CHAPTER

7a

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Risk Analysis in Engineering and Economics

RISK CONTROL METHODS

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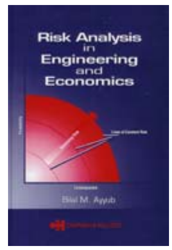

Risk Analysis for Engineering

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Introduction

- In general, risk management is performed within an economic framework with an objective of optimizing the allocation of available resources in support of a broader goal.
- Therefore, it requires the definition of acceptable risk, and comparative evaluation of options and/or alternatives for decision making.

Introduction

- ***Risk control*** has an objective to reduce risk to an acceptable level and/or prioritize resources based on comparative analysis.
- The objective of this chapter is to introduce fundamental concepts for risk control within an economic framework that include risk aversion, risk homeostasis, discounting procedures, decision analysis, tradeoff analysis, insurance models, and repair and maintainability issues.

Philosophies of Risk Control

- Risk control can be approached by an organization within a strategic, or a system-wide, or an organization-wide plan.
- A philosophy for risk control might be constructed based on recognizing that the occurrence of a consequence-inducing event is the tip of an iceberg representing a scenario.

Philosophies of Risk Control

- The domino theory for risk control was used in industrial accident prevention to eliminate injury-producing events by construction a domino sequence of events as demonstrated by the following sequence:
  - A personal injury as the final domino occurs only as a result of an accident.
  - An accident occurs only as a result of a human-related or mechanical hazard.

Philosophies of Risk Control

  - A human-related or mechanical hazard exits only as a result of human errors or degradation of equipment.
  - Human errors or degradation are inherited or acquired as a result of their environment.
  - An environment is defined by conditions into which individuals or processes are placed.
- This philosophy might be suitable for some applications such as manufacturing, construction, production, and material handling.

Philosophies of Risk Control

- A related philosophy for risk control is the cascading-failure theory for risk control according to which control strategies are identified by investigating cascading failures.
- For example, a loss of electric power to a facility might lead to the failures of other systems leading to the failure of additional systems, and so on.

Philosophies of Risk Control

- In this case, risk control can target increasing power availability as a solution.
- The following strategies can be adopted within this philosophy:
  - The creation of the hazard can be prevented in the first place during the concept development and design stages. For example, having no-smoking rules can be adopted to reduce the risk associated with fires, and the use of pressure relief valves are used to reduce risks associated with over-pressurizing vessels and tanks.

Philosophies of Risk Control

- The damage already done by the hazard can be countered and contained. For example, fire sprinkler systems and emergency response teams can be used to protect a facility.
- The object of damage can be repaired and rehabilitated. For example, injured workers, and salvage operations can be rehabilitated after an accident.

Philosophies of Risk Control

- A risk control Philosophy needs also to define the control measures, time of application, and target of the risk-control measures.
- The control measures can include pressure relief valves, firewalls, and emergency response teams.
- The timing of application identify when the measure is needed, such as, prior an event, at the time of an event, or after an event occurrence.

Philosophies of Risk Control

- The targets of the risk control measures could include workers, visitors, machinery, assets, or a population outside a plant.

Risk Aversion in Investment Decisions

- Risk control can be examined within an economic framework by constructing cash flows for available alternatives as investments.
- Selecting an optimal alternative can be based on the expected or average NPV as was demonstrated in decision-tree analyses.

Risk Aversion in Investment Decisions

- However, this selection criterion might not reflect the complexities involved in real decision situations.
- This section utilizes an example investment decision under uncertainty to introduce some key concepts and related complexities.

Risk Aversion in Investment Decisions

- A decision situation involves three alternatives A, B, and C that could lead to several scenarios each.
- These scenarios and their respective NPV values are shown in Table 1.
- The table shows that alternatives A and B have generally smaller returns and smaller spreads than alternative C.

Risk Aversion in Investment Decisions

Table 1. Scenarios for Three Alternatives

Quantity	Extremely Low	Very Low	Low	Good	High	Very High	Extremely High
Net Present Values (NPV)							
Alternative A ($)	100	200	300	400	500	600	700
Alternative B ($)	300	400	500	600	700	800	900
Alternative C ($)	0	200	400	600	800	1000	1200
Probabilities (*p*)							
Equally likely	1/7	1/7	1/7	1/7	1/7	1/7	1/7
Increasing Likelihood	1/28	2/28	3/28	4/28	5/28	6/28	7/28
Decreasing Likelihood	7/28	6/28	5/28	4/28	3/28	2/28	1/28

Risk Aversion in Investment Decisions

- Table 2 shows the descriptive statistics of the NPV of alternatives A, B and C using the three probability distributions for the scenarios of equally likely, increasing likelihood, and decreasing likelihood (p).
- The descriptive statistics were computed as follows:

$$\text{Expected value, } E(NPV) = \sum_{i=1}^{N=7} NPV_i p_i \qquad (1)$$

Risk Aversion in Investment Decisions

Table 2. Descriptive Statistics of the Net Present Values of Alternatives A and B

Quantity	Alternative A	Alternative B	Alternative C
Equally likely			
Expected *NPV* ($)	400	600	600
Standard Deviation of *NPV* ($)	200	200	400
Coefficient of Variation of *NPV*	0.5	0.333	0.667
Increasing Likelihood			
Expected *NPV* ($)	500	700	800
Standard Deviation of *NPV* ($)	173.21	173.21	346.41
Coefficient of Variation of *NPV*	0.346	0.247	0.433
Decreasing Likelihood			
Expected *NPV* ($)	300	500	400
Standard Deviation of *NPV* ($)	173.21	173.21	346.41
Coefficient of Variation of *NPV*	0.577	0.346	0.866

Risk Aversion in Investment Decisions

$$\text{Standard deviation, } \sigma(NPV) = \sqrt{\sum_{i=1}^{N=7} p_i \left(NPV_i - E(NPV)\right)^2} \qquad (2)$$

$$\text{Coefficient of variation, COV}(NPV) = \frac{\sigma(NPV)}{E(NPV)} \qquad (3)$$

Risk Aversion in Investment Decisions

- The inconclusive decision situation in this example can be attributed to the level of satisfaction, which an investor might reach based on each alternative.
- This level of satisfaction for each level of NPV (or generally called wealth W) that corresponds to each scenario is called utility (U) that represent the risk attitude of investors.

Risk Aversion in Investment Decisions

- The risk attitude of an investor or decision maker may be thought of as a decision maker's preference of taking a chance on an uncertain money payout of known probability versus accepting a sure money amount, i.e., with certainty.
- For example, a person having a choice between (1) accepting the outcome of a fair coin toss where heads means winning

Risk Aversion in Investment Decisions

$20,000 and tails means losing $10,000, and (2) accepting a certain cash amount of $4,000.

- The concept of utility under uncertainty is based on the following axioms:
  - Decision making is always rational;
  - Decision making takes into considerations all available alternatives; and
  - Decision makers prefer more consumption or wealth to less.

Risk Aversion in Investment Decisions

- These axioms define what is termed cardinal utility.
- For the purpose of illustration, a subjectively constructed utility function was used to produce the utility values shown in Table 3 for alternatives A, B and C.
- Decision-making can be viewed as all about maximizing utility rather maximizing wealth since maximizing utility leads to maximizing satisfaction.

Risk Aversion in Investment Decisions

Table 3. Utility Values for Net Present Values

Quantity	Extremely Low	Very Low	Low	Good	High	Very High	Extremely High
Alternative A ($)							
NPV ($)	100	200	300	400	500	600	700
Utility	77	148	213	272	325	372	413
Alternative B							
NPV ($)	300	400	500	600	700	800	900
Utility	213	272	325	372	413	448	477
Alternative C							
NPV ($)	0	200	400	600	800	1000	1200
Utility	0	148	272	372	448	500	528

Risk Aversion in Investment Decisions

- Commonly, an alternative with the highest expected utility $E(U)$ is identified and selected.
- The descriptive statistics of the utility for alternatives A, B and C using the three probability distributions for the scenarios of equally likely, increasing likelihood, and decreasing likelihood (p) are shown in Table 4.

Risk Aversion in Investment Decisions

Table 4. Descriptive Statistics for the Utility of Alternatives A and B

Quantity	Alternative A	Alternative B	Alternative B
Equally likely			
Expected Utility	260	360	324
Standard Deviation of Utility	112.48	88.61	180.84
Coefficient of Variation of Utility	0.433	0.246	0.558
Increasing Likelihood			
Expected Utility	316	404	412
Standard Deviation of Utility	92.24	71.58	136.47
Coefficient of Variation of Utility	0.292	0.177	0.331
Decreasing Likelihood			
Expected Utility	204	316	236
Standard Deviation of Utility	102.59	81.90	176.91
Coefficient of Variation of Utility	0.503	0.259	0.750

Risk Aversion in Investment Decisions

- The utility function of Table 4 reflects cautiousness of an investor or a decision maker.
- The values in Tables 2 and 4 reveal this impeded cautiousness of the investor based on the utility function.
- Considering alternative B for the equally likely scenarios as an example, the respective expected values of net present

Risk Aversion in Investment Decisions

and utility values, i.e., *E*(NPV) and *E*(*U*), respectively, are as follows:

$$E(NPV) = \$600 \qquad (4a)$$

$$E(U) = 360 \qquad (4b)$$

- The result of Eq. 4a and the utility function presented in Table 3 can be used to compute the utility of *E*(NPV) as follows:

$$U[E(NPV)] = U(600) = 372 \qquad (5)$$

Risk Aversion in Investment Decisions

- Since *U*(*E*(NPV)) > *E*(U) for alternative B, based on Eqs. 4b and 5, the investor in this case is cautious or called risk averse.
- The meaning of risk aversion in this case is that a certain NPV of $600 has a utility of 372 that is larger than the weighted utility of a risky project with an *E*(NPV) of $600 based on its *E*(*U*) of 360.

Risk Aversion in Investment Decisions

- An investor who could receive a certain NPV of $600 instead of an expected NPV with same value would be always more satisfied with higher utility.
- Therefore in this case, $U(E(\text{NPV}))$ is larger than $E(U)$ since any incremental increase in NPV results in a nonproportionally smaller increase in utility.

Risk Aversion in Investment Decisions

- Humans generally have an attitude towards risk where small stimuli over time and space are ignored, while the sum of these stimuli, if exerted instantly and locally, can cause a significant response.
- This attitude is called risk aversion.

Risk Aversion in Investment Decisions

- In general, risk aversion can be defined by the following relationship:

$$U[E(NPV)] > E[U(NPV)] \qquad (6a)$$

or

$$U[E(W)] > E[U(W)] \qquad (6b)$$

- The utility function used in the previous example is for a risk-averse investor as shown in Figure 1.

Risk Aversion in Investment Decisions

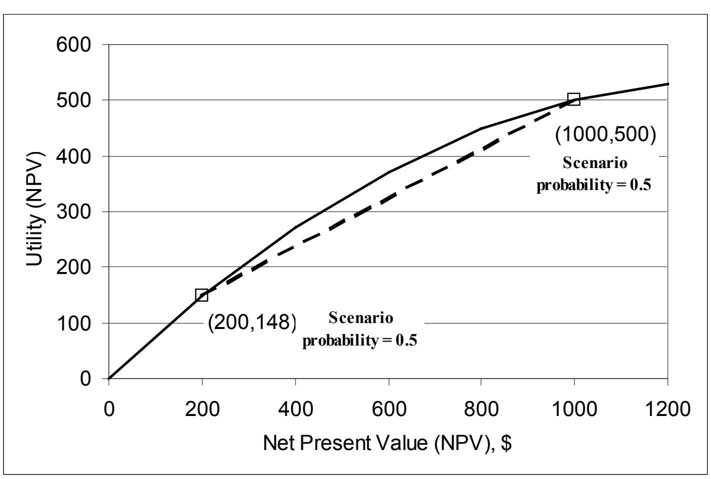

Figure 1. Utility Function for a Risk-Averse Investor

Risk Aversion in Investment Decisions

- The equation used to construct the utility function in Figure 1 for illustration purposes is given by

$$U(W) = 0.8W - 0.0003W^2 \qquad (7)$$

- The figure also shows two points that have the coordinates (NPV, *U*) of ($200, 148) and ($1000, 500).
- These two points represent two scenarios with, say, equal probabilities of 0.5 each.

Risk Aversion in Investment Decisions

- For these two scenarios, the following quantities can be computed:

$$E(NPV) = 0.5(\$200) + 0.5(\$1000) = \$600 \qquad (8a)$$

- The utility of this *E*(NPV) is

$$U\big(E(NPV)\big) = 0.8(600) - 0.0003(600)^2 = 372 \qquad (8b)$$

- The expected utility of the two points is

$$E\big(U(NPV)\big) = 0.5(148) + 0.5(500) = 324 \qquad (8c)$$

Risk Aversion in Investment Decisions

- Cases in which utility grows slower than wealth represent risk-aversive investors.
- The intensity of risk aversion depends the amount of curvature in the curve.
- The larger the curvature for this concave curve, the higher the risk aversion.
- Although is not as common, investors could display risk propensity, called risk-seeking investors.

Risk Aversion in Investment Decisions

- In this case, the utility function is convex as shown in Figure 2, and meets the following condition:

$$U(E(NPV)) < E(U(NPV)) \qquad \text{(9a)}$$

or

$$U(E(W)) < E(U(W)) \qquad \text{(9b)}$$

Risk Aversion in Investment Decisions

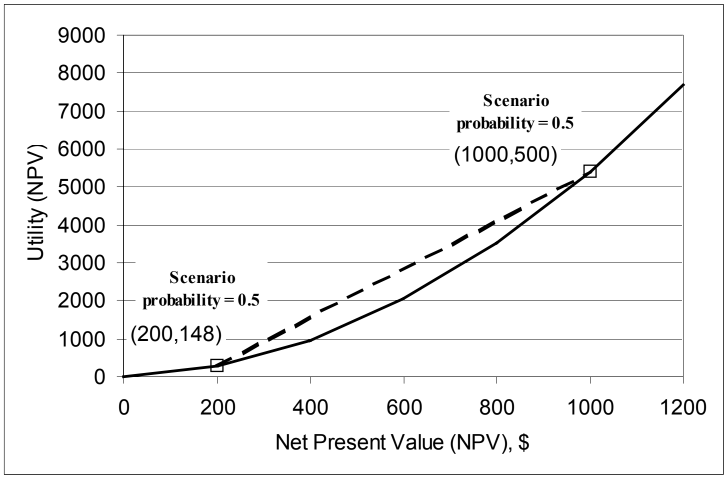

Figure 2. Utility Function for a Risk-Seeking Investor

Risk Aversion in Investment Decisions

- The utility function used in this case for a risk-seeking investor as shown in Figure 2 was constructed using the following utility function for illustration purposes:

$$U(W) = 0.4W + 0.005W^2 \qquad (10)$$

- The figure also shows two points that have the coordinates (NPV, *U*) of ($200, 280) and ($1000, 5400).

Risk Aversion in Investment Decisions

- These two points represent two scenarios with, say, equal probabilities of 0.5 each.
- For these two scenarios, the following quantities can be computed:

$$E(NPV) = 0.5(\$200) + 0.5(\$1000) = \$600 \quad (11a)$$

$$U\left[E(NPV)\right] = 0.4(600) + 0.005(600)^2 = 2040 \quad (11b)$$

$$E\left[U(NPV)\right] = 0.5(280) + 0.5(5400) = 2840 \quad (11c)$$

Risk Aversion in Investment Decisions

- The case of risk neutrality is a possibility as well, although it is common for governments and large corporation with relatively sizable resources.
- A risk neutral investor has a utility function without curvature as shown in Figure 3.
- In this case the utility function is linear, and meets the following condition:

$$U\left[E(NPV)\right] = E\left[U(NPV)\right] \quad (12a)$$

or

$$U\left[(E(W)\right] = E\left[U(W)\right] \quad (12b)$$

Risk Aversion in Investment Decisions

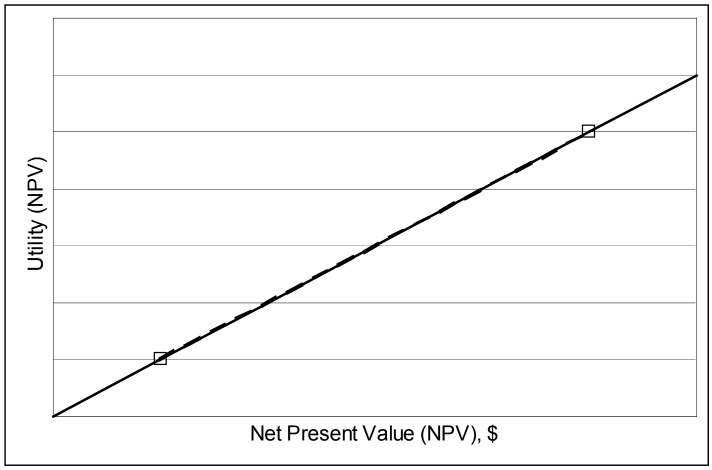

Figure 3. Utility Function for a Risk-Neutral Investor

Risk Aversion in Investment Decisions

- The larger the size of an initial investment, the smaller the rate of return for the same NPV.
- For this reason, the use of the rate of return might be needed in some applications.
- The expected values and standard deviations of NPV and U for investment alternatives are shown in Figure 4.

Risk Aversion in Investment Decisions

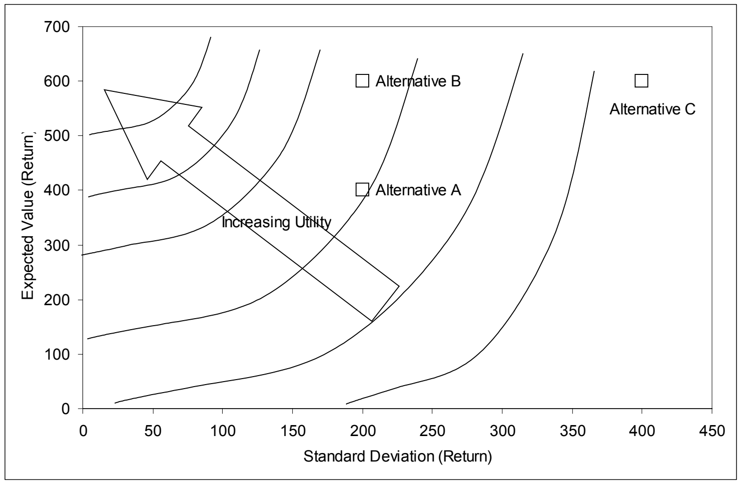

Figure 4. Indifference Curves for Risk Aversion

Risk Aversion in Investment Decisions

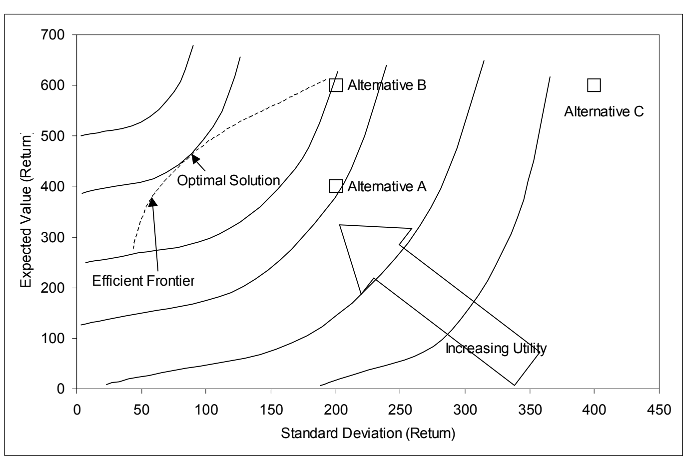

Figure 5. The Minimum Variance Frontier with an Indifference Curve for an Optimal Solution

Risk Aversion in Investment Decisions

- Portfolio of Investments
  - Investment decisions about portfolio might require treating the investments as multiple random variables that can be combined through a sum as follows for a portfolio of two investments:

$$NPV = NPV_1 + NPV_2 \tag{13}$$

$$E(NPV) = E(NPV_1) + E(NPV_2) \tag{14a}$$

$$\sigma(NPV) = \sqrt{(\sigma(NPV_1))^2 + (\sigma(NPV_2))^2 + 2Cov(NPV_1, NPV_2)} \tag{14b}$$

Risk Aversion in Investment Decisions

- Portfolio of Investments
  - where COV(NPV_1,NPV_2) is the covariance of NPV_1 and NPV_2 as a measure of correlation that is given by:

$$Cov(NPV_1, NPV_2) = \sum_i \sum_j (NPV_{1i} - E(NPV_1))(NPV_{2j} - E(NPV_2))p_{ij} \tag{15}$$

  - where p_{ij} is the joint probability of NPV_{1i} and NPV_{2j}. Sometimes, an approximate joint probability can be computed from the marginal probabilities as follows:

$$p_{ij} = p_{1i} p_{2j} \tag{16}$$

Risk Aversion in Investment Decisions

- **Example 1:** Construction of Utility Functions for Investment Decisions
  – Alternative A of Table 1 is used in this example to demonstrate the construction of utility functions.
  – A risk-averse investor and a risk seeking investor are as follows, respectively:

$$U_1(NPV) = 0.8NPV - 0.0003NPV^2 \qquad (17a)$$

$$U_2(NPV) = 0.4NPV - 0.0002NPV^2 \qquad (17b)$$

Risk Aversion in Investment Decisions

- Example 1 (cont'd):
  – The utility functions are evaluated in Table 5.
  – Figure 6 shows the two utility functions showing different slope characteristics for the two utility curves.
  – The curve for U_1, which is concave in shape, represents the risk aversion attitude of the investor; whereas the curve U_2, which is convex in shape, represents the risk-seeking attitude.

Risk Aversion in Investment Decisions

Table 5. Utility Values for Alternative A Based on Eqs. 17a and 17b

NPV ($)	100	200	300	400	500	600	700	Equations
$U_1(NPV)$	77	148	213	272	325	372	413	7-17a
$U_2(NPV)$	42	88	138	192	250	312	378	7-17b

Risk Aversion in Investment Decisions

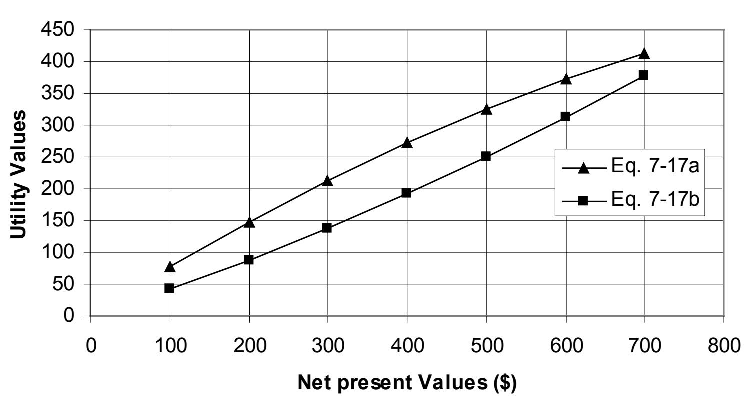

Figure 6. Utility and Net Present Value for Alternative A Based on Eqs.17a and 17b

Risk Aversion in Investment Decisions

- **Example 2:** Efficient Frontier for Screening Design Alternatives
  - An Architectural company developed six design alternatives for a new commercial structure, denoted as $D_1, D_2, \ldots,$ and D_6.
  - The company's management objective is to identify an optimal selection for implementation using economic-based efficient frontier analysis.

Risk Aversion in Investment Decisions

- Example 2 (cont'd):
  - The statistics of the NPV are presented in Table 6.
  - The efficient frontier can be identified based on the results of the six alternatives by plotting them as shown in Figure 7.
  - The figure clearly shows the efficient frontier as the alternatives that offer the largest expected NPV for any given standard deviation.

Risk Aversion in Investment Decisions

Table 6. Expected and Standard Deviation NPV for Design Alternatives

Design	D_1	D_2	D_3	D_4	D_5	D_6
Expected *NPV* ($1000)	100	42	66	66	88	118
Standard Deviation of *NPV* ($1000)	25	4	48	25	65	65

Risk Aversion in Investment Decisions

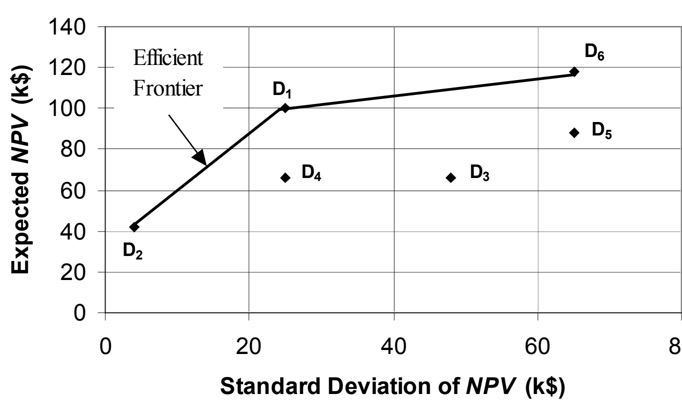

Figure 7. Efficient Frontier for Design Alternatives

Risk Aversion in Investment Decisions

- **Example 3:** Selecting Optimal Design Alternative Based on Different Risk Attitudes
  – Figure 8 shows two cases of a risk-averse management of the company and risk seeking management.
  – If the management is risk averse, the utility curves subjectively assigned in the space of the expected and standard deviation of NPV are shown on the left of the figure.

Risk Aversion in Investment Decisions

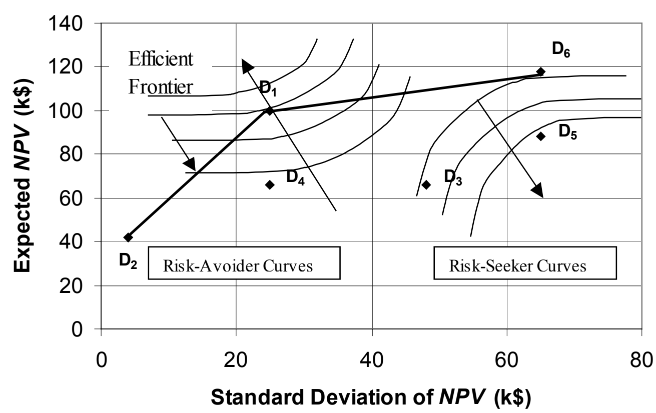

Figure 8. Efficient Frontier and Utilities for Design Alternatives

Risk Aversion in Investment Decisions

- Example 3 (cont'd):
  - These risk-averse curves lead the management to selecting alternative design D_1; while if they are risk-seekers as shown on the right of the figure then designs D_3 or D_5 or D_6 are among the appealing alternatives.
  - management might choose alternative D_6 despite its high level of risk.
  - if the management is risk neural, design D6 would be identified as one that gives the highest value of return in terms of expected NPV of $118,000 regardless of its high level of risk, i.e., a standard deviation of $65,000.

Risk Aversion in Investment Decisions

- **Example 4:** Efficient Frontier and Utility Values for Screening Car Product Alternatives
  - An automobile manufacturing company is considering five alternative product designs for its new generation of sedan cars.
  - The alternatives are denoted as A, B, C, D, and E.
  - For each design option, an analytical simulation was carried out to obtain the mean and standard deviation of the designs' marginal profits.

Risk Aversion in Investment Decisions

- Example 4 (cont'd):
  - The simulation results are presented in Table 7 showing the expected profit and standard deviation for the five design alternatives.
  - Comparing designs A and B, as shown in Table 7, reveals that they offer the same expected return, however design B is much riskier with a larger standard deviation of $225,000 than design A.

Risk Aversion in Investment Decisions

Table 7. Expected Value and Standard Deviation of Profits for Car Product Designs

Alternatives	Expected Profit ($1000)	Standard Deviation of Profit ($1000)
A	50	30
B	50	225
C	300	120
D	550	120
E	800	225

Risk Aversion in Investment Decisions

- Example 4 (cont'd):
  - To model the risk attitude of the decision maker, utility curves need to be constructed and used to identify the optimal choice among the alternative designs.
  - Assuming that the risk attitude of the manager can be expressed using the following utility function:

$$U(P) = 0.3P - 0.00015P^2$$

where U is the utility, and P is the profit

Risk Aversion in Investment Decisions

- Example 4 (cont'd):
  - The utility curve takes a concave shape as shown in Figure 9.
  - The utility function is tangent to the efficient frontier at design D.
  - Hence, product D with expected profit of \$550,000, and standard deviation of 120,000 is the optimal solution that maximizes profit and satisfies the risk level accepted by the decision maker

Risk Aversion in Investment Decisions

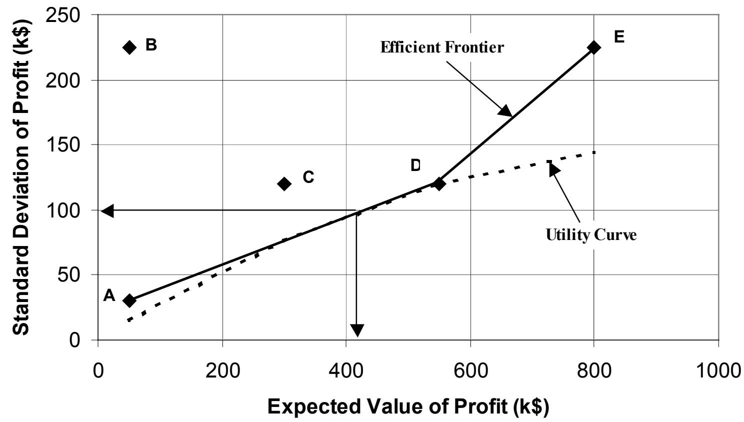

Figure 9. Efficient Frontier and Utility Curve for Design Alternatives

Risk Homeostasis

- People accept a certain level of risk in any activity.
- This risk level is subjectively estimated and accepted to their health, safety, and other things they value, in exchange for the benefits or satisfaction they hope to receive from that activity, such as transportation, work, eating, drinking, drug use, recreation, romance, sports, etc.

Risk Homeostasis

- ***Homeostasis*** is broadly defined as the tendency to maintain, or the maintenance of, normal, internal stability in a living species by coordinated responses of its relevant internal systems that automatically compensate for environmental changes.
- Risk homeostasis can be defined in a similar manner.

Risk Homeostasis

- ***Risk homeostasis*** can also explain the fact that a random selection of cigarette smokers who were advised to quit by their physician, did indeed reduce their cigarette consumption to a much greater extent than a comparison group.
- These former smokers had a lower frequency of smoking-related disease.
- However, they did not live longer, and their lives were a little shorter.

Risk Homeostasis

- Risk homeostasis could have a great implication on selecting risk mitigation actions.
- Traditional risk mitigation practices can therefore be called into question, such as prohibiting drinking and driving, or closing the borders to the illicit drug trade.
- Risk mitigation actions that are dependent on human conduct might not work or might not be effective in general.

Risk Homeostasis

- These conclusions emphasize the need to account for human behavior within risk mitigation actions, and devoting efforts to changing the behavior of humans, aimed at increasing people's desire to be safe and to live a healthy lifestyle.
- Thus, in addition to enforcement, educational, and engineering approaches, a motivational approach to prevention is needed.

Insurance for Loss Control and Risk Transfer

- Risk management including loss control is of central importance for insurers.
- Insurers typically perform rigorous studies and review before placing insurance on systems, followed by periodic site visits, and specialized studies.
- Some insurers utilize specialized methods and protocols for performance measurement and verification.

Insurance for Loss Control and Risk Transfer

- Loss Control
  - If insurers and insured systems are able to limit the frequency and/or intensity of losses, the cost of insurance can be lowered.
  - These loss control measures can range from requiring fire sprinklers in buildings to computer ergonomics training in workplaces.
  - The two primary approaches to implementing insurance loss control are *contractual* and *technical.*

Insurance for Loss Control and Risk Transfer

- Loss Control (cont'd)
  - ***Contractual methods*** include exclusions on the policy, or the ability to shift the loss cost to others. Insurance providers also limit claims through the use of deductibles and exclusions.
  - ***Technical methods*** for loss control include a host of quality-assurance techniques used during design, construction, and startup of a project.

Insurance for Loss Control and Risk Transfer

- Loss Control (cont'd)
  - *Technical methods* are captured within the set of tools known as system commissioning.
  - Measurement and diagnostics methods can be used to track actual performance, and make corrections before claims materialize.
  - Loss control specialists are used to help keep the number of accidents and losses to a minimum.

Insurance for Loss Control and Risk Transfer

- Loss Control (cont'd)
  - They visit factories, shop floors, and businesses to identify potential hazards and help to eliminate them.
  - In the health insurance area they might work with an organization to promote preventive health care in the workplace or to limit exposure to certain types of ailments.

Insurance for Loss Control and Risk Transfer

- Risk Actuaries and Insurance-Claim Models
  - The insurance industry utilizes analytical skills to assess risks and price of their insurance products.
  - Actuaries are used based on their analytical skills to assess risks of writing insurance policies on property, businesses and people's lives and health.

Insurance for Loss Control and Risk Transfer

- Risk Actuaries and Insurance-Claim Models (cont'd)
  - For example, automobile insurance cost is significantly higher for someone under the age of 25 than other age groups because actuaries determined that the risk of insuring automobiles is *highly age-dependent*.
  - ***Actuaries*** are a crucial part of the insurance process because they use statistical and mathematical analysis to assess the risks of providing coverage.

Insurance for Loss Control and Risk Transfer

- Risk Actuaries and Insurance-Claim Models (cont'd)
  - The annual frequency of events (λ) can be estimated as an interval, such [20%, 30%] or [20%, 90%].
  - The annual frequency can be modeled by a Poisson process with an estimated occurrence rate λ.
  - For simplicity, an elicited interval is assumed to be the mean annual frequency $\pm k\sigma$, where k is a given real value.

Insurance for Loss Control and Risk Transfer

- Risk Actuaries and Insurance-Claim Models (cont'd)
  - The annual frequency based on this model is a random variable distributed according to a continuous distribution with the PDF $f_\lambda(\lambda)$ which can be represented by a probability distribution such as
    - Gamma distribution
    - Beta distribution
    - Negative binomial distribution (Pascal distribution)

Insurance for Loss Control and Risk Transfer

- Risk Actuaries and Insurance-Claim Models (cont'd)
  - The Gamma distribution has two parameters, α and θ, defined as follows:

$$\alpha = \frac{\mu^2}{\sigma^2} \qquad (18a)$$

and

$$\theta = \frac{\sigma^2}{\mu} \qquad (18b)$$

Insurance for Loss Control and Risk Transfer

- Risk Actuaries and Insurance-Claim Models (cont'd)
  - The probability density function (f_λ) of the Gamma distribution is given by

$$f_\lambda(x) = \frac{x^{\alpha-1} e^{-x/\theta}}{\Gamma(\alpha)\theta^\alpha} \tag{19}$$

  - The severity of a claim is the second variable that needs to be examined in the assessment of insurance claims.

Insurance for Loss Control and Risk Transfer

- Risk Actuaries and Insurance-Claim Models (cont'd)
  - The severity of claims can be modeled using two lognormal distributions, representing lower and upper limits, based on expert opinion.
  - Means and standard deviations for both the lower and upper severity limits can be elicited.
  - Therefore, the event-occurrence severity is a random variable with the CDF, $F_S(s)$, taking on with equal probability of 0.5 one of the two lognormally distributed random variables, i.e., low and high estimates of the CDF.

Insurance for Loss Control and Risk Transfer

- Risk Actuaries and Insurance-Claim Models (cont'd)
  - Each of these distributions is defined by its mean and coefficient variation (COV).
  - The mean and standard deviation of the severity are designated as μ_s and σ_s respectively. Therefore,

$$\mu_y = \ln(\mu_s) - \frac{1}{2}\sigma_y^2 \qquad (20a)$$

$$\sigma_y = \sqrt{\ln\left(1 + \left[\frac{\sigma_s^2}{\mu_s}\right]^2\right)} \qquad (20b)$$

Insurance for Loss Control and Risk Transfer

- Risk Actuaries and Insurance-Claim Models (cont'd)
  - Having defined the normal-equivalent mean and standard deviation, the density function for the lognormal distribution may be shown as:

$$f_S(s) = \frac{1}{s\sigma_y\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\ln s - \mu_y}{\sigma_y}\right)^2} \qquad (21)$$

Insurance for Loss Control and Risk Transfer

- Risk Actuaries and Insurance-Claim Models (cont'd)
  - Having identified the major components for the modeling of the magnitude of the insurance claims, two cases are considered herein.
  - First,
    - the annual frequency of claims was regarded as nonrandom and
  - second,
    - as random.

Insurance for Loss Control and Risk Transfer

- Risk Actuaries and Insurance-Claim Models (cont'd)
  - Both cases examined the magnitude of claims over a time period t in years, e.g., t = [0,10].
  - A stochastic model is therefore gradually constructed in herein as provided under separate headings.

Insurance for Loss Control and Risk Transfer

- Risk Actuaries and Insurance-Claim Models (cont'd)
  - Modeling Loss Accumulation
    - The frequency (λ) is initially considered to be a nonrandom quantity.
    - The severity of each event is modeled using a continuous random variable with the cumulative distribution function $F_S(s)$.
    - The CDF of the accumulated damage (loss) during a non-random time interval [0, t] is given by

$$F(s;\, t, \lambda) = \sum_{n=0}^{\infty} e^{-\lambda t} \frac{(\lambda t)^n}{n!} F_S^{(n)}(s) \qquad (22)$$

Insurance for Loss Control and Risk Transfer

- Risk Actuaries and Insurance-Claim Models (cont'd)
  - Modeling Loss Accumulation (cont'd)
    - where $F_S^{(n)}(s)$ is the n-fold convolution of $F_S(s)$. In other words, $F_S^{(n)}(s)$ is the probability that the total loss accumulated over n events (during time t) does not exceed s.
    - For n = 0, F_S(0)(s) is defined as $F_S^{(0)}(s)$ =1, and for n = 1, $F_S^{(1)}(s) = F_S$, i.e., the cumulative distribution function of S using the mean (μ) and standard deviation (σ) of S.

Insurance for Loss Control and Risk Transfer

- Risk Actuaries and Insurance-Claim Models (cont'd)
  - Modeling Loss Accumulation (cont'd)
    - For n = 2, the 2-fold convolution $F_S^{(2)}(s)$ can be evaluated using conditional probabilities as

$$F_S^{(2)}(s) = P(S + S < s) = \int_0^{\infty} F_S(s - x) f_S(x)dx$$

    - where $f_S(s)$ is the density function of severity. This result can be expressed as

$$F_S^{(2)}(s) = \int_0^{\infty} F_S(s - x) dF_S(x)$$

Insurance for Loss Control and Risk Transfer

- Risk Actuaries and Insurance-Claim Models (cont'd)
  - Modeling Loss Accumulation (cont'd)
    - In the case of a normal probability distribution, the 2-fold convolution $F_S^{(2)}(s)$ can be evaluated as follows:

$$F_S^{(2)}(s) = P(S + S < s) = F_S(s; 2\mu, \sqrt{2}\sigma)$$

Insurance for Loss Control and Risk Transfer

- Risk Actuaries and Insurance-Claim Models (cont'd)
  - Modeling Loss Accumulation (cont'd)
    - The 3-fold convolution $F_S^{(3)}(s)$ is obtained as the convolution of the distributions of $F_S^{(2)}(s)$ and $F_S(s)$ for uncorrelated and identical severities represented by a normal probability distribution as follows:

$$F_S^{(3)}(s) = P(S + S + S < s) = F_S(s; 3\mu, \sqrt{3}\sigma)$$

Insurance for Loss Control and Risk Transfer

- Risk Actuaries and Insurance-Claim Models (cont'd)
  - Modeling Loss Accumulation (cont'd)
    - Higher-order convolution terms can be constructed in a similar manner for uncorrelated and identical severities represented by a normal probability distribution as follows:

$$F_S^{(n)}(s) = P(\overbrace{S + S + \cdots + S}^{n\ times} < s) = F_S(s; n\mu, \sqrt{n}\sigma)$$

Insurance for Loss Control and Risk Transfer

- Risk Actuaries and Insurance-Claim Models (cont'd)
  - Modeling Loss Accumulation (cont'd)
    - Therefore, Eq. 22 can be written for uncorrelated and identical severities represented by a normal probability distribution as follows:

$$F(s;\, t, \lambda) = \sum_{n=0}^{\infty} e^{-\lambda t} \frac{(\lambda t)^n}{n!} F_S(s; n\mu, \sqrt{n}\sigma)$$

    - If λ is random with the PDF $f_\lambda(\lambda)$, Eq. 22 can be modified to

$$F(s;t) = \int_0^{\infty} \left(\sum_{n=0}^{\infty} e^{-\lambda t} \frac{(\lambda t)^n}{n!} F_S^{(n)}(s) \right) f_\lambda(\lambda)\, d\lambda \qquad (23)$$

Insurance for Loss Control and Risk Transfer

- Risk Actuaries and Insurance-Claim Models (cont'd)
  - Subjective Severity Assessment
    - For the mixture of distributions, the accumulated damage (loss) cumulative distribution function during non-random time t is given by the following expression based on Eq. 23:

$$F(s;t) = \sum_{j=1}^{k} w_j F_j(s;t) \qquad (24)$$

    - where

$$\sum_{j=1}^{k} w_j = 1 \qquad F_j(s;t) = \int_0^{\infty} \left(\sum_{n=0}^{\infty} e^{-\lambda t} \frac{(\lambda t)^n}{n!} F_{Sj}^{(n)}(s) \right) f_{\lambda j}(\lambda)\, d\lambda$$

Insurance for Loss Control and Risk Transfer

- Risk Actuaries and Insurance-Claim Models (cont'd)
  - Subjective Severity Assessment (cont'd)
    - For the distribution of weighted average, the accumulated damage (loss) distribution is the k-fold weighted convolution of the distributions of Eq. 23:

$$F(s;t) = F_{wj}^{(k)}(s;t) \qquad (25)$$

    - where $F_{wj}(s, t) = F_j(w_j s, t)$ is associated with weight w_j and $\sum_{j=1}^{k} w_j = 1$

Insurance for Loss Control and Risk Transfer

- Risk Actuaries and Insurance-Claim Models (cont'd)
  - Subjective Severity Assessment (cont'd)
    - For equal weights, each weight is given by

$$w_j = \frac{1}{k} \qquad (26)$$

    - In this case, because the distributions $F_j(s, t)$ (j = 1, ..., k) are assumed to be independent, the mean μ_S and the standard deviation σ_S of $F(s, t)$ are expressed in terms of the mean μ_{Sj} and the standard deviation σ_{Sj} of the distributions $F_j(s, t)$ as

Insurance for Loss Control and Risk Transfer

- Risk Actuaries and Insurance-Claim Models (cont'd)
  - Subjective Severity Assessment (cont'd)

$$\mu_S = \sum_{j=1}^{k} \frac{\mu_{Sj}}{k} \qquad (27a)$$

$$\sigma_j = \frac{1}{k}\left(\sum_{j=1}^{k} \sigma^2{}_{Sj}\right)^{1/2} \qquad (27b)$$

Insurance for Loss Control and Risk Transfer

- Risk Actuaries and Insurance-Claim Models (cont'd)
  - Computational Procedures and Illustrations
    - Input data (k experts);
    - Distributions of λ and the distributions of damage (loss);
    - Evaluation of Eq. 23 for each expert; and
    - Combining the results from the previous steps in numerical and graphical forms of a mixed distribution solution based on Eq. 24, or an averaging distribution solution based on Eq. 25.

Insurance for Loss Control and Risk Transfer

- **Example 5:** One Expert and Nonrandom Event-Occurrence Rate
  - The numerical example presented in this section illustrates the case of one expert and nonrandom frequency λ.
  - The event rate (λ) is assumed to have a value of one event per year, and the loss as a result of one event occurrence is assumed to have the normal distribution with mean μ_S of 3 and standard deviation σ_S of 0.2 (in $1000).

Insurance for Loss Control and Risk Transfer

- Example 5 (cont'd):
  - The model given by Eq. 22 was evaluated for the following time intervals: t = 1, 2, 3, and 4 years.
  - In order to provide the accuracy acceptable for practical applications, the summation of Eq. 22 included 11 terms.
  - The normal distribution was used to evaluate the operation of convolution.

Insurance for Loss Control and Risk Transfer

- Example 5 (cont'd):
  - The n-fold convolution $F_S^{(n)}(s)$ is the normal distribution having the mean equal to the mean of the underlying distribution $F_S(s)$ multiplied by n, and the respective variance increased by the same factor n.
  - Selected computational steps of accumulated damage (loss) distributions are illustrated in Table 8.
  - The table shows sample computations for t = 1 year.

Insurance for Loss Control and Risk Transfer

Table 8. Accumulated Damage (Loss) Distribution Based on Eq. 22 for t = 1 year

Number of Events in t n	Occurrence Probability of n Events = $e^{-\lambda t}\frac{(\lambda t)^n}{n!}$	$e^{-\lambda t}\frac{(\lambda t)^n}{n!}F_S^{(n)}(s)$							
		s							
		0.3	0.6	…	3.3	…	6.6	…	36
0	0.367879	3.679 E-01	3.679 E-01		3.679 E-01		3.679 E-01		3.679 E-01
1	0.367879	0.000 E+00	0.000 E+00		3.433 E-01		3.679 E-01		3.679 E-01
2	0.18394	0.000 E+00	0.000 E+00		0.000 E+00		1.808 E-01		1.839 E-01
3	0.061313	0.000 E+00	0.000 E+00		0.000 E+00		1.315 E-13		6.131 E-02

Insurance for Loss Control and Risk Transfer

Table 8. (cont'd) Accumulated Damage (Loss) Distribution Based on Eq. 22 for t = 1 year

Number of Events in t n	Occurrence Probability of n Events = $e^{-\lambda t}\frac{(\lambda t)^n}{n!}$	$e^{-\lambda t}\frac{(\lambda t)^n}{n!}F_S^{(n)}(s)$							
		s							
		0.3	0.6	…	3.3	…	6.6	…	36
4	0.015328	0.000 E+00	0.000 E+00		0.000 E+00		0.000 E+00		1.533 E-02
5	0.003066	0.000 E+00	0.000 E+00		0.000 E+00		0.000 E+00		3.066 E-03
6	0.000511	0.000 E+00	0.000 E+00		0.000 E+00		0.000 E+00		5.109 E-04
7	7.3E-05	0.000 E+00	0.000 E+00		0.000 E+00		0.000 E+00		7.299 E-05
8	9.12E-06	0.000 E+00	0.000 E+00		0.000 E+00		0.000 E+00		9.124 E-06

Insurance for Loss Control and Risk Transfer

Table 8. (cont'd) Accumulated Damage (Loss) Distribution Based on Eq. 22 for t = 1 year

Number of Events in t n	Occurrence Probability of n Events = $e^{-\lambda t}\frac{(\lambda t)^n}{n!}$	$e^{-\lambda t}\frac{(\lambda t)^n}{n!}F_S^{(n)}(s)$							
		s							
		0.3	0.6	…	3.3	…	6.6	…	36
9	1.01E-06	0.000 E+00	0.000 E+00		0.000 E+00		0.000 E+00		1.014 E-06
10	1.01E-07	0.000 E+00	0.000 E+00		0.000 E+00		0.000 E+00		1.014 E-07
11	9.22E-09	0.000 E+00	0.000 E+00		0.000 E+00		0.000 E+00		9.216 E-09
$F(s;t,\lambda)=\sum_{n=0}^{11}e^{-\lambda t}\frac{(\lambda t)^n}{n!}F_S^{(n)}(s)=$		3.68 E-01	3.68 E-01		7.11 E-01		9.17 E-01		1.00

Insurance for Loss Control and Risk Transfer

- <u>Example 5 (cont'd):</u>
  - The accumulated damage (loss) distributions evaluated for all time intervals are shown in Figure 10.
  - The computational steps in each function are associated with the successive convolutions in the sum of Eq. 22.
  - The figure shows that the median of loss increases as the time exposure increases

Insurance for Loss Control and Risk Transfer

Figure 10. Cumulative Distribution Functions of the Accumulated Loss, $F(s, t; \lambda)$, with Nonrandom Annual Frequency

Benefit-Cost Analysis

- For cases involving several credible consequence scenarios, the risks associated with each can be assessed as the product of the corresponding probabilities and consequences.
- And the results summed up to obtain the total risk.
- If the risk is not acceptable, mitigation actions should be considered to reduce it.

Benefit-Cost Analysis

- Justification for these actions can be developed based on benefit-cost analysis.
- The costs in this case are associated with mitigation actions.
- On the other hand, the benefits that are associated with mitigation actions can be classified as follows:
  - Reduced severe accident frequency that lead to reduced fatalities, reduced injuries, and reduced property and environmental loss;

Benefit-Cost Analysis

- Reduced incidents, i.e., minor accidents, that lead to reduced injuries and reduced property and environmental losses;
- Reduced incidents and accident precursors leading to reduced errors and deviations, reduced equipment failures, reduced property and environmental losses, etc.
- Secondary and tertiary benefits as a result of intangibles.

Benefit-Cost Analysis

- Benefit assessment requires sometimes the development and use of categories of products and users in order to obtain meaningful results.
- An illustrative example of this requirement is the examination of survival data based on the use of personal flotation devices (PFDs) as provided in the following hypothetically constructed data:

Benefit-Cost Analysis

	Adults	Children	Adults & Children
Wearing PFDs	$\frac{98}{100} = 0.98$	$\frac{320}{400} = 0.80$	$\frac{418}{500} = 0.836$
Not wearing PFDs	$\frac{950}{1000} = 0.95$	$\frac{250}{400} = 0.625$	$\frac{1200}{1400} = 0.857$

Benefit-Cost Analysis

- The following are the most widely used present value comparison methods (as discussed in Chapter 6):
  - Net present value;
  - Benefit-cost ratio;
  - Internal rate of return; and
  - Payback period.

Benefit-Cost Analysis

- The NPV can be calculated as follows:

$$NPV = \sum_{t=0}^{k} \frac{(B-C)_t}{(1+r)^t} = \sum_{t=0}^{k} \frac{B_t}{(1+r)^t} - \sum_{t=0}^{k} \frac{C_t}{(1+r)^t} \qquad (28)$$

B = future annual benefits in constant dollars
C = future annual costs inconstant dollars
r = annual real discount rate
k = number of years from the base year over which the project will be evaluated, and t is an index running from 0 to k representing the year under consideration

Benefit-Cost Analysis

- The benefit of a risk mitigation action can be assessed as follows:

$$\text{Benefit} = \text{unmitigated risk} - \text{mitigated risk} \qquad (29)$$

$$\text{Benefit-to-cost ratio } (B/C) = \frac{\text{Benefit}}{\text{Cost}} = \frac{\text{Unmitigated Risk - Mitigated Risk}}{\text{Cost of Mitigation Action}} \qquad (30)$$

- Ratios greater than one are desirable. In general, the larger the ratio, the better the mitigation action.

Benefit-Cost Analysis

$$B/C = \frac{\sum_{t=0}^{k} \frac{B_t}{(1+r)^t}}{\sum_{t=0}^{k} \frac{C_t}{(1+r)^t}} \qquad (31)$$

- The internal rate of return (IRR) is defined as the discount rate that makes the present value of the stream of expected benefits in excess of expected costs zero.
- The payback period measures the number of years required for net undiscounted benefits to recover the initial investment in a project.

Benefit-Cost Analysis

- In some cases, benefit-cost analysis may reveal that a greater net benefit can be realized if a project is deferred for several years rather than implemented immediately.
- Such a situation has a higher likelihood of occurring if the following conditions are met:
  - The project benefit stream is heavily weighted to the later years of the project life;

Benefit-Cost Analysis

- The project is characterized by large, up-front capital costs; and
- Capital and land cost escalation can be contained through land banking or other means.

- For example, a project NPV can be calculated for the following two-time scenarios to assess delaying the start of the project by d years, without delay (NPV) and with delay (NPV_d):

$$NPV = \sum_{t=0}^{k} \frac{(B-C)_t}{(1+r)^t} \qquad (32a)$$

Benefit-Cost Analysis

$$NPV_d = \sum_{t=d}^{k+d} \frac{(B-C)_t}{(1+r)^t} \qquad (32b)$$

- Assuming B and C to normally distributed, a benefit-cost index ($\beta_{B/C}$) can be defined similar to Eq. 7 as follows:

$$\beta_{B/C} = \frac{\mu_B - \mu_C}{\sqrt{\sigma_B^2 + \sigma_C^2}} \qquad (33)$$

- The failure probability can be computed as

$$P_{f,B/C} = P(C > B) = 1 - \Phi(\beta) \qquad (34)$$

Benefit-Cost Analysis

- In the case of lognormally distributed B and C, the benefit-cost index ($\beta_{B/C}$) can be computed as

$$\beta_{B/C} = \frac{\ln\left(\frac{\mu_B}{\mu_C}\sqrt{\frac{\delta_C^2+1}{\delta_B^2+1}}\right)}{\sqrt{\ln[(\delta_B^2+1)(\delta_C^2+1)]}} \qquad (35)$$

Benefit-Cost Analysis

- **Example 6:** Protection of Critical Infrastructure
  - As an illustration, assume that there is a 0.01 probability of an attack on a facility containing hazardous material during the next year.
  - If the attack occurs, the probability of a serious release to the public is 0.01 with a total consequence of $100B.
  - The total consequence of an unsuccessful attack is negligible.

Benefit-Cost Analysis

- Example 6 (cont'd)
  - The unmitigated risk can therefore be computed as

$$\text{Unmitigated Risk} = 0.01(0.01)(\$100B) = \$10M$$

  - If armed guards are deployed at each facility, the probability of attack can be reduced to 0.001 and the probability of serious release if an attack occurs can be reduced to 0.001

Benefit-Cost Analysis

- Example 6 (cont'd)
  - The cost of the guards for all plants is assumed to be $100M per year.
  - The mitigated risk can therefore be computed as

$$\text{Mitigated Risk} = 0.001(0.001)(\$100B) = \$0.10M$$

  - The benefit in this case is

$$\text{Benefit} = \$10M - \$0.1M \text{ or} \sim \$10M$$

  - The benefit-to-cost ratio is about 0.1. Thus, the $100M cost might be difficult to justify.

Benefit-Cost Analysis

- **Example 7:** Efficient Frontier in Benefit-Cost Analysis for a Mode of Transportation
  - Four transportation modes are considered by the management of a warehousing company to supply components from the warehouse to one of its major customers in a foreign country.
  - The available alternatives for the modes of transport are (1) road and ferry (A_1), (2) rail and ferry (A_2), (3) sea (A_3), and (4) air (A_4).

Benefit-Cost Analysis

- Example 7 (cont'd):
  - The management team of the company was not certain of the cost and return values of the alternatives.
  - Probabilistic information for costs and revenues associated with each alternative was produced as shown in Table 9.
  - Table 10 shows the calculation of the benefits and the benefit to cost ratio (B/C) associated with each alternative.

Benefit-Cost Analysis

Table 9. Assessments of Modes of Transportation for Delivery to Foreign Clients

Alternatives	Cost		Revenue	
	Estimated *NPV* of Cost ($\$10^6$)	Probability	Estimated *NPV* of Revenue ($\$10^6$)	Probability
A_1: Road and Ferry	100	0.6	300	0.5
	90	0.3	250	0.4
	80	0.1	200	0.1
A_2: Rail and Ferry	80	0.4	210	0.3
	70	0.4	225	0.4
	35	0.2	240	0.3
A_3: Sea	100	0.6	140	0.5
	90	0.3	120	0.4
	80	0.1	110	0.1
A_4: Air	150	0.7	250	0.2
	120	0.2	150	0.4
	100	0.1	130	0.3
			100	0.1

Benefit-Cost Analysis

Table 10. Benefit to Cost Ratios for the Modes of Transportation

Alternatives	Cost ($\$10^6$)	Revenue ($\$10^6$)	Benefits ($\$10^6$)	*B*/*C*	Rank
A_1: Road and Ferry	95	270	175	1.84	3
A_2: Rail and Ferry	67	225	158	2.36	1
A_3: Sea	95	129	34	0.36	4
A_4: Air	34	109	75	2.21	2

Benefit-Cost Analysis

- Example 7 (cont'd):
  - From Table 10, the alternatives can be ranked based on the B/C ratios to produce that alternative A_2 is the best choice with the largest ratio of 2.36 followed in order by alternatives A_4, A_1, and A_3.
  - Figure 11 shows the results graphically along with the efficient frontier that include the most appealing alternatives A_1, A_2, and A_4

Benefit-Cost Analysis

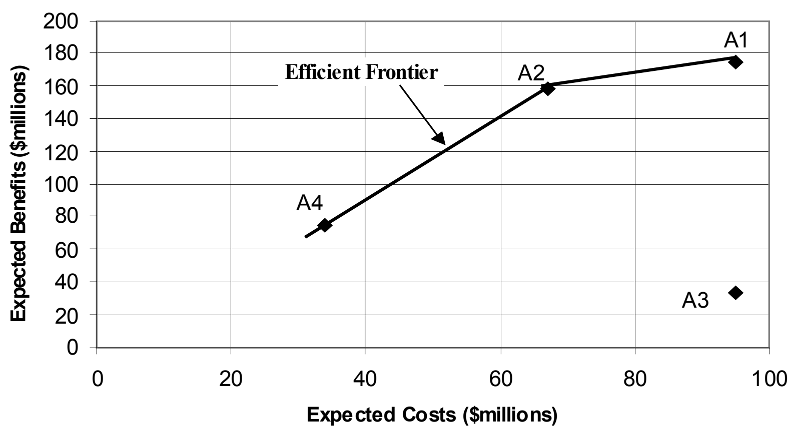

Figure 11. Efficient Frontier for the Benefit-Cost Analysis of Transportation Modes

Benefit-Cost Analysis

- Example 7 (cont'd):
  - Alternative A_3 is considered a risky alternative with low benefit value and high cost value in comparison to other alternatives.
  - Assuming that the management team is risk averse, then from Figure 11 alternative A_2 gives the highest benefit of $158 millions with the least cost of $67 millions.
  - which is in agreement with the selection based on its greatest B/C ratio of 2.36.