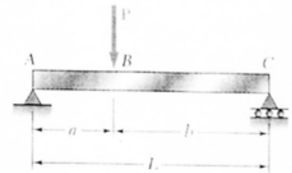
ENES 220 – Mechanics of Materials
Spring 2003

Solutions to Homework #7

Problem 5.1

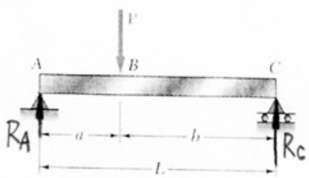
[Given]

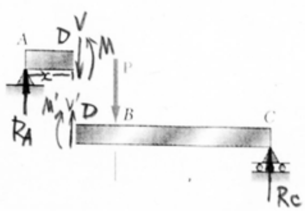


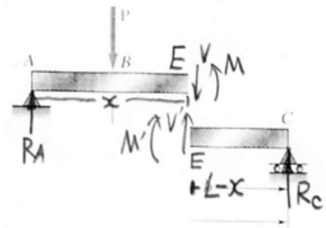
(Fig. 1)

[To Find] Draw the shear and bending-moment diagrams

[FBD]







(Fig. 2) **(Fig. 3)** **(Fig. 4)**

[FPU] $\sum F = 0 \; \sum M = 0$

[Solution]

(1) Determine the reactions at the supports from the FBD of the entire beam (Fig. 2)

$+\circlearrowleft \sum M_C = 0 \qquad (P)(b) - (R_A)(L) = 0 \;\Rightarrow\; R_A = \frac{b}{L}P$

$+\circlearrowleft \sum M_A = 0 \qquad (R_C)(L) - (P)(a) = 0 \;\Rightarrow\; R_C = \frac{a}{L}P$

(2) Cut the beam at a point D between A and B, draw the FBD (Fig. 3)

$+\uparrow \sum F_y = 0 \qquad R_A - V = 0 \;\Rightarrow\; V = R_A = \frac{b}{L}P$

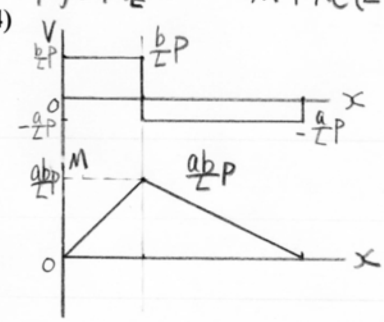
$+\circlearrowleft \sum M_D = 0 \qquad M - (R_A)(x) = 0 \;\Rightarrow\; M = R_A x = \frac{b}{L}Px$

(3) Cut the beam at a point E between B and C, draw the FBD (Fig. 4)

$+\uparrow \sum F_y = 0 \qquad V' + R_C = 0 \;\Rightarrow\; V' = -R_C = -\frac{a}{L}P, \qquad V = V' = -\frac{a}{L}P$

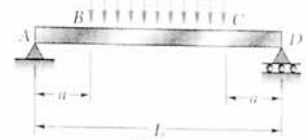
$+\circlearrowleft \sum M_E = 0 \qquad -M' + R_C(L-x) = 0 \;\Rightarrow\; M' = R_C(L-x) = \frac{a}{L}P(L-x) \qquad M = M' = \frac{a}{L}P(L-x)$

(4)



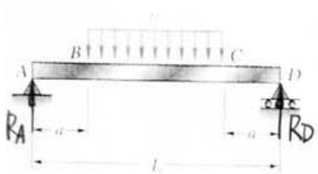
Problem 5.5

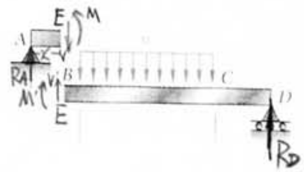
[Given]

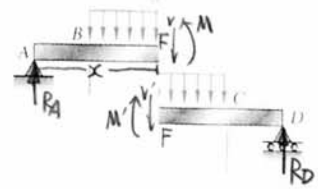

(Fig. 1)

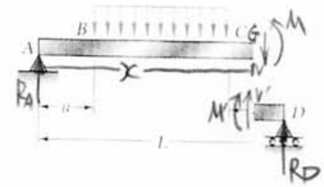
[To Find] Draw the shear and bending-moment diagrams

[FBD]


(Fig. 2)


(Fig. 3)


(Fig. 4)


(Fig. 5)

[FPU] $\sum F = 0 \quad \sum M = 0$

[Solution]

(1) Determine the reactions at the supports from the FBD of the entire beam (Fig. 2)

From statics and symmetry, obviously, $R_A = R_D = \frac{1}{2}w(L-2a)$

(2) Cut the beam at a point E between A and B, draw the FBD (Fig. 3)

$+\uparrow \Sigma F_y = 0 \quad R_A - V = 0 \Rightarrow V = R_A = \frac{1}{2}w(L-2a)$

$+\circlearrowleft \Sigma M_E = 0 \quad M - R_A x = 0 \Rightarrow M = R_A x = \frac{1}{2}w(L-2a)x$ @ $(0 \le x < a)$

(3) Cut the beam at a point F between B and C, draw the FBD (Fig. 4)

$+\uparrow \Sigma F_y = 0 \quad R_A - w(x-a) - V = 0 \Rightarrow V = R_A - w(x-a) = \frac{1}{2}w(L-2a) - w(x-a)$

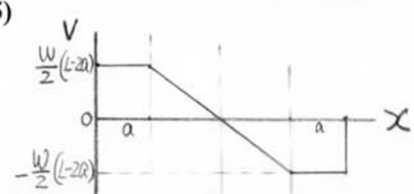
$+\circlearrowleft \Sigma M_F = 0 \quad M - R_A x + w(x-a)\cdot\left(\frac{x-a}{2}\right) = 0 \Rightarrow M = -\frac{w}{2}(x-a)^2 + \frac{1}{2}w(L-2a)x$ @ $(a \le x < L-a)$

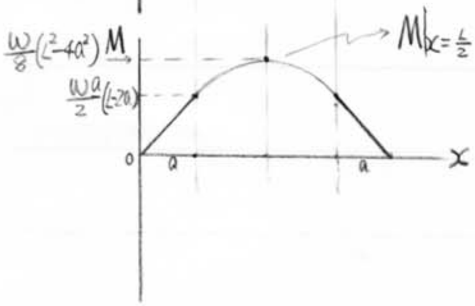
(4) Cut the beam at a point G between C and D, draw the FBD (Fig. 5)

$+\uparrow \Sigma F_y = 0 \quad V' + R_D = 0 \Rightarrow V' = -R_D = -\frac{1}{2}w(L-2a), \quad V = V' = -\frac{1}{2}w(L-2a)$

$+\circlearrowleft \Sigma M_G = 0 \quad -M' + R_D(L-x) = 0 \Rightarrow M' = R_D(L-x) = \frac{1}{2}w(L-2a)(L-x)$ (@ $L-a \le x \le L$)

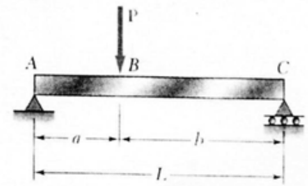
(5)





$M|_{x=\frac{L}{2}} = -\frac{w}{2}\left(\frac{L}{2}-a\right)^2 + \frac{1}{2}w(L-2a)\frac{L}{2} = \frac{w}{8}(L^2-4a^2)$

Problem 5.41

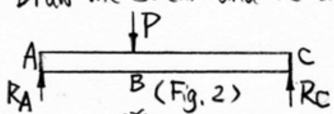


(Fig.1)

[Given]

[To Find] Draw the shear and bending-moment diagrams

[FBD]



[FPU] $V_D - V_C = -\int_{x_C}^{x_D} w\,dx$, $M_D - M_C = \int_{x_C}^{x_D} V\,dx$

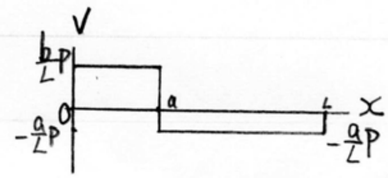
[Solution]

(1) Determine the reactions at the supports from the FBD of the entire beam (Fig.2)

$+\circlearrowleft \Sigma M_C = 0 \quad (P)(b) - (R_A)(L) = 0 \Rightarrow R_A = \frac{b}{L}P$

$+\circlearrowleft \Sigma M_A = 0 \quad (R_C)(L) - (P)(a) = 0 \Rightarrow R_C = \frac{a}{L}P$

(2) Shear :



$V_A = R_A = \frac{b}{L}P$

A to B : $V = \frac{b}{L}P$

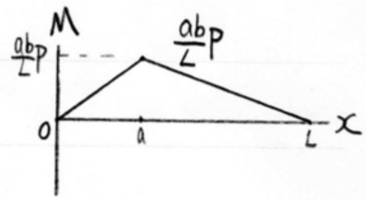
B to C : $V = \frac{b}{L}P - P = -\frac{a}{L}P$

(3) Areas of shear diagram

A to B : $\int_A^B V\,dx = (\frac{b}{L}P)(a) = \frac{ab}{L}P$

B to C : $\int_B^C V\,dx = (-\frac{a}{L}P)(b) = -\frac{ab}{L}P$

(4) Bending moments



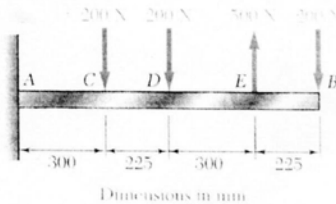
$M_A = 0$

$M_B = M_A + \int_A^B V\,dx = 0 + \frac{ab}{L}P = \frac{ab}{L}P$

$M_C = M_B + \int_B^C V\,dx = \frac{ab}{L}P - \frac{ab}{L}P = 0$

Problem 5.48

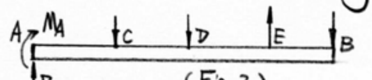
[Given]



(Fig. 1)

[To Find] Draw the shear and bending-moment diagrams

[FBD]



(Fig. 2)

[FPU] $V_D - V_C = -\int_{x_C}^{x_D} W\,dx$, $M_D - M_C = \int_{x_C}^{x_D} V\,dx$

[Solution]

(1) Determine the reactions at A from the FBD of the entire beam (Fig. 2)

$+\uparrow \Sigma F_y = 0 \quad R_A - 200N - 200N + 500N - 200N = 0 \quad \Rightarrow R_A = 100N$

$+\circlearrowleft \Sigma M_A = 0 \quad -M_A - (200N)(0.3m) - (200N)(0.525m) + (500N)(0.825m) - (200N)(1.05m) = 0$

$\Rightarrow M_A = 37.5\ N\cdot m$

(2) Shear

$V_A = R_A = 100N$

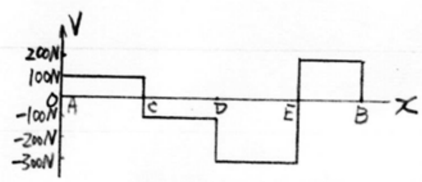
A to C : $V = 100N$

C to D : $V = 100N - 200N = -100N$

D to E : $V = -100N - 200N = -300N$

E to B : $V = -300N + 500N = 200N$

$V_B = 200N$



$\boxed{|V|_{max} = 300N}$

(3) Areas of shear diagram

A to C : $\int_A^C V\,dx = (100N)(0.3m) = 30\ N\cdot m$

C to D : $\int_C^D V\,dx = (-100N)(0.225m) = -22.5\ N\cdot m$

D to E : $\int_D^E V\,dx = (-300N)(0.3m) = -90\ N\cdot m$

E to B : $\int_E^B V\,dx = (200N)(0.225) = 45\ N\cdot m$

(4) Bending moments

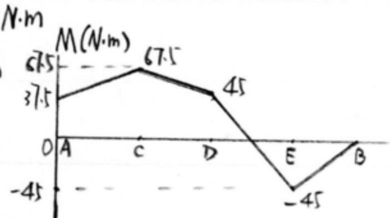
$M_A = 37.5\ N\cdot m$

$M_C = M_A + \int_A^C V\,dx = 37.5N\cdot m + 30N\cdot m = 67.5N\cdot m$

$M_D = M_C + \int_C^D V\,dx = 67.5N\cdot m - 22.5N\cdot m = 45N\cdot m$

$M_E = M_D + \int_D^E V\,dx = 45N\cdot m - 90N\cdot m = -45\ N\cdot m$

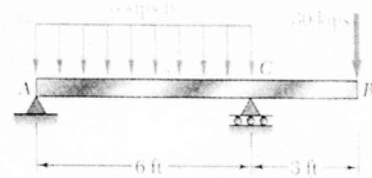
$M_B = M_E + \int_E^B V\,dx = -45N\cdot m + 45N\cdot m = 0$



$\boxed{|M|_{max} = 67.5\ N\cdot m}$

Problem 5.49

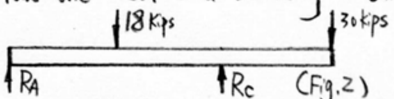
[Given]



(Fig. 1)

[To Find] Draw the shear and bending-moment diagrams

[FBD]



(Fig. 2)

[FPU] $V_D - V_C = -\int_{x_C}^{x_D} w\,dx$, $M_D - M_C = \int_{x_C}^{x_D} V\,dx$

[Solution]

(1) Determine the reactions at the supports from FBD of the entire beam (Fig. 2)

$+\circlearrowleft \Sigma M_A = 0 \quad -(18\text{ kips})(3\text{ft}) - (30\text{kips})(9\text{ft}) + R_C(6\text{ft}) = 0 \Rightarrow R_C = 54\text{ kips}$

$+\uparrow \Sigma F_y = 0 \quad R_A - 18\text{ kips} + R_C - 30\text{ kips} = 0 \Rightarrow R_A = -6\text{ kips}$

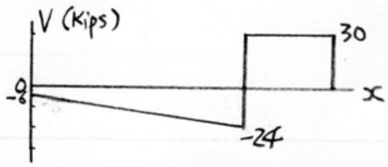
(2) Shear

$V_A = R_A = -6\text{ kips}$

$V_C = V_A - \int_A^C w\,dx = -6\text{kips} - (3\text{ kips/ft})(6\text{ft}) = -24\text{ kips}$

C to B: $V = -24\text{kips} + R_C = 30\text{ kips}$

$V_B = 30\text{ Kips}$.



$\boxed{|V|_{max} = 30\text{ Kips}}$

(3) Areas of shear diagram

A to C: $\int_A^C V\,dx = -\frac{(6+24)(6)}{2}\text{ kip}\cdot\text{ft} = -90\text{ kip}\cdot\text{ft}$

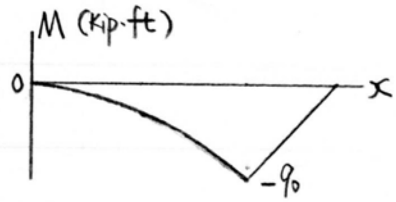
C to B: $\int_C^B V\,dx = 30\cdot 3\text{ kip}\cdot\text{ft} = 90\text{ kip}\cdot\text{ft}$

(4) Bending moments

$M_A = 0$

$M_C = M_A + \int_A^C V\,dx = 0 - 90\text{ kip}\cdot\text{ft} = -90\text{ kip}\cdot\text{ft}$

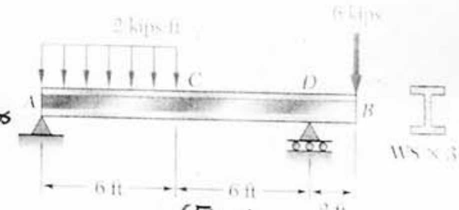
$M_B = M_C + \int_C^B V\,dx = -90\text{ kip}\cdot\text{ft} + 90\text{ kip}\cdot\text{ft} = 0$



$\boxed{|M|_{max} = 90\text{ kip}\cdot\text{ft}}$

Problem 5.64

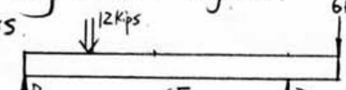
[Given]



(Fig. 1)

[To Find] Draw the shear and bending-moment diagrams
Maximum normal stress

[FBD]



(Fig. 2)

[FPU] $V_D - V_C = -\int_{x_C}^{x_D} w\,dx$, $M_D - M_C = \int_{x_C}^{x_D} V\,dx$, $\sigma =$

[Solution]

(1) Determine the reactions at the supports from the FBD of the entire beam (Fig. 2)

$+\circlearrowleft \Sigma M_A = 0 \quad -(12\text{Kips})(3\text{ft}) + R_D\cdot(12\text{ft}) - (6\text{Kips})(14\text{ft}) = 0 \Rightarrow R_D = 10\text{ Kips}$

$+\uparrow \Sigma F_y = 0 \quad R_A - 12\text{ Kips} + R_D - 6\text{ Kips} = 0 \Rightarrow R_A = 8\text{ Kips}$

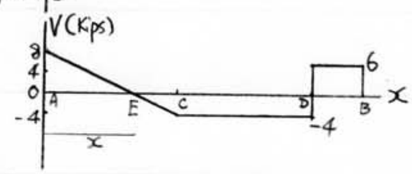
(2) Shear:

$V_A = R_A = 8\text{ Kips}$

$V_C = V_A - \int_A^C w\,dx = 8\text{ Kips} - (2\text{ Kips/ft})(6\text{ft}) = -4\text{ Kips}$

C to D: $V = -4\text{ Kips}$

D to B: $V = -4\text{ Kips} + 10\text{ Kips} = 6\text{ Kips}$



(3) Areas of shear diagram

$\boxed{|V|_{max} = 8\text{ Kips}}$

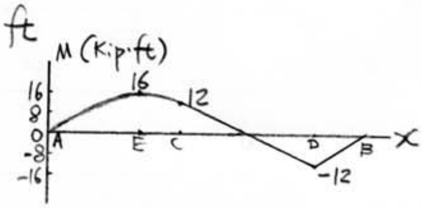
$V_E = V_A - \int_A^E w\,dx = 8\text{Kips} - (2\text{Kips/ft})x = 0 \Rightarrow x = 4\text{ ft}.$

A to E: $\int_A^E V\,dx = \frac{(8\text{ kips})(4\text{ft})}{2} = 16\text{ Kip}\cdot\text{ft}$

E to C: $\int_E^C V\,dx = \frac{-(4\text{Kips})(2\text{ft})}{2} = -4\text{ Kip}\cdot\text{ft}$

C to D: $\int_C^D V\,dx = -(4\text{Kips})(6\text{ft}) = -24\text{ Kip}\cdot\text{ft}$

D to B: $\int_D^B V\,dx = (6\text{Kips})(2\text{ft}) = 12\text{ Kip}\cdot\text{ft}$



(4) Bending moment

$M_A = 0$

$M_E = M_A + \int_A^E V\,dx = 0 + 16\text{Kip}\cdot\text{ft} = 16\text{Kip}\cdot\text{ft}$

$M_C = M_E + \int_E^C V\,dx = 16\text{Kip}\cdot\text{ft} - 4\text{ Kip}\cdot\text{ft} = 12\text{ Kip}\cdot\text{ft}$

$M_D = M_C + \int_C^D V\,dx = 12\text{Kip}\cdot\text{ft} - 24\text{Kip}\cdot\text{ft} = -12\text{ Kip}\cdot\text{ft}$

$\boxed{|M|_{max} = 16\text{ Kip}\cdot\text{ft}}$

$M_B = M_D + \int_D^B V\,dx = -12\text{ Kip}\cdot\text{ft} + 12\text{Kip}\cdot\text{ft} = 0$

(5) For W8x31, $S_x = 27.5\text{ in}^3$

Maximum normall stress $\sigma = \frac{|M|_{max}}{S_x} = \frac{(16\text{ Kip}\cdot\text{ft})(12\text{ in/ft})}{27.5\text{ in}^3} = \boxed{6.98\text{ ksi}}$