ENES 220 – Mechanics of Materials
Spring 2003

Solutions to Homework #10

Problem 9.51

[Given] $\longrightarrow$

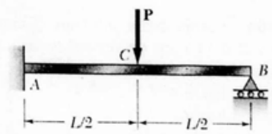
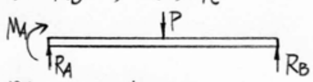


Fig. P9.51

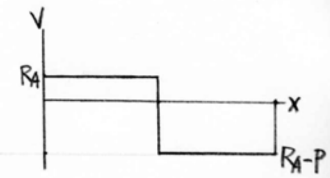
[To Find] (a) R_B ; (b) Y_c

[FBD]



[FPU] $EI\dfrac{d^2y}{dx^2} = M, \quad \dfrac{dM}{dx} = V$

[Solution]



(1) $+\uparrow \Sigma F_y = 0 \qquad R_A + R_B - P = 0 \Rightarrow R_A = P - R_B$ ①

$+\circlearrowleft \Sigma M_A = 0 \qquad -M_A - P\cdot\frac{L}{2} + R_B L = 0 \Rightarrow M_A = R_B L - \frac{1}{2}PL$ ②

(2) $\dfrac{dM}{dx} = V = R_A - P\langle x - \frac{L}{2}\rangle^0$

$EI\dfrac{d^2y}{dx^2} = M = M_A + R_A x - P\langle x - \frac{L}{2}\rangle^1$

$EI\dfrac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - \frac{1}{2}P\langle x - \frac{L}{2}\rangle^2 + C_1$

$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{1}{6}P\langle x - \frac{L}{2}\rangle^3 + C_1 x + C_2$

Boundary conditions:

$[x=0, y=0] \qquad 0 = 0 + 0 - 0 + 0 + C_2 \Rightarrow C_2 = 0$

$[x=0, \frac{dy}{dx}=0] \qquad 0 = 0 + 0 - 0 + C_1 \Rightarrow C_1 = 0$

$[x=L, y=0] \qquad \frac{1}{2}M_A L^2 + \frac{1}{6}R_A L^3 - \frac{1}{6}P\left(\frac{L}{2}\right)^3 + 0 + 0 = 0$

Plug ①, ② into above equation

$\frac{1}{2}(R_B L - \frac{1}{2}PL)L^2 + \frac{1}{6}(P - R_B)L^3 - \frac{1}{48}PL^3 = 0 \Rightarrow \boxed{R_B = \frac{5}{16}P \uparrow}$

(3) $R_A = P - R_B = P - \frac{5}{16}P = \frac{11}{16}P$

$M_A = R_B L - \frac{1}{2}PL = -\frac{3}{16}PL$

(4) Deflection at C

$Y_c = Y|_{x=\frac{L}{2}} = \frac{1}{EI}\left\{\frac{1}{2}M_A\left(\frac{L}{2}\right)^2 + \frac{1}{6}R_A\left(\frac{L}{2}\right)^3 - 0 + 0 + 0\right\}$

$= \frac{1}{EI}\left\{\frac{1}{2}\left(-\frac{3}{16}PL\right)\frac{L^2}{4} + \frac{1}{6}\left(\frac{11}{16}P\right)\frac{L^3}{8}\right\}$

$= -\frac{7}{768}\frac{PL^3}{EI} \qquad \boxed{Y_c = \frac{7}{768}\frac{PL^3}{EI} \downarrow}$

Problem 9.56

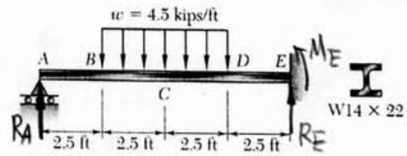


Fig. P9.56

[Given] ⟶ $E = 29\times10^6$ psi

[To Find] (a) R_A ; (b) y_C

[FBD] N/A

[FPU] $EI\frac{d^2y}{dx^2} = M,\ \frac{dM}{dx} = V,\ \frac{dV}{dx} = -w$

[Solution]

Units: Forces in kips, lengths in ft

$$w(x) = 4.5\langle x-2.5\rangle^0 - 4.5\langle x-7.5\rangle^0 \quad \text{kips/ft}$$

$$\frac{dV}{dx} = -w(x) = -4.5\langle x-2.5\rangle^0 + 4.5\langle x-7.5\rangle^0 \quad \text{kips/ft}$$

$$\frac{dM}{dx} = V = R_A - 4.5\langle x-2.5\rangle^1 + 4.5\langle x-7.5\rangle^1 \quad \text{kips}$$

$$EI\frac{d^2y}{dx^2} = M = R_A x - \frac{9}{4}\langle x-2.5\rangle^2 + \frac{9}{4}\langle x-7.5\rangle^2 \quad \text{kips}\cdot\text{ft}$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{3}{4}\langle x-\tfrac{5}{2}\rangle^3 + \frac{3}{4}\langle x-\tfrac{15}{2}\rangle^3 + C_1 \quad \text{kips}\cdot\text{ft}^2$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{3}{16}\langle x-\tfrac{5}{2}\rangle^4 + \frac{3}{16}\langle x-\tfrac{15}{2}\rangle^4 + C_1 x + C_2 \quad \text{kips}\cdot\text{ft}^3$$

$[x=0, y=0]$ $\quad 0-0+0+0+C_2 = 0 \Rightarrow C_2 = 0$

$[x=10, y=0]$ $\quad \frac{1}{6}R_A(10)^3 - \frac{3}{16}(\frac{15}{2})^4 + \frac{3}{16}(\frac{5}{2})^4 + 10C_1 = 0$

$$\Rightarrow C_1 = \frac{1875}{32} - \frac{50}{3}R_A \quad ①$$

$[x=10, \frac{dy}{dx}=0]$ $\quad \frac{1}{2}R_A(10)^2 - \frac{3}{4}(\frac{15}{2})^3 + \frac{3}{4}(\frac{5}{2})^3 + C_1 = 0$

$$\Rightarrow C_1 = \frac{9750}{32} - 50R_A \quad ②$$

(a) From ① and ②, we get $\begin{cases} R_A = \frac{945}{128}\text{ kips} = \boxed{7.383\text{ kips}}\ \uparrow \\ C_1 = -\frac{4125}{64}\text{ kips}\cdot\text{ft}^2 = -64.453\text{ kip}\cdot\text{ft}^2 \end{cases}$

(b) $y_C = y|_{x=5} = \frac{1}{EI}\left\{\frac{1}{6}R_A(5)^3 - \frac{3}{16}(\frac{5}{2})^4 + C_1(5)\right\}$

$$= \frac{1}{(29\times10^6\text{ psi})(199\text{ in}^4)}\left\{\left(\frac{1}{6}\right)\left(\frac{945}{128}\right)(125) - \left(\frac{3}{16}\right)\left(\frac{625}{16}\right) - \left(\frac{4125}{64}\right)(5)\right\}\text{ kips}\cdot\text{ft}^3$$

$$= \frac{1}{(29\times10^6\text{psi})(199\text{ in}^4)}(-175.78125)\text{kips}\cdot\text{ft}^2\cdot(1000\text{ lbs/kips})\cdot(12\text{ in/ft})^3$$

$$= -0.0526\text{ in} \qquad \boxed{y_C = 0.0526\text{ in}\ \downarrow}$$

Problem 9.58

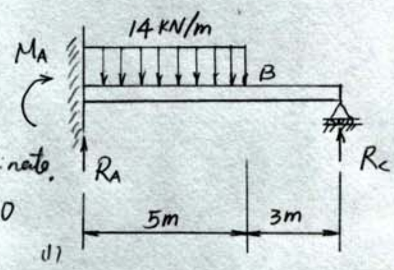
Given the beam and loading shown, the beam is W410×60, $E = 200\,GPa$

To determine (a) the reaction at C (b) the deflection at B.

FBD as shown.

FPU singularity function

Solution:



Reactions

Reactions are shown in the figure, we note that they are statically indeterminate.

$+\uparrow \Sigma F_y = 0 \quad R_A - (14\,KN/m)(5m) + R_c = 0$

$R_A = 70 - R_c \quad KN \quad (1)$

$+\circlearrowleft \Sigma M_A = 0 \quad -M_A - (14\,KN/m)(5m)(5/2\,m) + R_c(8m) = 0$

$M_A = 8R_c - 175 \quad KN\cdot m \quad (2)$

Bending moment

$w(x) = 14 - 14\langle x-5\rangle^0 \quad KN/m$

$\frac{dV}{dx} = -w = -14 + 14\langle x-5\rangle^0 \quad KN/m$

$\frac{dM}{dx} = V = R_A - 14x + 14\langle x-5\rangle^1 \quad KN$

$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - 7x^2 + 7\langle x-5\rangle^2 \quad KN\cdot m$

Equation of the elastic curve

$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - \frac{7}{3}x^3 + \frac{7}{3}\langle x-5\rangle^3 + C_1 \quad KN\cdot m^2$

$EI\,y = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{7}{12}x^4 + \frac{7}{12}\langle x-5\rangle^4 + C_1 x + C_2 \quad KN\cdot m^3$

Boundary conditions

$[x=0, \frac{dy}{dx}=0] \quad 0 = 0 + 0 - 0 + 0 + C_1 \quad \therefore C_1 = 0$

$[x=0, y=0] \quad 0 = 0 + 0 - 0 + 0 + 0 + C_2 \quad \therefore C_2 = 0$

$[x=8, y=0] \quad 0 = \frac{1}{2}M_A(8m)^2 + \frac{1}{6}R_A(8m)^3 - \frac{7}{12}(8m)^4 - \frac{7}{12}(3m)^4 + 0 + 0 \quad (3)$

Reaction at C

From (1) (2) (3), we have $R_c = 11.536\ KN$

$M_A = -82.715\ KN\cdot m$

$R_A = 58.464\ KN$

Deflection at B (y at $x = 5m$)

$\because$ W410×60, $\therefore I = 216\times10^6\,mm^4 = 216\times10^{-6}\,m^4$

$EI\,y_B = \frac{1}{2}(-82.715\,KN\cdot m)(5m)^2 + \frac{1}{6}(58.464)(5m)^3 - \frac{7}{12}(5m)^4 = -180.52\,KN\cdot m^3$

$y_B = \frac{-180.52\,KN\cdot m^3}{(200\times10^9\,Pa)(216\times10^{-6}\,m^4)} = -4.18\times10^{-3}\,m$

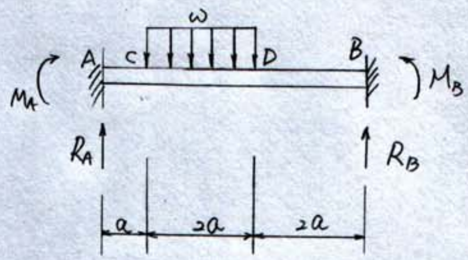
Problem 9.60

Given the beam and loading shown

To determine (a) the reaction at A.
(b) the deflection at D.

FBD as shown

FPU Singularity function



Solution

Reactions

The reactions are shown in the figure. They are statically indeterminate.

Bending Moment and equation of elastic curve

$$w(x) = w\langle x-a\rangle^0 - w\langle x-3a\rangle^0$$

$$\frac{dV}{dx} = -w(x) = -w\langle x-a\rangle^0 + w\langle x-3a\rangle^0$$

$$\frac{dM}{dx} = V = R_A - w\langle x-a\rangle^1 + w\langle x-3a\rangle^1$$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - \frac{1}{2}w\langle x-a\rangle^2 + \frac{1}{2}\langle w-3a\rangle^2$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - \frac{1}{6}w\langle x-a\rangle^3 + \frac{1}{6}\langle x-3a\rangle^3 + C_1$$

$$EI\,y = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{1}{24}w\langle x-a\rangle^4 + \frac{1}{24}\langle x-3a\rangle^4 + C_1 x + C_2$$

Boundary condition

$[x=0,\ \frac{dy}{dx}=0]$ $\quad 0 = 0+0-0+0+C_1 \qquad C_1 = 0$

$[x=0,\ y=0]$ $\quad 0 = 0+0-0+0+0+C_2 \qquad C_2 = 0$

$[x=5a,\ \frac{dy}{dx}=0]$ $\quad 0 = M_A(5a) + \frac{1}{2}R_A(5a)^2 - \frac{1}{6}w(4a)^3 + \frac{1}{6}(2a)^3 \qquad (1)$

$[x=5a,\ y=0]$ $\quad 0 = \frac{1}{2}M_A(5a)^2 + \frac{1}{6}R_A(5a)^3 - \frac{1}{24}w(4a)^4 + \frac{1}{24}w(2a)^4 \qquad (2)$

Reaction at A

From (1) (2), we have $\quad R_A = 1.280\,wa$

$\quad M_A = -1.3333\,wa^2$

Deflection at D

$$y = \frac{1}{EI}\left[\frac{1}{2}(-1.3333wa^2)x^2 + \frac{1}{6}(1.280wa)x^3 - \frac{1}{24}w\langle x-a\rangle^4 + \frac{1}{24}\langle x-3a\rangle^4\right]$$

$$y_D = y|_{x=3a} = \frac{1}{EI}\left[\frac{1}{2}(-1.3333wa^2)(3a)^2 + \frac{1}{6}(1.280wa)(3a)^3 - \frac{1}{24}w(2a)^4 + 0\right]$$

$$= -0.907\frac{wa^4}{EI}$$

Problem 9.68

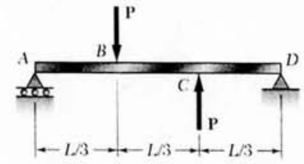


Fig. P9.68

[Given] ⟶

[To Find] (a) y_C ; (b) θ_A

[FBD] N/A

[FPU] Appendix D. Method of Superposition

[Solution]

(1) Loading I : Downward load P at B

Use case 5 of Appendix D with:

$$P=P,\quad a=\frac{L}{3},\quad b=\frac{2}{3}L,\quad L=L,\quad x=\frac{2}{3}L$$

For $x<a$, $y=\dfrac{Pb}{6EIL}\left[x^3-(L^2-b^2)x\right]$ ①

For $x>a$, $y=\dfrac{Pa}{6EIL}\left[(L-x)^3-(L^2-a^2)(L-x)\right]$ ② (replace x by L-x, interchange a and b in ①)

At $x=\frac{2}{3}L$ (point C),

$$y_C=y\Big|_{x=\frac{2}{3}L}=\frac{P\left(\frac{L}{3}\right)}{6EIL}\left[\left(\frac{L}{3}\right)^3-\left(L^2-\frac{L^2}{9}\right)\left(\frac{L}{3}\right)\right]=-\frac{7}{486}\frac{PL^3}{EI}$$

At $x=\frac{1}{3}L$ (point A)

$$\theta_A=-\frac{Pb\,(L^2-b^2)}{6EIL}=-\frac{P\left(\frac{2L}{3}\right)\left(L^2-\frac{4}{9}L^2\right)}{6EIL}=-\frac{5}{81}\frac{PL^2}{EI}$$

(2) Loading II : Upward load at C

Use case 5 of Appendix D with:

$$P=-P,\quad a=\frac{2L}{3},\quad b=\frac{L}{3},\quad L=L,\quad x=\frac{2}{3}L=a$$

$$y_C=y\Big|_{x=a=\frac{2}{3}L}=\frac{-(-P)\left(\frac{2}{3}L\right)^2\left(\frac{L}{3}\right)^2}{3EIL}=\frac{4}{243}\frac{PL^3}{EI}$$

$$\theta_A=-\frac{(-P)\left(\frac{L}{3}\right)\left(L^2-\frac{1}{9}L^2\right)}{6EIL}=\frac{4}{81}\frac{PL^2}{EI}$$

(a) Deflection at C : $y_C=y_C^{I}+y_C^{II}=-\dfrac{7}{486}\dfrac{PL^3}{EI}+\dfrac{4}{243}\dfrac{PL^3}{EI}=\boxed{\dfrac{1}{486}\dfrac{PL^3}{EI}\ \uparrow}$

(b) slope at A : $\theta_A=\theta_A^{I}+\theta_A^{II}=-\dfrac{5}{81}\dfrac{PL^2}{EI}+\dfrac{4}{81}\dfrac{PL^2}{EI}=-\dfrac{1}{81}\dfrac{PL^2}{EI}$

i.e., $\boxed{\theta_A=\dfrac{1}{81}\dfrac{PL^2}{EI}}$

Problem 9.74

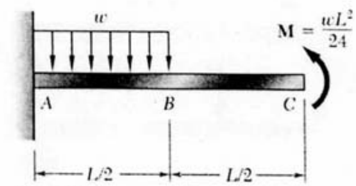


Fig. P9.74

[Given] ⟶

[To Find] θ_C, y_C

[FBD] N/A

[FPU] Appendix D. Method of Superposition.

[Solution]

(1) Loading I : Downward distributed load w applied to portion AB

Use case 2 of Appendix D applied to portion AB

$$\theta_B = -\frac{w\left(\frac{L}{2}\right)^3}{6EI} = -\frac{wL^3}{48EI}$$

$$y_B = -\frac{w\left(\frac{L}{2}\right)^4}{8EI} = -\frac{wL^4}{128EI}$$

Portion BC remains straight

$$\theta_C = \theta_B = -\frac{wL^3}{48EI}$$

$$y_C = y_B + \left(\frac{L}{2}\right)\theta_B = -\frac{wL^4}{128EI} - \frac{wL^4}{96EI} = -\frac{7}{384}\frac{wL^4}{EI}$$

(2) Loading II : counterclockwise couple $\frac{wL^2}{24}$ applied at C

Use case 3 of Appendix D

$$\theta_C = \frac{\left(\frac{wL^2}{24}\right)L}{EI} = \frac{wL^3}{24EI}$$

$$y_C = \frac{\left(\frac{wL^2}{24}\right)L^2}{2EI} = \frac{wL^4}{48EI}$$

(a) $$\theta_C = \theta_C^{I} + \theta_C^{II} = -\frac{wL^3}{48EI} + \frac{wL^3}{24EI} = \boxed{\frac{wL^3}{48EI}}$$

(b) $$y_C = y_C^{I} + y_C^{II} = -\frac{7wL^4}{384EI} + \frac{wL^4}{48EI} = \boxed{\frac{wL^4}{384EI} \uparrow}$$

Problem 9.84

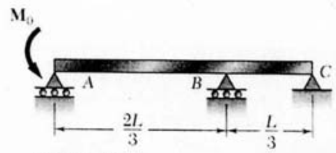
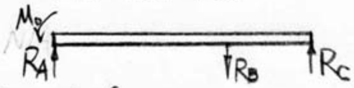


Fig. P9.84

[Given] ⟶

[To Find] R_A, R_B, R_C

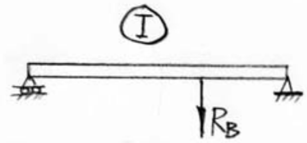
[FBD]



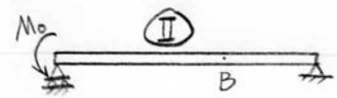
[FPU] Method of Superposition, Appendix D

[Solution]

Consider R_B to be the redundant reaction



(1) Loading I : Downward load R_B at B.



Use case 5 of Appendix D : $P = R_B$, $a = \frac{1}{3}L$, $b = \frac{2}{3}L$

$$y_B^{I} = -\frac{Pa^2b^2}{3EIL} = -\frac{R_B\left(\frac{L}{3}\right)^2\left(\frac{2L}{3}\right)^2}{3EIL} = -\frac{4}{243}\frac{R_BL^3}{EI}$$

(2) Loading II: counter-clockwise couple M_0 at A.

Use case 7 of Appendix D: $M = M_0$, $x = \frac{L}{3}$

$$y_B^{II} = -\frac{M}{6EIL}\left(x^3 - L^2x\right) = -\frac{M_0}{6EIL}\left[\left(\frac{L}{3}\right)^3 - L^2\left(\frac{L}{3}\right)\right] = \frac{4}{81}\frac{M_0L^2}{EI}$$

(3) Superposition and constraint

$$y_B = y_B^{I} + y_B^{II} = 0$$

$$-\frac{4}{243}\frac{R_BL^3}{EI} + \frac{4}{81}\frac{M_0L^2}{EI} = 0 \quad \Rightarrow \quad \boxed{R_B = 3\frac{M_0}{L} \downarrow}$$

(4) Statics

$$+\circlearrowleft \Sigma M_C = 0 \quad -R_AL + M_0 + R_B\cdot\frac{L}{3} = 0 \Rightarrow R_A = \frac{M_0}{L} + \frac{R_B}{3} = \boxed{2\frac{M_0}{L} \uparrow}$$

$$+\uparrow \Sigma F_y = 0 \quad R_A - R_B + R_C = 0 \Rightarrow R_C = R_B - R_A = \boxed{\frac{M_0}{L} \uparrow}$$

Problem 9.92

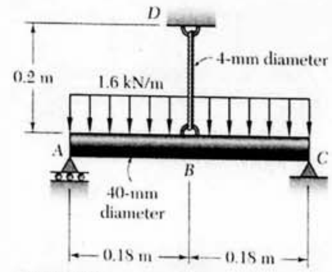
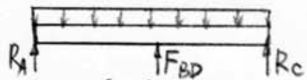


Fig. P9.92

[Given] →

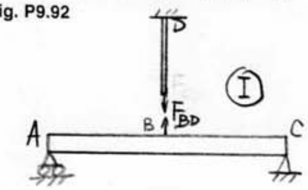
[To Find] (a) y_B ; (b) R_A

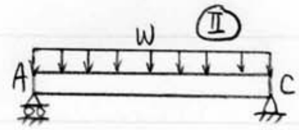
[FBD]



[FPU] Method of Superposition, Appendix D

[Solution]





(1) Let F_{BD} be the tension in wire BD. The elongation of the wire is

$$\delta_{BD} = \frac{F_{BD}\, l}{EA} \quad , \quad y_B = \delta_{BD}$$

(2) Loading I: Upward load F_{BD} at B.

Use case 4 of Appendix D: $P = -F_{BD}$

$$y_B^{I} = -\frac{PL^3}{48EI} = \frac{F_{BD}L^3}{48EI}$$

(3) Loading II: Downward distributed load W applied to the beam

Use case 6 of Appendix D:

$$y_B^{II} = -\frac{5WL^4}{384EI}$$

(4) Deflection at B

$$y_B = y_B^{I} + y_B^{II} = \frac{F_{BD}L^3}{48EI} - \frac{5WL^4}{384EI} = -\delta_{BD} = -\frac{F_{BD}\, l}{EA}$$

$$\therefore F_{BD} = \frac{\frac{5}{384}\frac{WL^4}{EI}}{\frac{l}{EA} + \frac{L^3}{48EI}} = \frac{5WL^4}{384\frac{lI}{A} + 8L^3}$$

$$= \frac{(5)(1.6\times10^3\,N/m)(0.36\,m)^4}{(384)\frac{(0.2\,m)\left[\frac{\pi}{4}(0.02\,m)^4\right]}{\frac{\pi}{4}(0.004\,m)^2} + 8(0.36\,m)^3} = 117.74\ N$$

(a) $$y_B = \frac{F_{BD}\, l}{EA} = \frac{(117.74\,N)(0.2\,m)}{(200\times10^9\,Pa)\left[\frac{\pi}{4}(0.004\,m)^2\right]} = 9.374\times10^{-6}\ m = 0.00937\ mm \downarrow$$

(b) From Symmetry, $R_A = R_C$

$$+\uparrow \Sigma F_y = 0 \quad R_A + R_C + F_{BD} - WL = 0 \Rightarrow R_A = R_C = \frac{1}{2}\left[WL - F_{BD}\right]$$

$$= \frac{1}{2}\left[(1.6\times10^3\,N/m)(0.36\,m) - 117.74\,N\right]$$

$$= \boxed{229.13\ N \uparrow}$$