ENES 220 – Mechanics of Materials
Spring 2003

Solutions to Homework #13

Problem 7.119

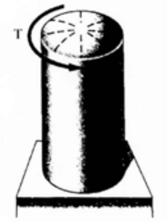


Fig. P7.118 and *P7.119*

[Given] $d = 180\,mm$, $t = 12\,mm$, $P = 8\,MPa$, $\sigma_{max} = 75\,MPa$

[To Find] T

[FBD] N/A

[FPU] $\sigma_1 = \frac{pr}{t}$, $\sigma_2 = \frac{pr}{2t}$, $\tau = \frac{Tc}{J}$

[Solution]

(1) $r = \frac{1}{2}d = \left(\frac{1}{2}\right)(180\,mm) = 90\,mm$, $t = 12\,mm$

$$\sigma_1 = \frac{pr}{t} = \frac{(8\,MPa)(90\,mm)}{12\,mm} = 60\,MPa$$

$$\sigma_2 = \frac{pr}{2t} = \frac{(8\,MPa)(90\,mm)}{(2)(12\,mm)} = 30\,MPa$$

$$\sigma_{ave} = \frac{\sigma_1 + \sigma_2}{2} = \frac{60\,MPa + 30\,MPa}{2} = 45\,MPa,$$

$$\sigma_{max} = 75\,MPa$$

$$\therefore R = \sigma_{max} - \sigma_{ave} = 75\,MPa - 45\,MPa = 30\,MPa$$

$$R = \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau_{xy}^2}$$

$$\therefore \tau_{xy} = \sqrt{R^2 - \left(\frac{\sigma_1 - \sigma_2}{2}\right)^2} = \sqrt{(30\,MPa)^2 - \left(\frac{60\,MPa - 30\,MPa}{2}\right)^2} = 25.98\,MPa$$

(2) $c_1 = 90\,mm$, $c_2 = 90\,mm + 12\,mm = 102\,mm$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}\left[(102\,mm)^4 - (90\,mm)^4\right] = 66.968 \times 10^6\,mm^4 = 66.968 \times 10^{-6}\,m^4$$

$$\tau_{xy} = \frac{Tc_2}{J}$$

$$T = \frac{J\tau_{xy}}{c_2} = \frac{(66.968 \times 10^{-6}\,m^4)(25.98 \times 10^6\,Pa)}{102 \times 10^{-3}\,m} = 17.06 \times 10^3\,N \cdot m = \boxed{17.06\,kN \cdot m}$$

Problem 8.31

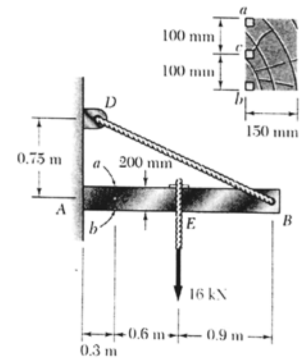
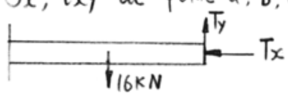


Fig. P8.31

[Given] $T_{BD} = 10.4$ KN.

[To Find] σ_x, τ_{xy} at point a, b, c.

[FBD]



[FPU] $\sigma = \frac{P}{A} - \frac{My}{I}$, $\tau = \frac{VQ}{It}$ $\left(\tau = \frac{3}{2}\frac{V}{A}\right)$

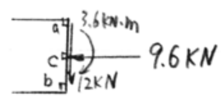
[Solution]

(1) $\overline{BD} = \sqrt{\overline{AD}^2 + \overline{AB}^2} = \sqrt{(0.75\,m)^2 + (1.8\,m)^2} = 1.95$

Vertical component of T_{BD}: $T_y = \frac{0.75}{1.95} T_{BD} = 4$ kN ↑

Horizontal component of T_{BD}: $T_x = \frac{1.8}{1.95} T_{BD} = 9.6$ KN ←

(2) At section containing points a, b, c



$P = -9.6$ KN $\quad V = 16\,KN - 4\,kN = 12\,kN$

$M = (T_y)(1.5\,m) - (16\,KN)(0.6\,m) = 6\,KN\cdot m - 9.6\,KN\cdot m = -3.6\,KN\cdot m$

Section properties

$A = (0.15\,m)(0.2\,m) = 0.03\,m^2$

$I = \frac{1}{12}(0.15\,m)(0.2\,m)^3 = 1\times10^{-4}\,m^4$

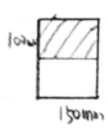
(3) At point a: $\sigma_x = \frac{P}{A} - \frac{My_a}{I} = \frac{-9.6\times10^3 N}{0.03\,m^2} - \frac{(-3.6\times10^3 N\cdot m)(0.1\,m)}{1\times10^{-4}\,m^4} = 3.28\times10^6\,Pa = \boxed{3.28\ MPa}$

$\boxed{\tau_{xy} = 0}$

At point b: $\sigma_x = \frac{P}{A} - \frac{My_b}{I} = \frac{-9.6\times10^3 N}{0.03\,m^2} - \frac{(-3.6\times10^3 N\cdot m)(-0.1\,m)}{1\times10^{-4}\,m^4} = -3.92\times10^6\,Pa = \boxed{-3.92\ MPa}$

$\boxed{\tau_{xy} = 0}$

At point c: $\sigma_x = \frac{P}{A} = \frac{-9.6\times10^3 N}{0.03\,m^2} = -0.32\times10^6\,Pa = \boxed{-0.32\ MPa}$



$Q = (150\,mm)(100\,mm)(50\,mm) = 75\times10^4\,mm^3 = 7.5\times10^{-4}\,m^3$

$\tau_{xy} = \frac{VQ}{It} = \frac{(12\times10^3 N)(7.5\times10^{-4}\,m^3)}{(1\times10^{-4}\,m^4)(0.15\,m)} = 600\times10^3\,Pa = \boxed{0.6\ MPa}$

or $\left(\tau_{xy} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\cdot\frac{12\times10^3 N}{0.03\,m^2} = 6\times10^5\,Pa = 0.6\,MPa\right)$

Problem 8.32

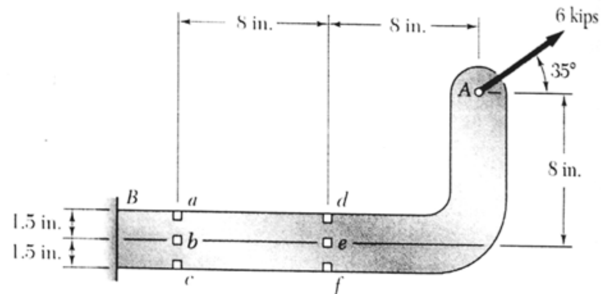


Fig. P8.32 and P8.33

[Given] $t = 0.8$ in

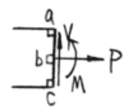
[To Find] σ_x, τ_{xy} @ points a, b, c

[FBD] N/A

[FPU] $\sigma = \frac{P}{A} - \frac{My}{I}$, $\tau = \frac{3}{2}\frac{V}{A}$

[Solution]

(1) At the section containing points a, b, and c.



$P = (6\text{kips})(\cos 35°) = 4.9149$ kips

$V = (6\text{Kips})(\sin 35°) = 3.4415$ kips

$M = (6\sin 35°\ \text{kips})(16\text{in}) - (6\cos 35°\ \text{Kips})(8\text{in}) = 15.744$ Kip·in

$A = (0.8\text{ in})(3.0\text{ in}) = 2.4\text{ in}^2$

$I = \frac{1}{12}bh^3 = \frac{1}{12}(0.8\text{ in})(3.0\text{ in})^3 = 1.8\text{ in}^4$

(2) At point a: $\sigma_x = \frac{P}{A} - \frac{My}{I} = \frac{4.9149\text{ Kips}}{2.4\text{ in}^2} - \frac{(15.744\text{ Kip·in})(1.5\text{ in})}{1.8\text{ in}^4} = \boxed{-11.07\text{ ksi}}$

$\boxed{\tau_{xy} = 0}$

At point c: $\sigma_x = \frac{P}{A} - \frac{My}{I} = \frac{4.9149\text{ Kips}}{2.4\text{ in}^2} - \frac{(15.744\text{ Kip·in})(-1.5\text{in})}{1.8\text{ in}^4} = \boxed{15.17\text{ ksi}}$

$\boxed{\tau_{xy} = 0}$

At point b: $\sigma_x = \frac{P}{A} = \frac{4.9149\text{ Kips}}{2.4\text{ in}^2} = \boxed{2.05\text{ ksi}}$

$\tau_{xy} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\cdot\frac{3.4415\text{ Kips}}{2.4\text{ in}^2} = \boxed{2.15\text{ ksi}}$

Problem 8.37

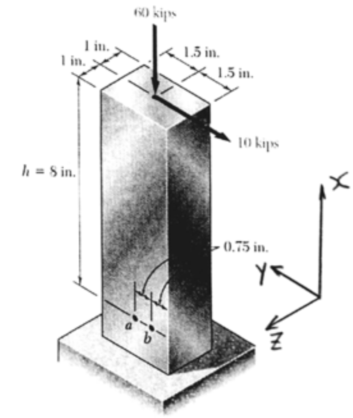


Fig. P8.37 and P8.38

[Given] $\rightarrow$

[To Find] (a) principal stresses and planes ; (b) τ_{max} at point a.

[FBD] N/A

[FPU] $\sigma = \frac{P}{A}$, $\tau = \frac{3}{2}\frac{V}{A}$

[Solution]

(1) At the section containing point a

$P = -60$ kips,

$V = 10$ kips

$M = -(10 \text{ kips})(8 \text{ in}) = -80 \text{ kip} \cdot \text{in}$

$A = (2 \text{ in})(3 \text{ in}) = 6 \text{ in}^2$

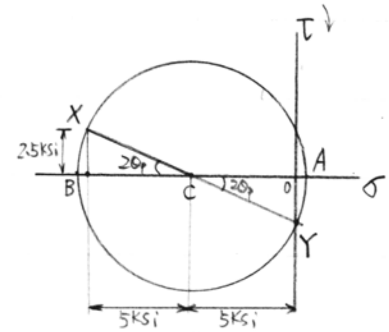
$I = \frac{1}{12}(2 \text{ in})(3 \text{ in})^3 = 4.5 \text{ kip} \cdot \text{in}$

(2) At point a:

$\sigma_x = \frac{P}{A} = \frac{(-60 \text{ kip})}{6 \text{ in}^2} = -10 \text{ ksi}$, $\sigma_y = 0$

$\tau = \frac{3}{2}\frac{V}{A} = \frac{3}{2} \cdot \frac{10 \text{ kip}}{6 \text{ in}^2} = 2.5 \text{ ksi}$

(3) Use Mohr's circle



$\sigma_C = -5 \text{ ksi}$

$R = \sqrt{(5 \text{ ksi})^2 + (2.5 \text{ ksi})^2} = 5.590 \text{ ksi}$

$\sigma_A = \sigma_C + R = -5 \text{ ksi} + 5.590 \text{ ksi} = \boxed{0.590 \text{ ksi}}$

$\sigma_B = \sigma_C - R = -5 \text{ ksi} - 5.590 \text{ ksi} = \boxed{-10.590 \text{ ksi}}$

$\tan 2\theta_p = \frac{2.5}{5} = 0.5$

$\therefore 2\theta_p = 26.565°$

$\boxed{\theta_p = 13.3°, \quad 103.3°}$

$\tau_{max} = R = \boxed{5.590 \text{ ksi}}$

Problem 8.39

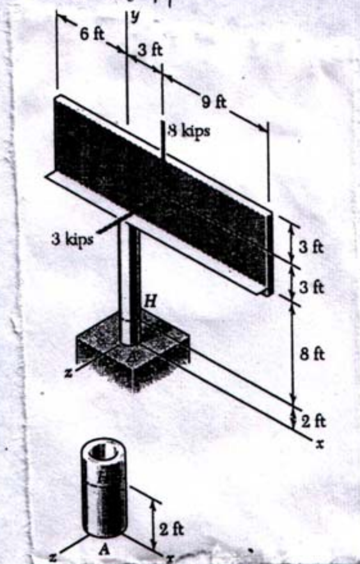
Given $P = 8\text{ kips}$ $d_o = 15\text{ in}$ $t = 0.5\text{ in}$ $F_w = 3\text{ kips}$

To determine the normal and shearing stress at H

FBD as shown

FPU stresses under combined loading

Solution:



Internal forces in section containing H

$P = 8\text{ Kips (compression)}$

$T = (3\text{ Kips})\cdot(3\text{ ft}) = 9\text{ Kip}\cdot\text{ft} = 108\text{ Kip}\cdot\text{in}$

$M_x = -(3\text{ Kips})(11\text{ ft}) = -33\text{ Kip}\cdot\text{ft} = -396\text{ Kip}\cdot\text{in}$

$M_z = -(8\text{ Kips})(3\text{ ft}) = -24\text{ Kip}\cdot\text{ft} = -288\text{ Kip}\cdot\text{in}$

$V = 3\text{ Kip}$

Section property

$d_o = 15\text{ in}$ $c_o = \frac{1}{2}d_o = 7.5\text{ in}$

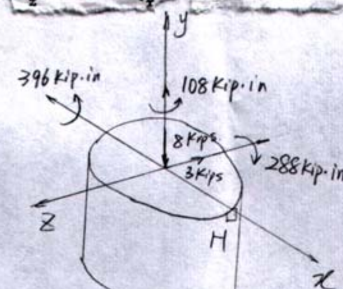
$c_i = c_o - t = (7.5\text{ in}) - (0.5\text{ in}) = 7.0\text{ in}$

$A = \pi(c_o^2 - c_i^2) = \pi\left((7.5\text{ in})^2 - (7.0\text{ in})^2\right) = 22.777\text{ in}^2$

$I = \frac{\pi}{4}(c_o^2 - c_i^2) = \frac{\pi}{4}((7.5\text{ in})^4 - (7.0\text{ in})^4) = 599.31\text{ in}^4$

$J = 2I = 2\cdot(599.31\text{ in}^4) = 1198.62\text{ in}^4$

$Q = \frac{2}{3}(c_o^3 - c_i^3) = \frac{2}{3}((7.5\text{ in})^3 - (7.0\text{ in})^3) = 52.583\text{ in}^3$



Normal stress

$$\sigma = -\frac{P}{A} - \frac{MC}{I} = -\frac{8\text{ Kips}}{22.777\text{ in}^2} - \frac{(288\text{ Kip}\cdot\text{in})\cdot(7.5\text{ in})}{599.31\text{ in}^4}$$

$$= -3.96\text{ ksi}$$

Shearing stress

$$\tau = \frac{TC}{J} + \frac{VQ}{It} = \frac{(108\text{ Kip}\cdot\text{in})\cdot(7.5\text{ in})}{1198.62\text{ in}^4} + \frac{(3\text{ Kips})(52.583\text{ in}^3)}{(599.31\text{ in}^4)(1\text{ in})}$$

$$= 0.938\text{ ksi}$$

Problem 8.42

Given $V = 1.5$ $T = 9\ \text{kip}\cdot\text{in}$ $d = 2.5\ \text{in}$

To determine the normal and shearing stress at point H and K

FBD as shown

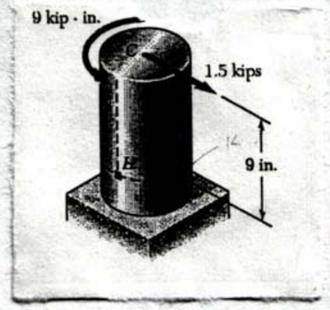
FPU stress under combined loading

Solution

Internal forces in Section containing H and K

$P = 0$ $V = 1.5\ \text{kips}$ $T = 9\ \text{kip}\cdot\text{in}$

$M = (1.5\ \text{kips})(9\cdot\text{in}) = 13.5\ \text{kip}\cdot\text{in}$



Section property

$A = \pi c^2 = \pi\cdot(1.25\ \text{in})^2 = 4.909\ \text{in}^2$

$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(1.25\ \text{in})^4 = 1.9175\ \text{in}^4$

$J = 2I = 2\cdot(1.9175\ \text{in}^4) = 3.835\ \text{in}^4$

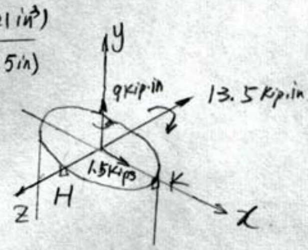
$Q = \frac{2}{3}\pi c^3 = \frac{2}{3}\pi(1.25\ \text{in})^3 = 1.3021\ \text{in}^3$

Stresses at H

$\sigma_H = 0$

$$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(9\ \text{kip}\cdot\text{in})(1.25\ \text{in})}{3.835\ \text{in}^4} + \frac{(1.5\ \text{kip})(1.3021\ \text{in}^3)}{(1.9175\ \text{in}^4)(2.5\ \text{in})}$$

$$= 3.34\ \text{ksi}$$



Stresses at K

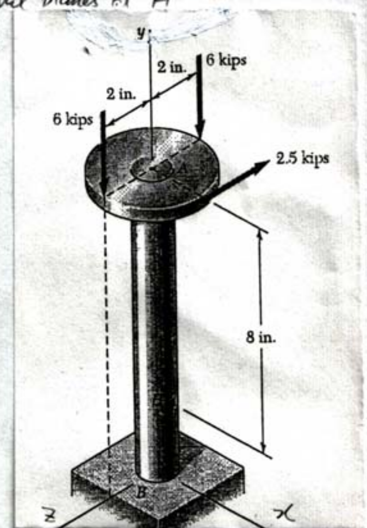
$$\sigma_K = -\frac{Mc}{I} = -\frac{(13.5\ \text{kip}\cdot\text{in})(1.25\ \text{in})}{1.9175\ \text{in}^4} = -8.80\ \text{ksi}$$

$$\tau_K = \frac{Tc}{J} = \frac{(9\ \text{kip}\cdot\text{in})(1.25\ \text{in})}{3.835\ \text{in}^4} = 2.93\ \text{ksi}$$

Problem 8.47

Given: three forces applied to a plate as shown. $d = 1.8$ in

To: determine (a) the principal stresses and principal planes at H
(b) the maximum shearing stress



FBD: as shown

FPU: stresses under combined loading

Solution:

<u>Internal forces in the section containing H</u>

$P = (6\,\text{kips}) + (6\,\text{kips}) = 12\,\text{kips} \qquad V = 2.5\,\text{kips}$

$M = (2.5\,\text{kips})\cdot(8\,\text{in}) = 20\,\text{kip}\cdot\text{in}$

$T = (2.5\,\text{kips})(2\,\text{in}) = 5\,\text{kip}\cdot\text{in}$

<u>Section property</u>

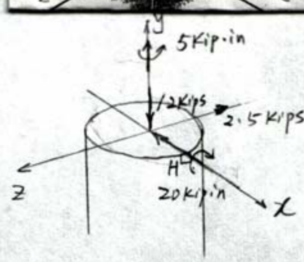
$d = 1.8\,\text{in} \qquad C = \frac{1}{2}d = 0.9\,\text{in}$

$A = \pi C^2 = \pi(0.9\,\text{in})^2 = 2.545\,\text{in}^2$

$I = \frac{\pi}{4}C^4 = \frac{\pi}{4}(0.9\,\text{in})^4 = 0.5153\,\text{in}^4$

$J = 2I = 2\cdot(0.5153\,\text{in}^4) = 1.0306\,\text{in}^4$

$Q = \frac{2}{3}C^3 = \frac{2}{3}(0.9\,\text{in})^3 = 0.486\,\text{in}^3$

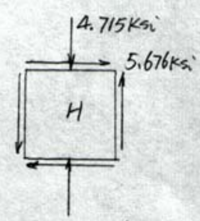


<u>Stresses at H</u>

$\sigma_H = -\frac{P}{A} = -\frac{12\,\text{Kips}}{2.545\,\text{in}^2} = -4.715\,\text{Ksi}$

$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(5\,\text{Kip}\cdot\text{in})(0.9\,\text{in})}{1.0306\,\text{in}^4} + \frac{(2.5\,\text{Kips})(0.486\,\text{in}^3)}{(0.5153\,\text{in}^4)(1.8\,\text{in})}$

$= 5.676\,\text{Ksi}$



(a) <u>Principal stresses and principal planes</u>

We draw Mohr's circle for the stresses at H

$\tan 2\theta_p = \frac{(5.676\,\text{Ksi})}{(2.3575\,\text{Ksi})} = 2.408$

$\therefore\ \theta_p = 33.7^\circ$

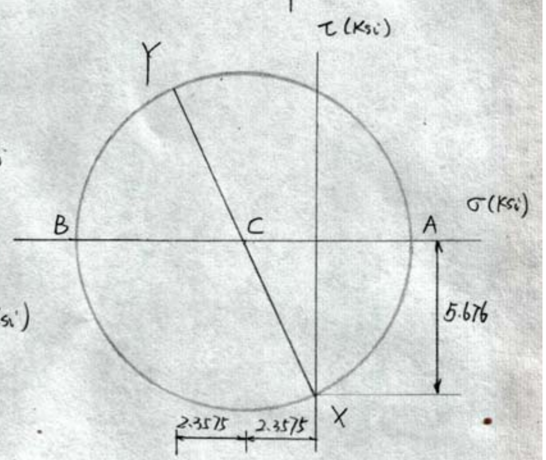
$R = \sqrt{(2.3575\,\text{Ksi})^2 + (5.676\,\text{Ksi})^2} = 6.1461\,\text{Ksi}$

$\sigma_{max} = R - OC = (6.1461\,\text{Ksi}) - (2.3575\,\text{Ksi})$

$= 3.79\,\text{Ksi}$

$\sigma_{min} = -R - OC = (-6.1461\,\text{Ksi}) - (2.3575\,\text{Ksi})$

$= -8.50\,\text{Ksi}$



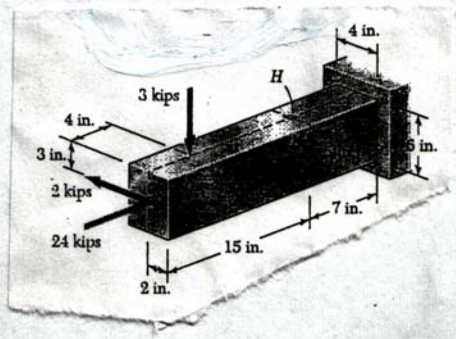
(b) The maximum shearing stress

$\tau_{max} = R = 6.15\,\text{Ksi}$

Problem 8.51

Given: the beam and loading shown

To: determine the principal stresses and the maximum shearing stress at point H



FBD: as shown

FPU: stress under combined loading

Solution:

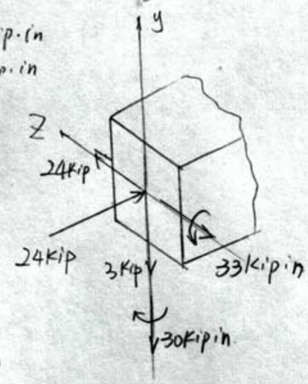
Internal forces in section containing H

$P = 24\,\text{kips}$ $\quad V = \begin{cases} 3\,\text{Kips} & \text{vertical;} \\ 2\,\text{Kips} & \text{horizontal} \end{cases}$

The bending moment components are

about horizontal axis: $M = (3\,\text{Kip})(11\,\text{in}) = 33\,\text{Kip}\cdot\text{in}$

about vertical axis $M = (2\,\text{Kip})(15\,\text{in}) = 30\,\text{Kip}\cdot\text{in}$



Section property

$A = (4\,\text{in})(6\,\text{in}) = 24\,\text{in}^2$

$I_z = \frac{1}{12}(4\,\text{in})(6\,\text{in})^3 = 72\,\text{in}^4$

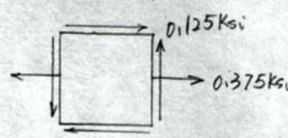
$I_y = \frac{1}{12}(6\,\text{in})(4\,\text{in})^3 = 32\,\text{in}^4$

Stresses at point H

$\sigma_H = -\frac{P}{A} + \frac{Mc}{I} = -\frac{24\,\text{Kips}}{24\,\text{in}^2} + \frac{(33\,\text{Kip}\cdot\text{in})\cdot(3\,\text{in})}{72\,\text{in}^4}$

$= 0.375\,\text{Ksi}$

$\tau_H = \frac{3V}{2A} = \frac{3\cdot(2\,\text{Kips})}{2\cdot(24\,\text{in}^2)} = 0.125\,\text{Ksi}$



Principal stresses

we draw Mohr's circle for the stress at H

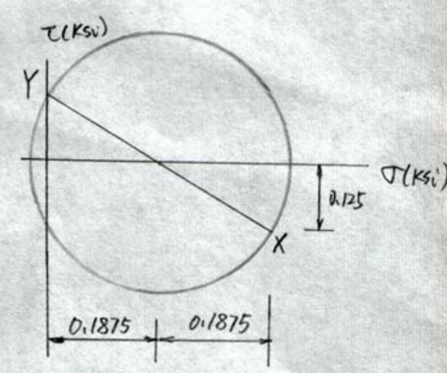
$R = \sqrt{(0.1875\,\text{Ksi})^2 + (0.125\,\text{Ksi})^2} = 0.2253\,\text{Ksi}$

$\sigma_{max} = OC + R = (0.1875\,\text{Ksi}) + (0.2253\,\text{Ksi})$

$= 0.413\,\text{Ksi}$

$\sigma_{min} = OC - R = (0.1875\,\text{Ksi}) - (0.2253\,\text{Ksi})$

$= -0.0378\,\text{Ksi}$



Maximum shearing stress

$\tau_{max} = R = 0.2253\,\text{Ksi}$