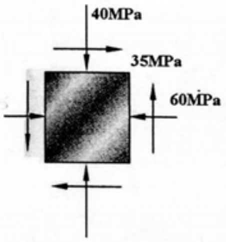
**ENES 220 – Mechanics of Materials**
**Spring 2003**

**Solutions to Homework #12**

**Problem 7.31**




**[Given]** $\longrightarrow$

**[To Find]** (a) principal planes ; (b) principal stresses ; (c) maximum shearing stress and the orientation of the planes ; (d) corresponding normal stress.

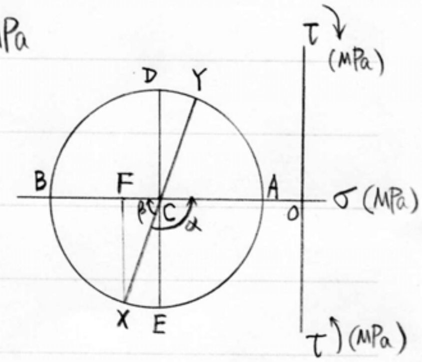
**[FBD]** N/A

**[FPU]** Mohr's circle

**[Solution]**

① $\sigma_x = -60\,MPa, \quad \sigma_y = -40\,MPa, \quad \tau_{xy} = 35\,MPa$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = \frac{(-60MPa)+(-40MPa)}{2} = -50\,MPa$$




② points:

X: $(\sigma_x, -\tau_{xy}) = (-60MPa, -35MPa)$

Y: $(\sigma_y, \tau_{xy}) = (-40MPa, 35MPa)$

C: $(\sigma_{ave}, 0) = (-50MPa, 0)$

③ $\tan\beta = \frac{FX}{CF} = \frac{35MPa}{10MPa} = 3.5$

$\Rightarrow \beta = 74.05°, \quad \alpha = 180° - \beta = 105.95°$

$\therefore \theta_B = -\frac{1}{2}\beta = -37.03°$ ◄

$\theta_A = \frac{1}{2}\alpha = 52.97°$ ◄

$R = CX = \sqrt{CF^2 + FX^2} = \sqrt{(10MPa)^2 + (35MPa)^2} = 36.4\,MPa$

$\sigma_{min} = \sigma_B = \sigma_{ave} - R = -50MPa - 36.4MPa = -86.4\,MPa$ ◄

$\sigma_{max} = \sigma_A = \sigma_{ave} + R = -50MPa + 36.4MPa = -13.6\,MPa$ ◄

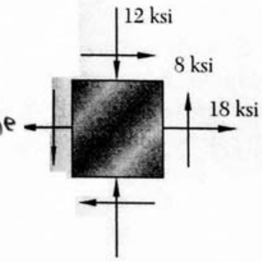
④ $\theta_D = \theta_A + 45° = 97.97°$ ◄

$\theta_E = \theta_B + 45° = 7.97°$ ◄

$\tau_{max} = R = 36.4\,MPa$ ◄

$\sigma' = \sigma_{ave} = -50\,MPa$ ◄

**Problem 7.33**



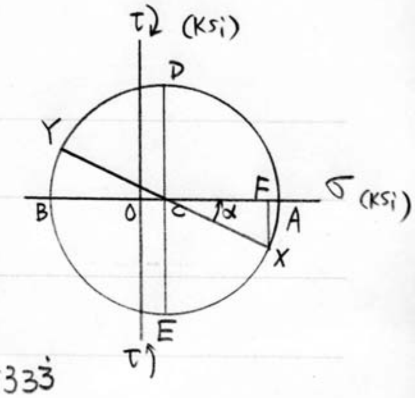

**[Given]** ⟶

**[To Find]** (a) Maximum in-plane shearing stress and the orientation of the planes; (b) The corresponding normal stress.

**[FBD]** N/A

**[FPU]** Mohr's circle

**[Solution]**




① $\sigma_x = 18\text{ ksi}$, $\sigma_y = -12\text{ ksi}$, $\tau_{xy} = 8\text{ ksi}$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = \frac{18\text{ksi} - 12\text{ksi}}{2} = 3\text{ ksi}$$

② Points:

X: $(\sigma_x, -\tau_{xy}) = (18\text{ksi}, -8\text{ksi})$

Y: $(\sigma_y, \tau_{xy}) = (-12\text{ksi}, 8\text{ksi})$

C: $(\sigma_{ave}, 0) = (3\text{ksi}, 0)$

③ $\tan\alpha = \dfrac{FX}{CF} = \dfrac{FX}{OF - OC} = \dfrac{8\text{ ksi}}{18\text{ksi} - 3\text{ksi}} = \dfrac{8}{15} = 0.533\dot{3}$

$\alpha = 28.07°$

$\theta_A = \frac{1}{2}\alpha = 14.04°$

④ $\theta_D = \theta_A + 45° = 59.04°$ ◀

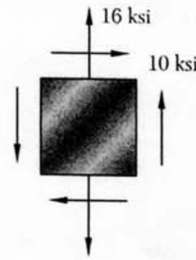
$\theta_E = \theta_A - 45° = -30.96°$ ◀

$R = \sqrt{(FX)^2 + (CF)^2} = \sqrt{(8\text{ksi})^2 + (15\text{ksi})^2} = 17\text{ ksi}$ ◀

$\sigma' = \sigma_{ave} = 3\text{ ksi}$ ◀

**Problem 7.38**

**[Given]** $\longrightarrow$




**[To Find]** $\sigma_x, \sigma_y, \tau_{xy}$ @ (a) $25^\circ \downarrow$ (b) $10^\circ \uparrow$

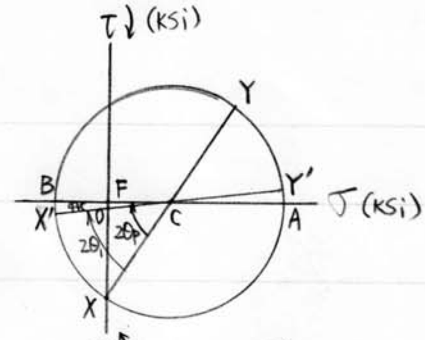
**[FBD]** N/A

**[FPU]** Mohr's circle

**[Solution]**

① $\sigma_x = 0, \quad \sigma_y = 16\ \text{ksi}, \quad \tau_{xy} = 10\ \text{ksi}$

$\sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = \dfrac{0 + 16\ \text{ksi}}{2} = 8\ \text{ksi}$




②
$X: (\sigma_x, -\tau_{xy}) = (0, -10\ \text{ksi})$

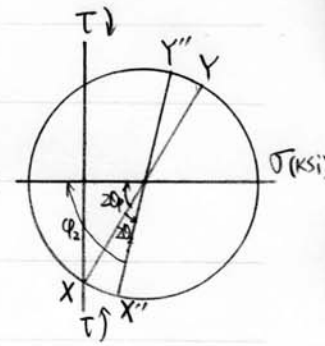
$Y: (\sigma_y, \tau_{xy}) = (16\ \text{ksi}, 10\ \text{ksi})$

$C: (\sigma_{ave}, 0) = (8\ \text{ksi}, 0)$

③ $\tan 2\theta_p = \dfrac{FX}{CF} = \dfrac{10\ \text{ksi}}{8\ \text{ksi}} = 1.25$

$2\theta_p = 51.34^\circ$

$R = \sqrt{(CF)^2 + (FX)^2} = \sqrt{(8\ \text{ksi})^2 + (10\ \text{ksi})^2} = 12.81\ \text{ksi}$




④ $\theta_1 = 25^\circ \downarrow \quad 2\theta_1 = 50^\circ \downarrow$

$\varphi_1 = 2\theta_p - 2\theta_1 = 51.34^\circ - 50^\circ = 1.34^\circ$

$\sigma_{x'} = \sigma_{ave} - R\cos\varphi_1 = 8\ \text{ksi} - (12.81\ \text{ksi})\cdot\cos(1.34^\circ) = -4.81\ \text{ksi}$ ◄

$\sigma_{y'} = \sigma_{ave} + R\cos\varphi_1 = 8\ \text{ksi} + (12.81\ \text{ksi})\cdot\cos(1.34^\circ) = 20.81\ \text{ksi}$ ◄

$\tau_{x'y'} = R\sin\varphi_1 = (12.81\ \text{ksi})\cdot\sin(1.34^\circ) = 0.30\ \text{ksi}$ ◄

⑤ $\theta = 10^\circ \uparrow \quad 2\theta = 20^\circ \uparrow$

$\varphi_2 = 2\theta_p + 2\theta_2 = 51.34^\circ + 20^\circ = 71.34^\circ$

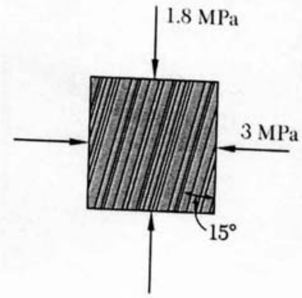
$\sigma_{x''} = \sigma_{ave} - R\cos\varphi_2 = 8\ \text{ksi} - (12.81\ \text{ksi})\cdot\cos(71.34^\circ) = 3.90\ \text{ksi}$ ◄

$\sigma_{y''} = \sigma_{ave} + R\cos\varphi_2 = 8\ \text{ksi} + (12.81\ \text{ksi})\cdot\cos(71.34^\circ) = 12.10\ \text{ksi}$ ◄

$\tau_{x''y''} = R\sin\varphi_2 = (12.81\ \text{ksi})\cdot\sin(71.34^\circ) = 12.14\ \text{ksi}$ ◄

**Problem 7.39**

**[Given]** ⟶




**[To Find]** $\tau_{x'y'}$, $\sigma_{x'}$

**[FBD]** N/A

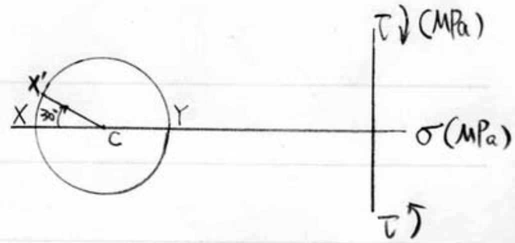
**[FPU]** Mohr's circle.

**[Solution]**

① $\sigma_x = -3\text{MPa}, \quad \sigma_y = -1.8\text{MPa}, \quad \tau_{xy} = 0$

$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = -2.4\text{MPa}$

② Points




X: $(\sigma_x, -\tau_{xy}) = (-3\text{MPa}, 0)$

Y: $(\sigma_y, \tau_{xy}) = (-1.8\text{MPa}, 0)$

C: $(\sigma_{ave}, 0) = (-2.4\text{MPa}, 0)$

③ $R = \overline{CX} = (-2.4\text{MPa}) - (-3\text{MPa}) = 0.6\text{MPa}$

④ $\theta = -15°, \quad 2\theta = -30°$

$\sigma_{x'} = \sigma_{ave} - R\cos 30° = -2.4\text{MPa} - (0.6\text{MPa})\cos 30° = -2.92\text{ MPa}$ ◄

$\tau_{x'y'} = -R\sin 30° = -(0.6\text{MPa})\sin 30° = -0.3\text{MPa}$ ◄

Problem 7.69

Given the state of plane stress. $\tau_{xy} = 8$ Ksi

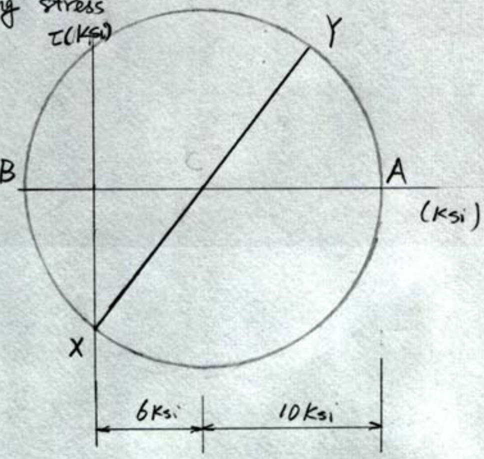
(a) $\sigma_x = 0$ and $\sigma_y = 12$ Ksi (b) $\sigma_x = 21$ Ksi and $\sigma_y = 9$ Ksi

To determine the maximum shearing stress

FBD as shown

FPU Mohr's circle

Solution




(a) $\sigma_x = 0$, $\sigma_y = 12$ Ksi $\tau_{xy} = 8$ Ksi

Principal planes and principal stresses

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}(0 + 12\text{Ksi}) = 6\text{Ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{0\text{Ksi} - 12\text{Ksi}}{2}\right)^2 + (8\text{Ksi})^2} = 10\text{Ksi}$$

$$\sigma_{max} = \sigma_{ave} + R = 16\text{Ksi}$$

$$\sigma_{min} = \sigma_{ave} - R = -4\text{Ksi}$$

Maximum shearing stress

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = \frac{1}{2}(16\text{Ksi} - (-4\text{Ksi})) = 10\text{Ksi}$$

(b) $\sigma_x = 21$ Ksi $\sigma_y = 9$ Ksi $\tau_{xy} = 8$ Ksi

Principal planes and principal stresses

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}(21\text{Ksi} + 9\text{Ksi}) = 15\text{Ksi}$$

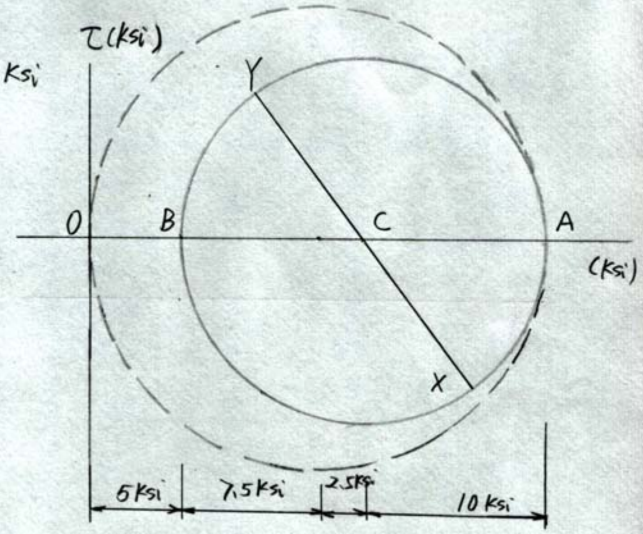
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{21\text{Ksi} - 9\text{Ksi}}{2}\right)^2 + (8\text{Ksi})^2} = 10\text{Ksi}$$

$$\sigma_{max} = \sigma_{ave} + R = 25\text{Ksi}$$

$$\sigma_{min} = 0$$

Maximum shearing stress

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = \frac{1}{2}(25\text{Ksi} - 0) = 12.5\text{Ksi}$$



Problem 7.70

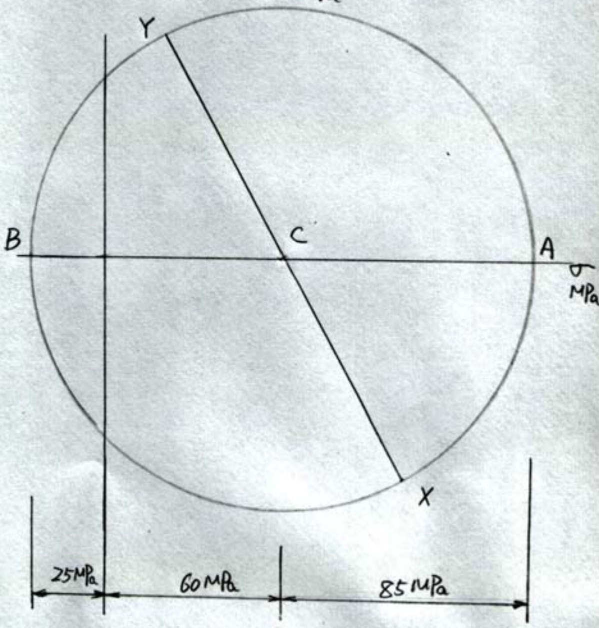
Given the state of stress: $\sigma_x = 100$ MPa $\sigma_y = 20$ MPa, $\tau_{xy} = 75$ MPa

To determine the maximum shearing stress when
(a) $\sigma_z = 0$, (b) $\sigma_z = +45$ MPa (c) $\sigma_z = -45$ MPa

FBD as shown

FPU Mohr's circle

Solution




Mohr's circle

$\sigma_x = 100$ MPa, $\sigma_y = 20$ MPa

$\tau_{xy} = 75$ MPa

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}(100\,\text{MPa} + 20\,\text{MPa}) = 60\,\text{MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{100\,\text{MPa} - 20\,\text{MPa}}{2}\right)^2 + (75\,\text{MPa})^2} = 85\,\text{MPa}$$

$$\sigma_A = \sigma_{ave} + R = 145\,\text{MPa}$$

$$\sigma_B = \sigma_{ave} - R = -25\,\text{MPa}$$

(a) $\sigma_z = 0$

$\sigma_{max} = 145$ MPa, $\sigma_{min} = -25$ MPa

$$\therefore \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = \frac{1}{2}(145\,\text{MPa} - (-25\,\text{MPa})) = 85\,\text{MPa}$$

(b) $\sigma_z = 45$ MPa

$\sigma_{max} = 145$ MPa, $\sigma_{min} = -25$ MPa

$$\therefore \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = \frac{1}{2}(145\,\text{MPa} - (-25\,\text{MPa})) = 85\,\text{MPa}$$

(c) $\sigma_z = -45$ MPa

$\sigma_{max} = 145$ MPa, $\sigma_{min} = -45$ MPa

$$\therefore \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = \frac{1}{2}(145\,\text{MPa} - (-45\,\text{MPa})) = 95\,\text{MPa}$$

Problem 7.100

Given a spherical steel pressure vessel $d = 250\,mm$ $t = 6\,mm$

Maximum gage pressure $p = 8\,MPa$, $\sigma_U = 400\,MPa$

To determine the factor of safety with respect to tensile failure.

FBD not required

FPU $\sigma_1 = \sigma_2 = \dfrac{pr}{2t}$

Solution:

$P = 8\,MPa = 8 \times 10^6\,Pa$

$t = 6\,mm = 6 \times 10^{-3}\,m$

$r = \frac{1}{2}d - t = \frac{1}{2}(250\,mm) - (6\,mm) = 119\,mm = 0.119\,m$

$$\sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{(8 \times 10^6\,Pa)(0.119\,m)}{2 \cdot (6 \times 10^{-3}\,m)} = 79.33 \times 10^6\,Pa = 79.33\,MPa$$

$$F.S. = \frac{\sigma_U}{\sigma_1} = \frac{400\,MPa}{79.33\,MPa} = 5.04$$

Problem 7.114

Given the pressure tank $t = \frac{3}{8}$ in $d = 5$ ft

$\beta = 25°$ $\sigma_w = 18$ Ksi $\tau_w = 10$ Ksi

To determine the largest allowable gage pressure

FBD as show

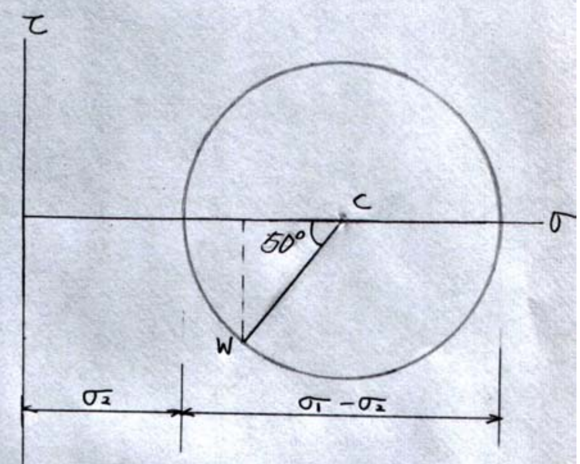
FPU Mohr's cycle. $\sigma_1 = \frac{pr}{t}$ $\sigma_2 = \frac{pr}{2t}$

Solution;

Stress at the weld.

$\sigma_w = 18$ Ksi

$\tau_w = 10$ Ksi




Mohr's cycle

$\sigma_1 = \frac{pr}{t}$

$\sigma_2 = \frac{pr}{2t}$

$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4}\frac{pr}{t}$

$R = \frac{1}{2}(\sigma_1 - \sigma_2) = \frac{1}{4}\frac{pr}{t}$

$r = \frac{1}{2}d - t = \frac{1}{2}(5 \cdot 12)\text{ in} - 0.375\text{ in} = 29.625\text{ in}$

From $\sigma_w = 18$ Ksi

$\sigma_w = \sigma_{ave} - R\cos 50° = \frac{3pr}{4t} - \frac{pr}{4t}\cos 50° = 0.5893\frac{pr}{t}$

$p = \frac{\sigma_w t}{0.5893\, r} = \frac{(18\text{ksi})(0.375\text{ in})}{0.5893\,(29.625\text{ in})} = 0.387\text{ Ksi} = 387\text{ psi}$

From $\tau_w = 10$ Ksi

$\tau_w = R\sin 50° = 0.19151\frac{pr}{t}$

$p = \frac{\tau_w t}{0.19151\, r} = \frac{(10\text{Ksi})(0.375\text{in})}{0.19151\,(39.625\text{in})} = 0.661\text{ Ksi} = 661\text{psi}$

∴ Allowable gage pressure is the smaller value,

that is $p = 387$ psi