = \left|\vec{C}\right| = \sqrt{(-226)^2 + (288)^2 + (-85)^2} = 376$$

$$\theta_x = \cos^{-1}\frac{C_x}{\left|\vec{C}\right|} = \cos^{-1}\frac{-226}{376} = 126.9^0$$

$$\theta_{yx} = \cos^{-1}\frac{C_y}{\left|\vec{C}\right|} = \cos^{-1}\frac{288}{376} = 40.0^0$$

$$\theta_{zx} = \cos^{-1}\frac{C_z}{\left|\vec{C}\right|} = \cos^{-1}\frac{-85.0}{376} = 103.1^0$$

Introduction

- Objectives

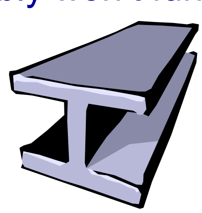
Mechanics of Materials answers two questions:

Is the material strong enough?

Is the material stiff enough?

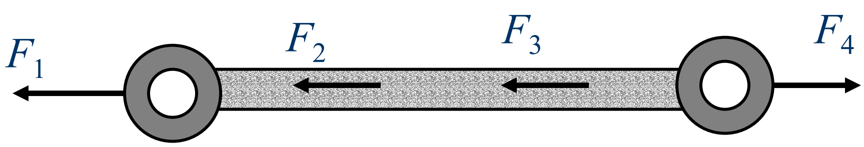
Introduction

- Objectives
  - If the material is not strong enough, your design will break.
  - If the material isn't stiff enough, your design probably won't function the way it's intended to.



Internal Forces for Axially Loaded Members

- Analysis of Internal Forces



Assume that F_1 = 2 k, F_3 = 5 k, and F_4 = 8 k

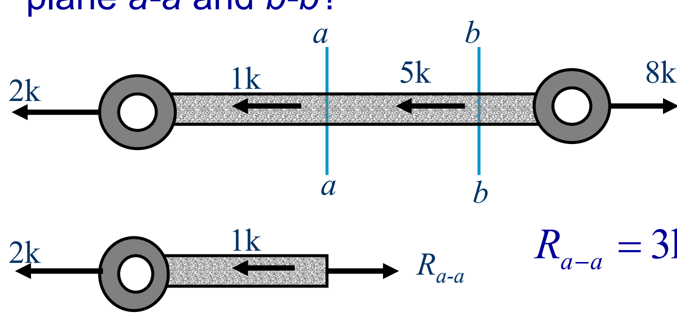
Then

$$\rightarrow + \sum -F_1 - F_2 - F_3 + F_4 = 0; \Rightarrow F_2 = F_4 - F_1 - F_3$$

or $F_2 = 8 - 2 - 5 = 1\,\text{k}$

Internal Forces for Axially Loaded Members

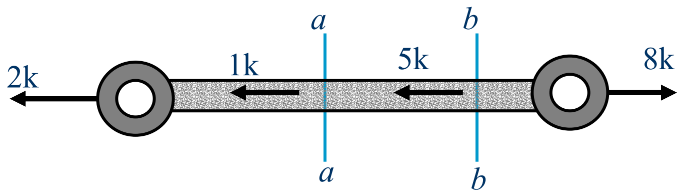
- Analysis of Internal Forces
  - What is the internal force developed on plane *a-a* and *b-b*?



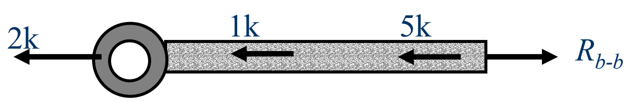
$$R_{a-a} = 3\text{k}$$

Internal Forces for Axially Loaded Members

- Analysis of Internal Forces

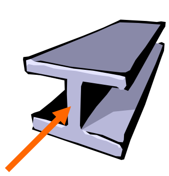


$$R_{b-b} = 8\text{k}$$



Axial Loading: Normal Stress

- Stress
  - Stress is the intensity of internal force.
  - It can also be defined as force per unit area, or intensity of the forces distributed over a given section.



$$\boxed{\text{Stress} = \frac{\text{Force}}{\text{Area}}} \qquad (1)$$

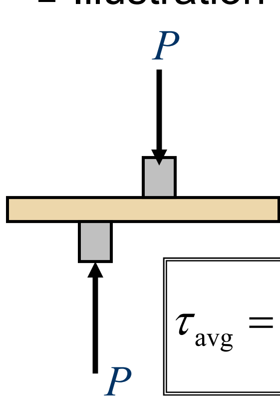
Axial Loading: Normal Stress

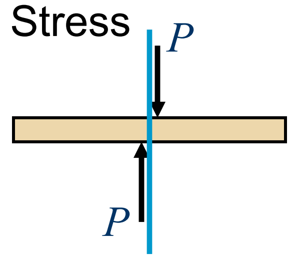
- Normal Stress
  - *F* = magnitude of the force ***F***
  - *A* = area of the cross sectional area of the eye bar.
- Units of Stress

SI System	U.S. Customary Units
1 kPa = 10^3 Pa = 10^3 N/m^2 1 MPa = 10^6 Pa = 10^6 N/m^2 1 GPa = 10^9 Pa = 10^9 N/m^2	lb/in^2 = psi Kip/in^2 = ksi = 1000 psi

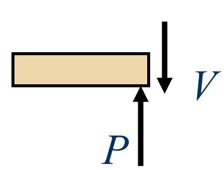
Shearing Stress

- Illustration of Shearing Stress



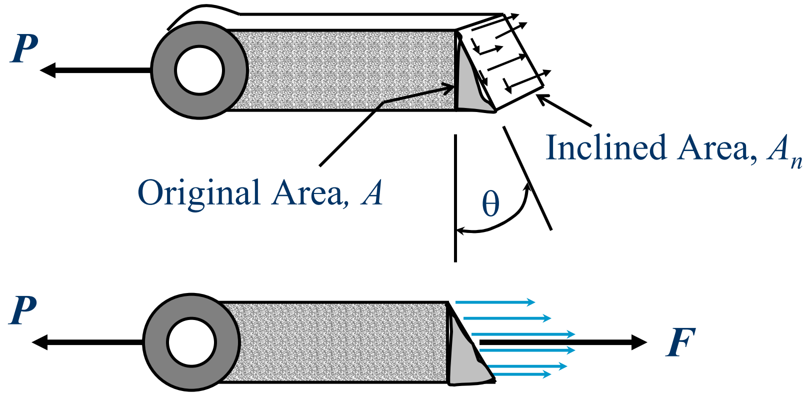


$$\tau_{\text{avg}} = \frac{V}{A_s} = \frac{P}{A_s}$$



Stresses on an Inclined Plane in an Axially Loaded Member

- Illustration



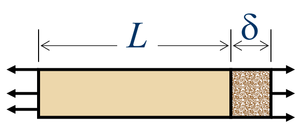
Design Loads, Working Stresses, and Factor of Safety (FS)

- Factor of Safety
  - The factor of safety (FS) can be defined as the ratio of the ultimate stress of the material to the allowable stress

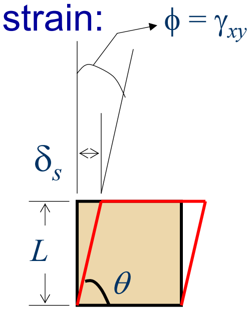
$$\boxed{\text{FS} = \frac{\text{ultimate stress}}{\text{allowable stress}}}$$

Displacement, Deformation, and Strain

- Strain
  - Two general types of strain:
    - Axial (normal) Strain
    - Shearing Strain



Normal Strain

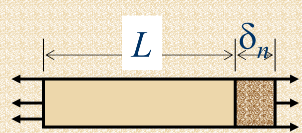


Shearing Strain

Displacement, Deformation, and Strain

- Average Axial Strain

$$\varepsilon_{\text{avg}} = \frac{\delta_n}{L}$$



- True Axial Strain

$$\varepsilon(p) = \lim_{\Delta L \to 0} \frac{\Delta \delta_n}{\Delta L} = \frac{d\delta_n}{dL}$$

Displacement, Deformation, and Strain

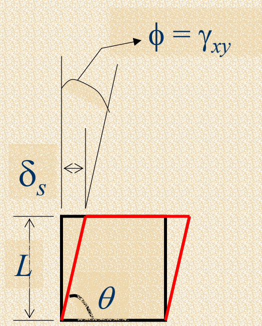
- Average Shearing Strain

$$\gamma_{\text{avg}} = \tan \phi$$

Since δ_s is vary small,

$\sin \phi = \tan \phi = \phi$, therefore,

$$\gamma_{\text{avg}} = \frac{\delta_s}{L}$$



Displacement, Deformation, and Strain

- Example

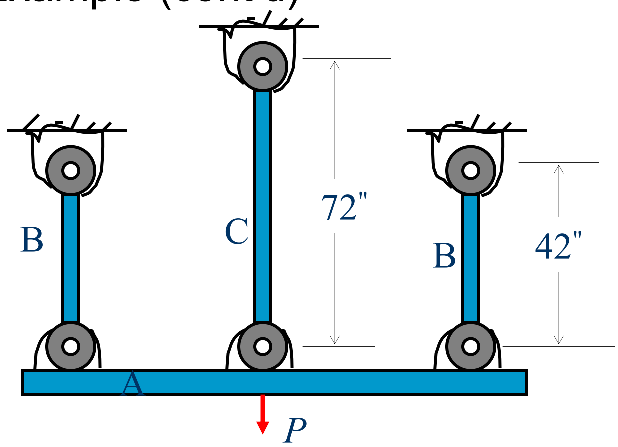
A rigid steel plate A is supported by three rods as shown. There is no strain in the rods before the load *P* is applied. After *P* is applied, the axial strain in rod *C* is 900μ in/in. Determine

(a) The axial strain in rods B.

(b) The axial strain in rods B if there is a 0.006-in clearance in the connections between A and B before the load is applied.

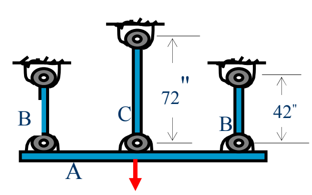
Displacement, Deformation, and Strain

- Example (cont'd)



Displacement, Deformation, and Strain

- Example (cont'd)



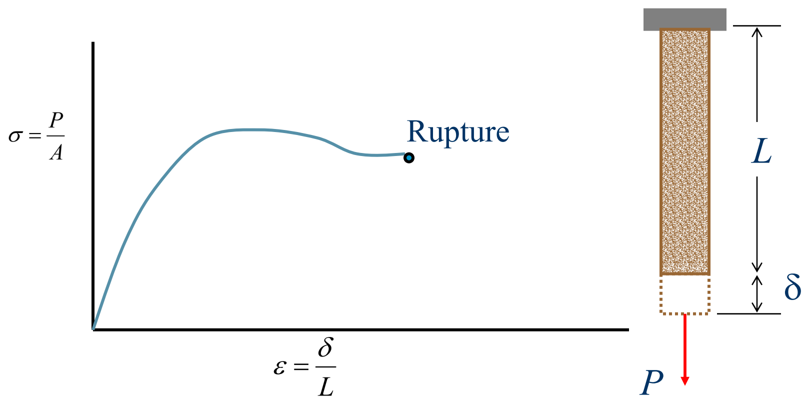
$$\varepsilon_C = \frac{\delta_C}{L_C} \Rightarrow \delta_C = \varepsilon_C L_C = 900\times10^{-6}(72) = 0.0648\text{ in}$$

a) $$\varepsilon_B = \frac{\delta_B}{L_B} = \frac{0.0648}{42} = 0.001543 = 1543\mu$$

a) $$\varepsilon_B = \frac{\delta_B}{L_B} = \frac{0.0648-0.006}{42} = 0.001400 = 1400\mu$$

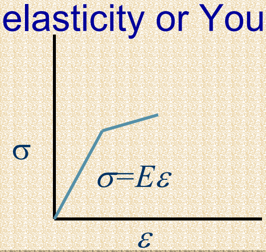
Stress-Strain-Temperature Relationships

- General Stress-Strain Diagram



Stress-Strain-Temperature Relationships

- Modulus of Elasticity, *E*
  - The initial portion of the stress-strain curve (diagram) is a straight line. The equation for this straight line is called the modulus of elasticity or Young's Modulus *E*



$$\sigma = E\varepsilon$$

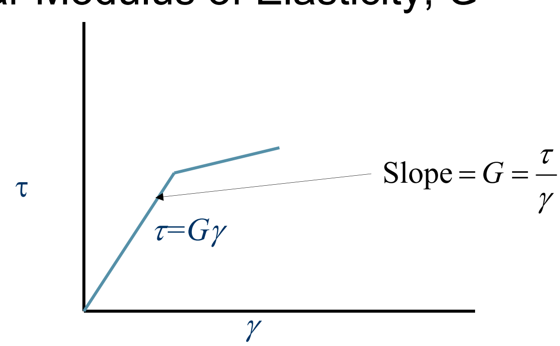
Stress-Strain-Temperature Relationships

- Shear Modulus of Elasticity, *G*
  - The shear modulus is similar to the modulus of elasticity. However it is applied to shear stress-strain.

$$\tau = G\gamma$$

Stress-Strain-Temperature Relationships

Shear Modulus of Elasticity, *G*



Stress-Strain-Temperature Relationships

- Poisson's Ratio
  - A material loaded in one direction will undergo strains perpendicular to the direction of the load in addition to those parallel to the load. The ratio of the lateral or perpendicular strain to the longitudinal or axial strain is called Poisson's ratio.

$$\nu = \frac{\varepsilon_{\text{lat}}}{\varepsilon_{\text{long}}} = \frac{\varepsilon_l}{\varepsilon_a} \qquad E = 2(1+\nu)G$$

Stress-Strain-Temperature Relationships

- Thermal Strain
  - The thermal strain due a temperature change of ΔT degrees is given by

$$\varepsilon_T = \alpha \Delta T$$

Stress-Strain-Temperature Relationships

- Total Strain
  - The sum of the normal strain caused by the loads and the thermal strain is called the total strain, and it is given by

$$\varepsilon_{\text{total}} = \varepsilon_{\sigma} + \varepsilon_{T} = \frac{\sigma}{E} + \alpha\,\Delta T$$

Rods: Stress Concentrations

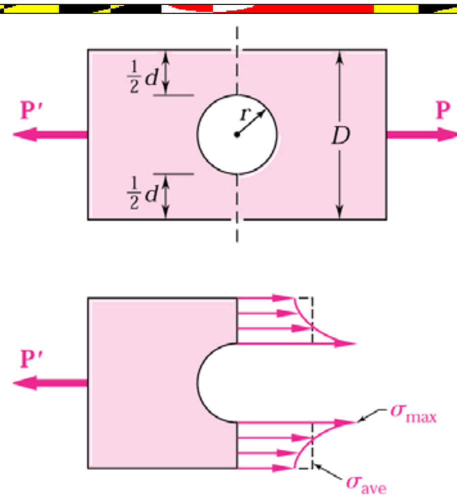


Fig. 1. Stress distribution near circular hole in flat bar under axial loading

Rods: Stress Concentrations

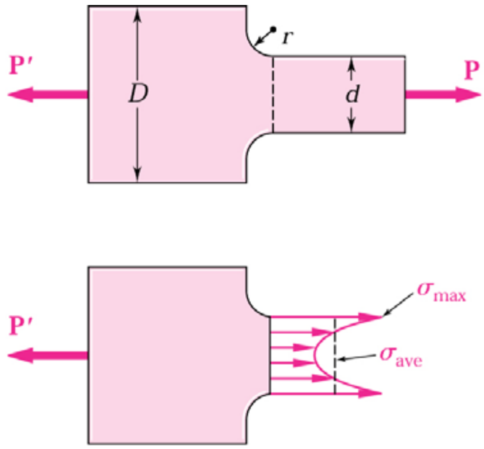
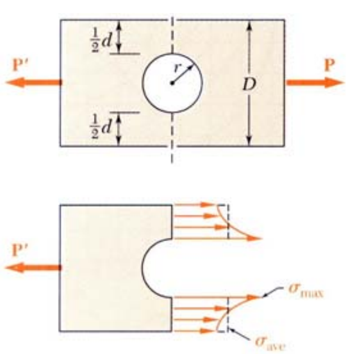


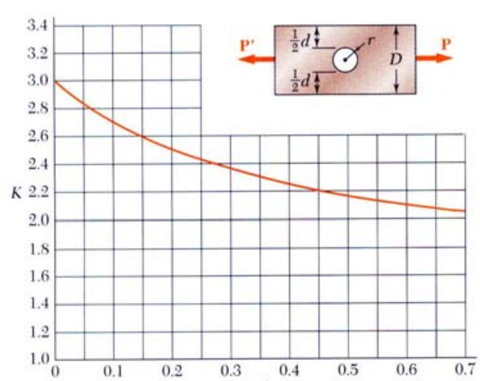
Fig. 2. Stress distribution near fillets in flat bar under axial loading

Rods: Stress Concentrations

- Hole

Fig. 3





(a) Flat bars with holes

Discontinuities of cross section may result in high localized or *concentrated* stresses.

$$K = \frac{\sigma_{max}}{\sigma_{ave}}$$

Rods: Stress Concentrations

- Fillet

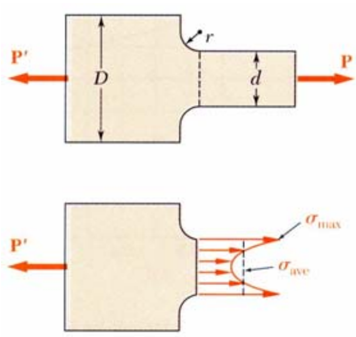
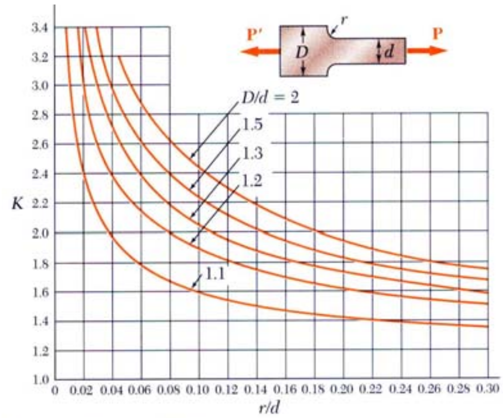


Fig. 4



(b) Flat bars with fillets

Rods: Stress Concentrations

- To determine the maximum stress occurring near discontinuity in a given member subjected to a given axial load *P*, it is only required that the average stress $\sigma_{ave} = P/A$ be computed in the critical section, and the result be multiplied by the appropriate value of the stress-concentration factor *K*.
- It is to be noted that this procedure is valid as long as $\sigma_{max} \leq \sigma_y$

Deformations of Members under Axial Loading

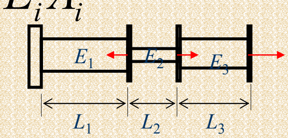
- Uniform Member
  – The deflection (deformation), δ, of the uniform member subjected to axial loading *P* is given by

$$\delta = \frac{PL}{EA} \qquad (1)$$

Deformations of Members under Axial Loading

- Multiple Loads/Sizes
  – The deformation of of various parts of a rod or uniform member can be given by

$$\delta = \sum_{i=1}^{n} \delta_i = \sum_{i=1}^{n} \frac{P_i L_i}{E_i A_i} \qquad (2)$$



Deformations of Members under Axial Loading

- Relative Deformation
  - On the other hand, since both ends of bars *AB* move, the deformation of *AB* is measured by the difference between the displacements δ_A and δ_B of points *A* and *B*.
  - That is by *relative displacement* of *B* with respect to *A*, or

$$\boxed{\delta_{B/A} = \delta_B - \delta_A = \frac{PL}{EA}} \quad (3)$$

Statically Indeterminate Structures

- Determinacy of Beams
  - For a coplanar (two-dimensional) structure, there are at most three equilibrium equations for each part, so that if there is a total of *n* parts and *r* reactions, we have

$$\boxed{\begin{aligned} r &= 3n, \quad \Rightarrow \text{statically determinate} \\ r &> 3n, \quad \Rightarrow \text{statically indeterminate} \end{aligned}} \quad (4)$$

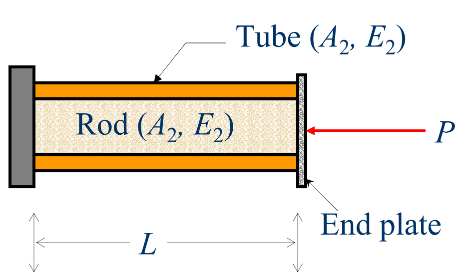
Statically Indeterminate Structures

- Determinacy of Trusses
  - For a coplanar (two-dimensional) truss, there are at most two equilibrium equations for each joint *j*, so that if there is a total of *b* members and *r* reactions, we have

$$\boxed{\begin{array}{l} b+r=2j, \quad \Rightarrow \text{statically determinate} \\ b+r>2j, \quad \Rightarrow \text{statically indeterminate} \end{array}} \qquad (5)$$

Statically Indeterminate Axially Loaded Members

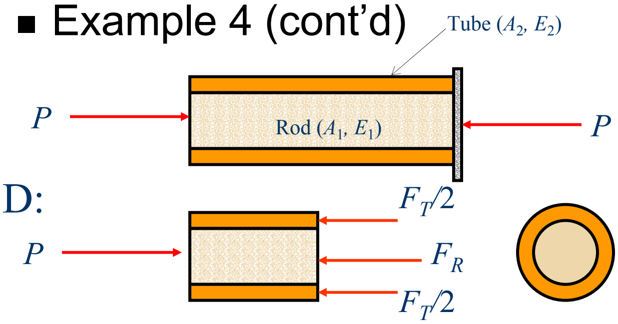
- Example 4 (cont'd)



Statically Indeterminate Axially Loaded Members

- Example 4 (cont'd)

FBD:



$$\rightarrow + \sum F_x = 0; \; P - \frac{F_T}{2} - \frac{F_T}{2} - F_R = 0$$

$$F_R + F_T = P$$

Torsional Loading

- Introduction

Cylindrical members

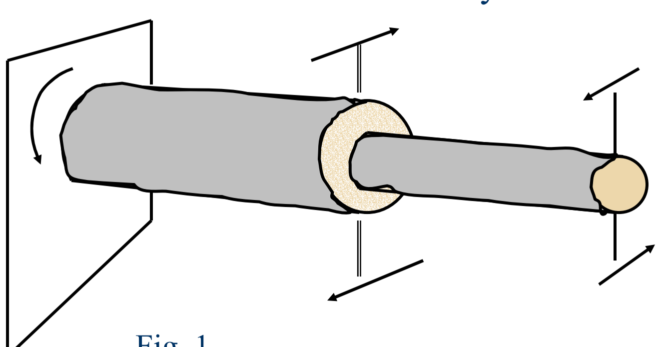


Fig. 1

Torsional Loading

- Stresses in Circular Shaft due to Torsion

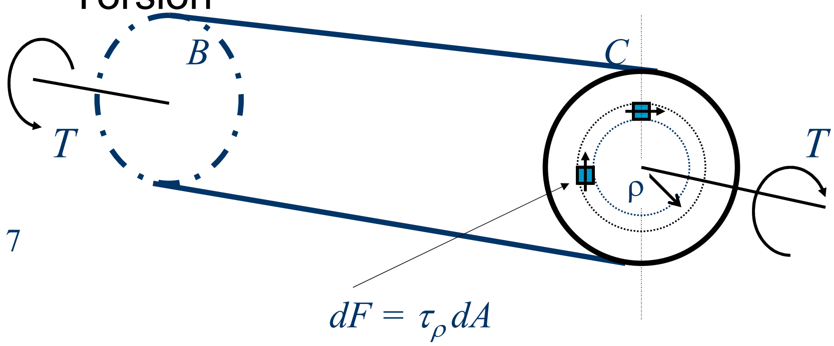


Fig. 7

Torsional Shearing Strain

- Shearing Strain

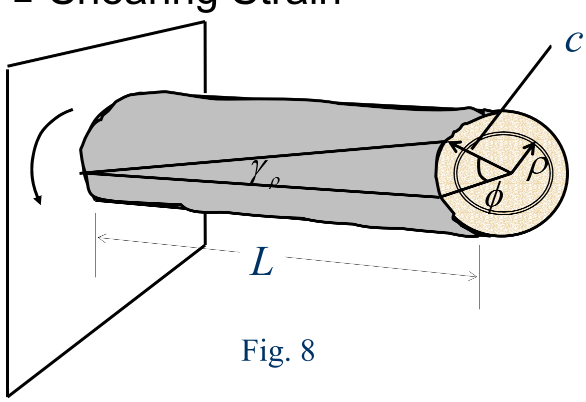


Fig. 8

Torsional Shearing Strain

- Shearing Strain

For radius ρ, the shearing strain for circular shaft is

$$\boxed{\gamma_\rho = \frac{\rho\phi}{L}} \qquad (6)$$

For radius c, the shearing strain for circular shaft is

$$\boxed{\gamma_c = \frac{c\phi}{L}} \qquad (7)$$

Torsional Shearing Strain

- Polar Moment of Inertia

The integral of equation 12 is called the polar moment of inertia (polar second moment of area).

It is given the symbol J. For a solid circular shaft, the polar moment of inertia is given by

$$J = \int \rho^2 \mathrm{d}A = \int_0^c \rho^2 (2\pi\rho\,\mathrm{d}\rho) = \frac{\pi c^4}{2} \qquad (13)$$

Torsional Shearing Strain

- Polar Moment of Inertia

The integral of equation 12 is called the polar moment of inertia (polar second moment of area).

It is given the symbol *J*. For a solid circular shaft, the polar moment of inertia is given by

$$J = \int \rho^2 \mathrm{d}A = \int_0^c \rho^2 \left(2\pi\rho\, \mathrm{d}\rho\right) = \frac{\pi c^4}{2} \qquad (13)$$

Torsional Shearing Strain

- Shearing Stress in Terms of Torque and Polar Moment of Inertia

$$\tau_{\max} = \frac{Tc}{J} \qquad (17a)$$

$$\tau_{\rho} = \frac{T\rho}{J} \qquad (18a)$$

τ= shearing stress, T = applied torque
ρ = radius, and J = polar moment on inertia

Torsional Displacements

- Angle of Twist in the Elastic Range

  The angle of twist for a circular uniform shaft subjected to external torque *T* is given by

$$\theta = \frac{TL}{GJ} \qquad (22)$$

Torsional Displacements

- Multiple Torques/Sizes $\theta = \sum_{i=1}^{n} \theta_i = \sum_{i=1}^{n} \frac{T_i L_i}{G_i J_i}$

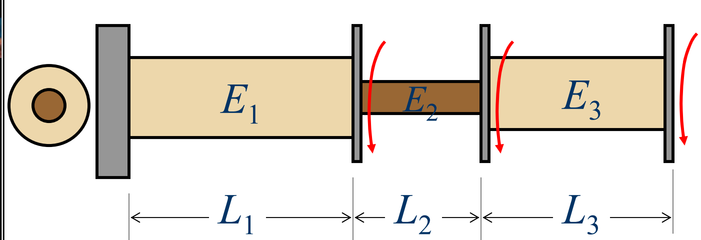


Fig. 12 Circular Shafts

Torsional Displacements

- Angle of Twist in the Elastic Range

  The angle of twist of various parts of a shaft of uniform member can be given by

$$\theta = \sum_{i=1}^{n} \theta_i = \sum_{i=1}^{n} \frac{T_i L_i}{G_i J_i} \qquad (24)$$

Torsional Displacements

- Angle of Twist in the Elastic Range

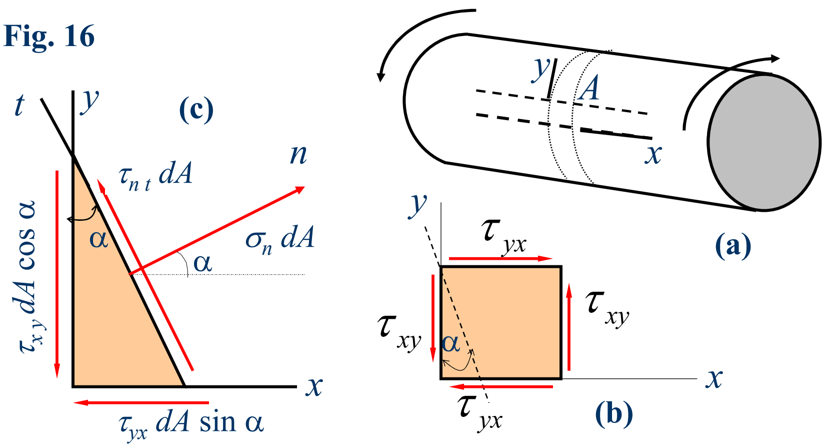
  If the properties (*T, G,* or *J*) of the shaft are functions of the length of the shaft, then

$$\theta = \int_0^L \frac{T}{GJ}\,dx \qquad (25)$$

Stresses in Oblique Planes

- Other Stresses Induced By Torsion

Fig. 16



Stresses in Oblique Planes

- Other Stresses Induced By Torsion

$$+\nwarrow \sum F_t = 0$$

$$\tau_{nt}dA - \tau_{xy}(dA\cos\alpha)\cos\alpha + \tau_{yx}(dA\sin\alpha)\sin\alpha = 0$$

From which

$$\tau_{nt} = \tau_{xy}\left(\cos^2\alpha - \sin^2\alpha\right) = \tau_{xy}\cos 2\alpha \qquad (29)$$

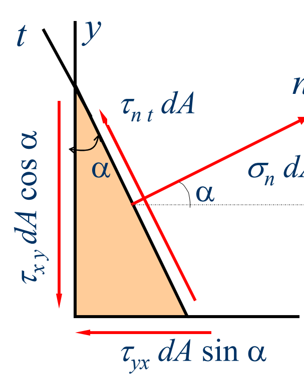


Fig. 16c

Stresses in Oblique Planes

- Other Stresses Induced By Torsion

$$+\nearrow \sum F_t = 0$$

$$\sigma_n dA - \tau_{xy}(dA\cos\alpha)\sin\alpha - \tau_{yx}(dA\sin\alpha)\cos\alpha = 0$$

From which

$$\sigma_n = 2\tau_{xy}\sin\alpha\cos\alpha = \tau_{xy}\sin 2\alpha \qquad (31)$$

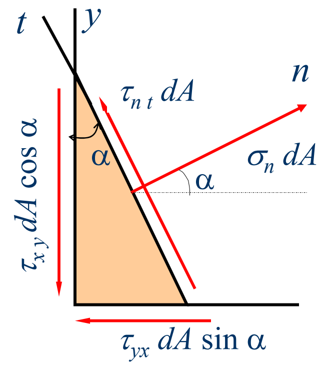


Fig. 16c

Stresses in Oblique Planes

- Maximum Normal Stress due to Torsion on Circular Shaft

  The maximum compressive normal stress σ_{max} can be computed from

$$\sigma_{\max} = \tau_{\max} = \frac{T_{\max} c}{J} \qquad (32)$$

Power Transmission

- Power Transmission by Torsional Shaft
  - But $\omega = 2\pi f$, where f = frequency. The unit of frequency is 1/s and is called *hertz* (Hz).
  - If this is the case, then the power is given by

$$\text{Power} = 2\pi f T$$

or

$$T = \frac{\text{Power}}{2\pi f} \qquad (37)$$

Power Transmission

- Power Transmission by Torsional Shaft
  - Units of Power

SI	US Customary
watt (1 N·m/s)	hp (33,000 ft·lb/min)

Power Transmission

- Power Transmission by Torsional Shaft
  - Some useful relations

$$1\ \text{rpm} = \frac{1}{60} s^{-1} = \frac{1}{60}\ \text{Hz}$$
$$1\ \text{hp} = 550\ \text{ft} \cdot \text{lb/s} = 6600\ \text{in} \cdot \text{lb/s}$$

rpm = revolution per minute

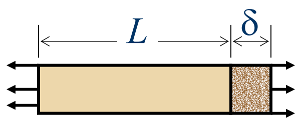
Summary

- Axially Loaded Versus Torsionally Loaded Members

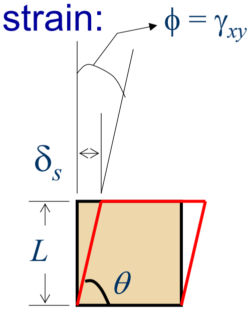
	Axial	Torsion
Stress	$\sigma = \frac{P}{A}$	$\tau_\rho = \frac{T\rho}{J}$
Deformation	$\delta = \frac{PL}{EA}$	$\theta = \frac{TL}{GJ}$

Summary

- Strain
  - Two general types of strain:
    - Axial (normal) Strain
    - Shearing Strain



Normal Strain



Shearing Strain