ENES 220 – Mechanics of Materials
Spring 2003

Solutions to Homework #5

Problem 3.72

Given : $L = 125\,ft$ $\quad d_1 = 16\,in$ $\quad d_2 = 8\,in$ $\quad \tau_{all} = 8500\,psi$

$G = 11.2 \times 10^6\,psi$ $\quad f = 165\,rpm$

To Find: (a) the maximum power that can be transmitted P

(b) the angle of twist of the shaft φ

FBD : not required

FPU : $T = \frac{J\tau}{c}$, $P = 2\pi f T$, $\varphi = \frac{TL}{GJ}$

Solution : Polar moment of inertia J

$$C_2 = \tfrac{1}{2} d_2 = 8\,in, \quad C_1 = \tfrac{1}{2} d_1 = 4\,in$$

$$J = \frac{\pi}{2}(C_2^4 - C_1^4) = \frac{\pi}{2}((8in)^4 - (4in)^4) = 6031.86\,in^4$$

Allowable Torque T

$$T = \frac{J\,\tau_{all}}{C_2} = \frac{(6031.86in^4)\cdot(8500\,psi)}{8\,in} = 6.4088 \times 10^6\,lb\cdot in$$

(a) Maximum power that can be transmitted P

$$f = \frac{165\,rpm}{60\,sec.} = 2.75\,Hz$$

$$P = 2\pi f T = 2\pi\,(2.75\,Hz)(6.4088 \times 10^6\,lb\cdot in)$$

$$= 110.74 \times 10^6\,lb\cdot in/s = \underline{\underline{16.78 \times 10^3\,hp}}.$$

(b) Twist angle of the shaft φ

$$L = 125\,ft = 1500\,in$$

$$\varphi = \frac{TL}{GJ} = \frac{(6.4088 \times 10^6\,lb\cdot in)\cdot(1500\,in)}{(11.2 \times 10^6\,psi)(6031.86\,in^4)}$$

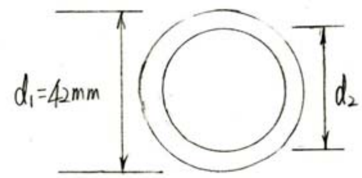
$$= 0.1423\,rad = \underline{\underline{8.15^\circ}}$$

Problem 3.85

Given : $d_1 = 42mm$ $\tau_{all} = 75\,MPa$ $G = 77\,GPa$ $T = 900\,N\cdot m$

$\varphi_{all} = 4^\circ$ $L = 1.6m$

To Find: d_2



FBD : not required

FPU : $\tau = \frac{TC}{J}$ $\varphi = \frac{TL}{GJ}$

Solution: Polar moment of Inertia J

Based on stress limit $\tau_{all} = 75MPa = 75 \times 10^6 Pa$

$$C_1 = \frac{1}{2} d_1 = 0.021m$$

$$\tau = \frac{TC}{J}$$

$$\therefore J = \frac{TC_1}{\tau_{all}} = \frac{(900N.m)(0.021m)}{75 \times 10^6 Pa} = 252 \times 10^{-9} m^4$$

Based on angle of twist limit $\varphi = 4^\circ = 69.813 \times 10^{-3}$ rad

$$\varphi = \frac{TL}{GJ} \quad , \quad G = 77GPa = 77 \times 10^9 Pa$$

$$\therefore J = \frac{TL}{G\varphi} = \frac{(900\,N\cdot m)(1.6m)}{(77 \times 10^9 Pa)(69.813 \times 10^{-3} rad)} = 267.88 \times 10^{-4} m^4$$

Larger value for J govern, $\therefore J = 267.88 \times 10^{-4} m^4$

Inner diameter d_2

$$J = \frac{\pi}{2}(C_1^4 - C_2^4)$$

$$\therefore C_2 = \left(C_1^4 - \frac{2J}{\pi}\right)^{\frac{1}{4}} = \left((0.021\,m)^4 - \frac{2 \cdot (267.88 \times 10^{-4} m^4)}{\pi}\right)^{\frac{1}{4}}$$

$$= 12.44 \times 10^{-3} m = 12.44mm$$

$$\therefore d_2 = 2C_2 = \underline{\underline{24.9\,mm}}$$

Problem 3.90

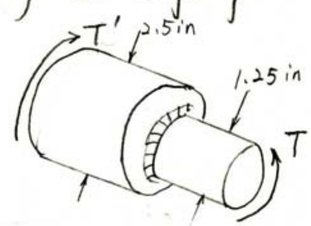
Given: $P = 60hp \quad \tau_{all} = 6000\,psi \quad r = 0.25\,in$

$D = 2.5\,in \quad d = 1.25\,in$

To Find: the smallest permissible speed of the shaft f

FBD: not required

FPU: $\tau = \frac{KTc}{J} \qquad P = 2\pi f T$



Solution:

Stress concentration factor K

$$\frac{r}{d} = \frac{0.25\,in}{1.25\,in} = 0.200$$

$$\frac{D}{d} = \frac{2.5\,in}{1.25\,in} = 2.00$$

From Figure 3.32 $K = 1.26$

Allowable Torque T

For smaller shaft $c = \frac{1}{2}d = \frac{1}{2}(1.25\,in) = 0.625\,in$

$$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$$

$$T = \frac{\pi c^3 \tau_{all}}{2K} = \frac{\pi\,(0.625\,in)^3\,(6000\,psi)}{2 \cdot (1.26)} = 1.826 \times 10^3\,lb \cdot in$$

The smallest permissible speed of the shaft f

$$P = 60hp = (60\,hp)\,(6600\,lb \cdot in/s \cdot hp) = 396 \times 10^3\,lb \cdot in/s$$

$$P = 2\pi f T$$

$$\therefore f = \frac{P}{2\pi T} = \frac{396 \times 10^3\,lb \cdot in/s}{2\pi\,(1.826 \times 10^3\,lb \cdot in)} = 34.5\,Hz$$

$$= \underline{\underline{2071\,rpm}}$$

Problem 3.140

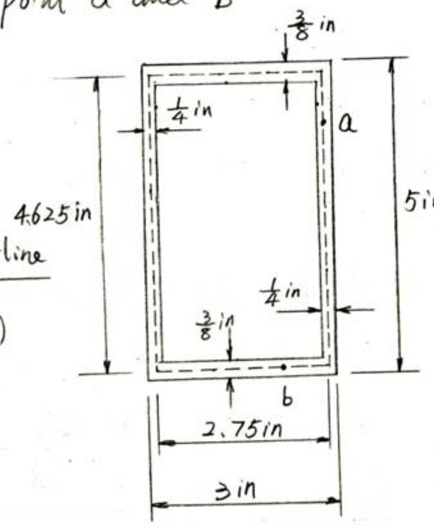
Given: $T = 50\ kip \cdot in$ $\quad t_a = 0.25\ in \quad t_b = 0.375\ in$

$h = 5\ in \quad w = 3\ in$

To Find: shearing stress at point a and b

FBD: not required

FPU: $\tau = \dfrac{T}{2t\mathfrak{a}}$



Solution:

Area bounded by centerline

$$\mathfrak{a} = (4.625\ in)(2.75\ in) = 12.719\ in^2$$

Shearing Stresses

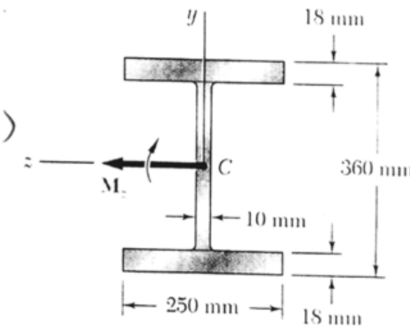
At point a $\quad t_a = 0.25\ in$

$$\tau = \frac{T}{2t_a\mathfrak{a}} = \frac{50\ kip \cdot in}{2 \cdot (0.25\ in)(12.719\ in^2)} = \underline{\underline{7.86\ ksi}}$$

At point b $\quad t_b = 0.375\ in$

$$\tau = \frac{T}{2t_b\mathfrak{a}} = \frac{50\ kip \cdot in}{2 \cdot (0.375\ in)(12.719\ in^2)} = \underline{\underline{5.24\ ksi}}$$

Problem 4.4



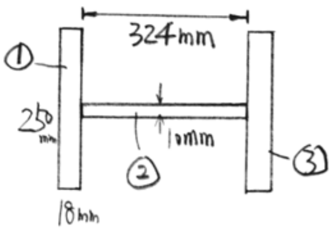
[Given] $\sigma_Y = 345$ MPa, $\sigma_U = 450$ MPa, F.S. = 3.0

[To Find] largest couple can be applied (bent about y axis)

[FBD] Not required

[FPU] $I = \frac{1}{12}bh^3$, $\sigma_{all} = \frac{\sigma_U}{F.S.}$ $M = \frac{\sigma I}{C}$

[Solution]



$$I_1 = \frac{1}{12}b_1h_1^3 = \frac{1}{12}(18\text{mm})(250\text{mm})^3 = 23437500\text{mm}^4$$
$$= 2.34375 \times 10^{-5}\ \text{m}^4$$

$$I_3 = I_1 = 2.34375 \times 10^{-5}\ \text{m}^4$$

$$I_2 = \frac{1}{12}b_2h_2^3 = \frac{1}{12}(324\text{mm})(10\text{mm})^3 = 27000\ \text{mm}^4 = 2.7 \times 10^{-8}\ \text{m}^4$$

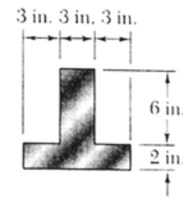
$$I = I_1 + I_2 + I_3 = 2.34375 \times 10^{-5}\text{m}^4 + 2.7 \times 10^{-8}\text{m}^4 + 2.34375 \times 10^{-5}\text{m}^4$$
$$= 4.6902 \times 10^{-5}\ \text{m}^4$$

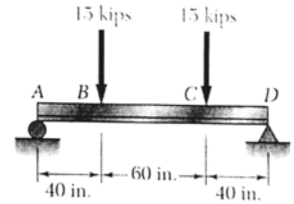
$$\sigma = \frac{Mc}{I}, \quad \text{where } c = \frac{250\text{ mm}}{2} = 125\text{ mm} = 0.125\text{ m}$$

$$\sigma_{all} = \frac{\sigma_U}{F.S.} = \frac{450\text{ MPa}}{3.0} = 150 \times 10^6\ \text{Pa}$$

$$\therefore M_{all} = \frac{\sigma_{all} I}{c} = \frac{(150 \times 10^6\text{ Pa})(4.6902 \times 10^{-5}\text{ m}^4)}{0.125\text{ m}} = 56282.4\ \text{N}\cdot\text{m}$$
$$= \boxed{56.3\ \text{kN}\cdot\text{m}}$$

Problem 4.10

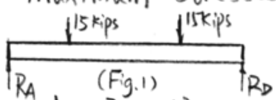


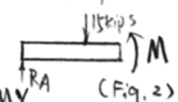


[Given]

[To Find] Maximum stresses in BC portion.

[FBD]

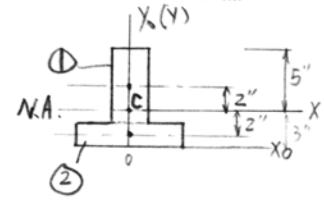




[FPU] $I = \frac{1}{12}bh^3 + Ad^2$, $\sigma = -\frac{My}{I}$.

[Solution]

(1)



	$A(in^2)$	$\bar{y}_0$ (in)	$A\bar{y}_0$ (in^3)
①	$3 \cdot 6 = 18$	$3+2=5$	$18 \cdot 5 = 90$
②	$9 \cdot 2 = 18$	1	$18 \cdot 1 = 18$
Σ	36		108

$$\bar{Y}_0 = \frac{108\,in^3}{36\,in^2} = 3\,in$$

$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(3in)(6in)^3 + (18in^2)(2in)^2 = 126\,in^4$$

$$I_2 = \frac{1}{12}b_2h_2^3 + A_2d_2^2 = \frac{1}{12}(9in)(2in)^3 + (18in^2)(2in)^2 = 78\,in^4$$

$$I = I_1 + I_2 = 126\,in^4 + 78\,in^4 = 204\,in^4$$

$$y_{top} = 3in + 2in = 5\,in, \quad y_{bot} = -(1in + 2in) = -3\,in$$

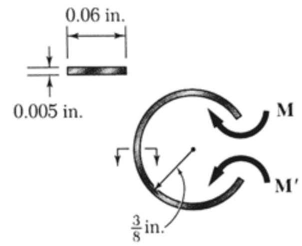
(2) From statics, using the symmetry feature $R_A = R_D = 15\,kips$

From Fig. 2, $+\circlearrowleft \Sigma M_A = 0 \quad M - P(40in) = 0 \Rightarrow M = (15kips)(40in) = 600\,kip \cdot in$

$$\sigma_{top} = -\frac{My_{top}}{I} = -\frac{(600\,kip \cdot in)(5\,in)}{204\,in^4} = \boxed{-14.71\,ksi}$$

$$\sigma_{bot} = -\frac{My_{bot}}{I} = -\frac{(600\,kip \cdot in)(-3\,in)}{204\,in^4} = \boxed{8.82\,ksi}$$

Problem 4.27



[Given] $E = 29 \times 10^6$ psi, $\rho = \frac{3}{8}$ in

[To Find] σ_{max}, M

[FBD] Not required

[FPU] $\frac{1}{\rho} = \frac{\sigma_m}{Ec}$, $\frac{1}{\rho} = \frac{M}{EI}$, $I = \frac{1}{12}bh^3$

[Solution]

(1) $$c = \frac{0.005\text{ in}}{2} = 0.0025\text{ in}$$

$$\frac{1}{\rho} = \frac{\sigma_m}{Ec}$$

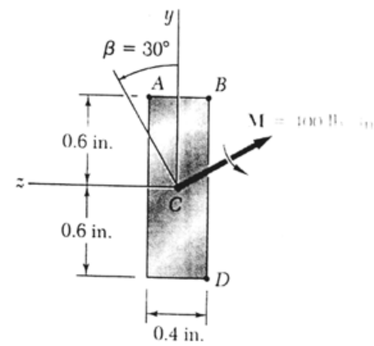
$$\therefore \sigma_m = \frac{Ec}{\rho} = \frac{(29 \times 10^6\text{ psi})(0.0025\text{ in})}{(0.375\text{ in})} = 193333.3\text{ psi} = \boxed{193.33\text{ ksi}}$$

(2) $$I = \frac{1}{12}bh^3 = \frac{1}{12}(0.06\text{ in})(0.005\text{ in})^3 = 6.25 \times 10^{-10}\text{ in}^4$$

$$\frac{1}{\rho} = \frac{M}{EI}$$

$$\therefore M = \frac{EI}{\rho} = \frac{(29 \times 10^6\text{ psi})(6.25 \times 10^{-10}\text{ in}^4)}{0.375\text{ in}} = \boxed{4.833 \times 10^{-2}\text{ lb}\cdot\text{in}}$$

Problem 4.145



[Given] →

[To Find] σ_A, σ_B, σ_D

[FBD] Not required

[FPU] $\sigma = -\dfrac{M_z y}{I_z} + \dfrac{M_y z}{I_y}$

[Solution]

$$I_z = \frac{1}{12}(0.4\,in)(1.2\,in)^3 = 0.0576\,in^4$$

$$I_y = \frac{1}{12}(1.2\,in)(0.4\,in)^3 = 0.0064\,in^4$$

$$M_z = -400\cos 30° = -346.41\ lb\cdot in$$

$$M_y = 400\sin 30° = 200\ lb\cdot in$$

(a) $y_A = 0.6\,in$, $z_A = 0.2\,in$

$$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(-346.41\,lb\cdot in)(0.6\,in)}{(0.0576\,in^4)} + \frac{(200\,lb\cdot in)(0.2\,in)}{(0.0064\,in^4)}$$

$$= 9858.4\,psi = \boxed{9.86\,ksi}$$

(b) $y_B = 0.6\,in$, $z_B = -0.2\,in$

$$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(-346.41\,lb\cdot in)(0.6\,in)}{(0.0576\,in^4)} + \frac{(200\,lb\cdot in)(-0.2\,in)}{(0.0064\,in^4)}$$

$$= -2641.6\,psi = \boxed{-2.64\,ksi}$$

(c) $y_D = -0.6\,in$, $z_D = -0.2\,in$

$$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(-346.41\,lb\cdot in)(-0.6\,in)}{(0.0576\,in^4)} + \frac{(200\,lb\cdot in)(-0.2\,in)}{(0.0064\,in^4)}$$

$$= -9858.4\,psi = \boxed{-9.86\,ksi}$$