

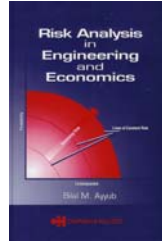


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# ENGINEERING ECONOMICS AND FINANCE

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## Risk Analysis for Engineering

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## Introduction

- Need for Economics
  - Engineers sometimes are faced with nontechnological barriers that limit what can be done to solve a problem.
  - In addition to designing and building systems, engineers must meet other constraints, such as
    - Budgets
    - Regulations





## Introduction

- Need for Economics (cont'd)
  - For example, natural resources necessary to build systems are becoming scarcer and more expensive than ever before.
  - Also, engineers and economists are aware of the potential negative side effects of engineering innovations, such as air pollution from automobiles.
  - They must ask themselves if a particular project would offer some net benefit to individuals or society as a whole.



## Introduction

- Need for Economics (cont'd)
  - Therefore, a net benefit assessment of a particular project is required.
  - The net benefit assessment should include severities associated with failure consequences due to hazards, plus the cost of consuming natural resources.
  - Risk analysis require engineers and economists to work closely together to
    - Develop new system,
    - Solve problems that face society, and
    - Meet societal needs.



# Introduction

- Need for Economics (cont'd)
  - Therefore, results from risk assessment should feed into economic models, and economic model might drive technological innovations and solutions.
  - The development of such economic framework is as important as the physical laws and sciences defining technologies that determine what can be accomplished with engineering.
  - Figure 1 shows how problem solving is composed of physical and economic components.



# Introduction

- Need for Economics (cont'd)

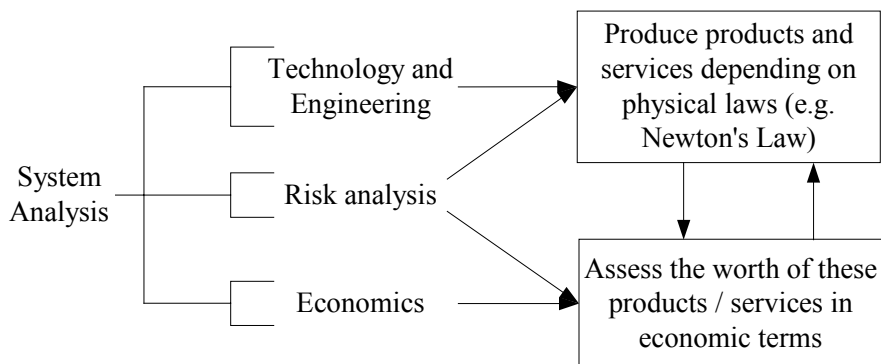


Figure 1. Systems Framework for Risk Analysis



## Introduction

- Need for Economics (cont'd)
  - Engineers and economists are concerned with two types of efficiency:
    1. physical, and
    2. economic efficiencies.
  - The physical efficiency takes the following form:

$$\text{Physical efficiency} = \frac{\text{System output(s)}}{\text{System input(s)}} \quad (1)$$



## Introduction

- Need for Economics (cont'd)
  - The other form of efficiency of interest herein is economic efficiency, which takes the following form:

$$\text{Economic efficiency} = \frac{\text{System worth}}{\text{System cost}} \quad (2)$$

- This ratio is also commonly known as the benefit-cost ratio.



## Introduction

- Need for Economics (cont'd)
  - Both terms for this ratio are assumed to be of monetary units, such as dollars.
  - In contrast to physical efficiency, economic efficiency can exceed unity.
  - In fact it should if a project is to be economically desirable and feasible.
  - In the final evaluation of most ventures, economic efficiency takes precedence over physical efficiency.



## Introduction

- Need for Economics (cont'd)
  - The reason for this is that projects cannot be approved, regardless of their physical efficiency, if there is no conceived demand for them among the public.
  - That is, if they are economically infeasible, or if they do not constitute a wise use of those resources that they require.
  - Example:
    - A proposal to purify water needed by a large city by boiling it and collecting it again through condensation. ..TOO COSTLY..





## Introduction

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- Role of Uncertainty and Risk in Engineering Economics
  - Engineering economic analysis might require the assumption of knowing the
    - Benefits
    - Costs
    - Physical quantitieswith high degree of confidence.
  - This degree of confidence is sometimes called *assumed certainty*.



## Introduction

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- Role of Uncertainty and Risk in Engineering Economics (cont'd)
  - In virtually all situations, however, some doubt as to the ultimate values of various quantities exists.
  - Both risk and uncertainty in decision-making activities are caused by a lack of
    - precise knowledge,
    - Incomplete knowledge, or
    - fallacy in knowledge regarding future conditions.



## Introduction

- Role of Uncertainty and Risk in Engineering Economics (cont'd)
  - Decisions under risk are decisions in which the analyst models the decision problem in terms of assumed possible future outcomes whose probabilities of occurrence and severities can be estimated.
  - Decisions under uncertainty, by contrast, could also include decision problems characterized by several unknown outcomes, or outcomes for which probabilities of occurrence cannot be estimated.



## Introduction

- Engineering and Economic Studies
  - Engineering activities dealing with elements of the physical environment take place to meet human needs that could arise in an economic setting.
  - The engineering process employed from the time a particular need is recognized until it is satisfied may be divided into the following five phases:
    1. determination of objectives,
    2. identification of strategic factors,



## Introduction

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- Engineering and Economic Studies
  3. determination of means (engineering proposals)
  4. evaluation of engineering proposals, and
  5. assistance in decision making.
- These steps can also be presented within an economic framework.
- The creative step involves people with vision and initiative adopting the premise that better opportunities exist than do now.
- The definition step involves developing system alternatives with specific economic



## Introduction

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- Engineering and Economic Studies and physical requirements for particular inputs and outputs.
- The conversion step involves converting the attributes of system alternatives to a common measure so that systems can be compared. Future cash flows are assigned to each alternative to account for the time value of money.





## Introduction

- Engineering and Economic Studies
  - The *decision step* involves evaluating the qualitative and quantitative inputs and outputs to and from each system as the basis for system comparison and decision making.
  - Decisions among system alternatives should be made on the basis of their differences in regard to accounting for uncertainties and risks.



## Fundamental Economic Concepts

- Economics as a field can be defined as the science that deals with the production, distribution, and consumption of wealth, and with the various related problems of labor, finance, and taxation.
- It is the study of how human beings allocate scarce resources to produce various commodities and how those commodities are distributed for consumption among the people in society.



## Fundamental Economic Concepts

- The essence of economics lies in the fact that resources are scarce, or at least limited, and that not all human needs and desires can be met.
- The principal concern of economists is how to distribute these resources in the most efficient and equitable way.
- Economists currently are employed in large numbers in private industry, government, and educational institutions.



## Fundamental Economic Concepts

- **Utility**
  - Utility is the power of a good or service to satisfy human needs.
  - Value designates the worth that a person attaches to an object or service.
  - It is also a measure or appraisal of utility in some medium of exchange, and is not the same as cost or price.
  - Consumer goods are the goods and services that directly satisfy human wants, for example, television sets, shoes, and houses.



## Fundamental Economic Concepts

- On the other hand, producer goods are the goods and services that satisfy human wants indirectly as a part of the production or construction processes, for example, factory equipment, and industrial chemicals and materials.

### ■ Economy of exchange

- Economy of exchange occurs when two or more people exchange utilities since consumers evaluate utilities subjectively representing mutual benefit and persuasion in exchange.



## Fundamental Economic Concepts

- On the other hand, economy of organization can be attained more economically by labor savings and efficiency in manufacturing or capital use.
- Economy of exchange is also greatly affected by supply and demand that respectively express available number of units in a market for meeting some utility or need, and number of units that a market demand of such units.
- The supply and demand can be expressed using respective curves.



## Fundamental Economic Concepts

- The exchange price is defined by the intersection of the two curves.
- Elasticity of demand involves price changes and their effect on demand changes.
- It depends on whether the consumer product is a necessity or a luxury.
- **The law of diminishing return**
  - The law of diminishing return for a process states that the process can be improved at a rate with a diminishing return, for example the cost of inspection to reduce cost of repair and lost production.



## Fundamental Economic Concepts

- **Interest**
  - Interest is a rental amount, expressed on an annual basis, and charged by financial institutions for the use of money.
  - It is also called the rate of capital growth, or the rate of gain received from an investment.
  - For the lender, it consists of
    - (1) risk of loss,
    - (2) administrative expenses, and
    - (3) profit or pure gain.



# Fundamental Economic Concepts

- For the borrower, it is the cost of using a capital for immediately meeting his or her needs.

## ■ The time-value of money

- The time-value of money is the relationship between interest and time, i.e., money has time-value because the purchasing power of a dollar changes with time.
- Figure 2 illustrates the time-value of money.



# Fundamental Economic Concepts

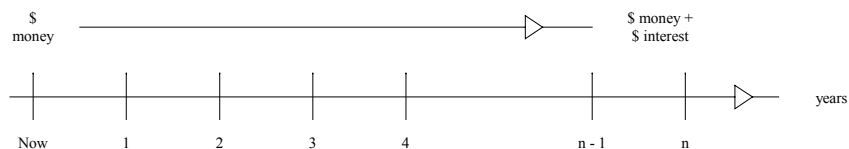


Figure 2. Time-Value of Money



## Fundamental Economic Concepts

### ■ The earning power of money

- The earning power of money represents funds borrowed for the prospect of gain.
- Often these funds will be exchanged for goods, services, or production tools, which in turn can be employed to generate an economic gain.
- On the other hand, the earning power of money involves prices of goods and services that can move upward or downward, where the purchasing power of money can change with time.



## Fundamental Economic Concepts

- Both price reductions and price increases can occur where reductions are caused by increases in productivity and availability of goods, and increases are caused by government policies, price support schemes, and deficit financing.



## Cash Flow Diagrams

- Cash flow diagrams are a means of visualizing and/or simplifying the flow of receipts and disbursements, for the acquisition and operation of items in an enterprise.
- A cash flow diagram convention involves a horizontal axis that is marked off in equal increments, one per period, up to the duration of the project.



## Cash Flow Diagrams

- It also involves revenues and disbursements, where revenues or receipts are represented by upward-pointing arrows, and disbursements or payments are represented by downward-pointing arrows.
- All disbursements and receipts (i.e., cash flows) are assumed to take place at the end of the year in which they occur.
- This is known as the “end-of-year” convention.





## Cash Flow Diagrams

- Arrow lengths are approximately proportional to the magnitude of the cash flow.
- Expenses incurred before time = 0 are sunk costs and are not relevant to the problem.
- Because there are two parties to every transaction, it is important to note that cash flow directions in cash-flow diagrams depend upon the point view taken.



## Cash Flow Diagrams

- **Example 1: Cash-Flow Diagram**
  - Figure 3 shows cash flow diagrams for a transaction spanning five years.
  - The transaction begins with a \$1,000 loan.
  - For years two, three and four, the borrower pays the lender \$120 interest.
  - At year five, the borrower pays the lender \$120 interest plus the \$1,000 principal.





## Cash Flow Diagrams

### ■ Example 1 (cont'd):

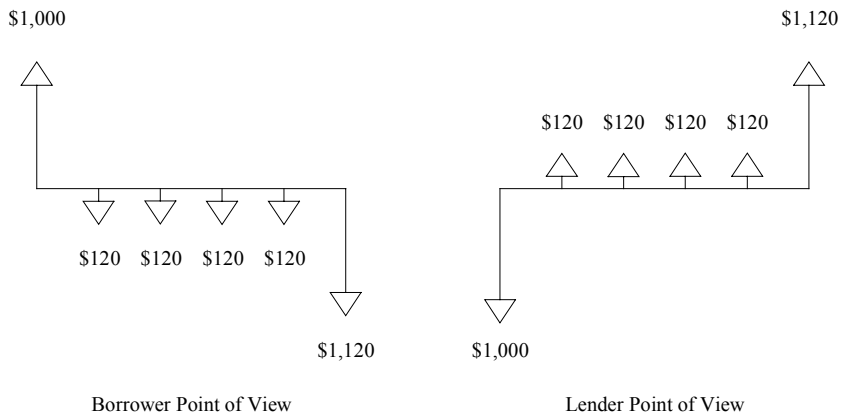


Figure 3. Typical Cash Flow Diagrams



## Cash Flow Diagrams

### ■ Example 1 (cont'd):

- The figure shows two types of cash-flow arrows.
- A cash flow over time involves upward arrow means positive flow, while a downward arrow means negative flow.
- Each problem has two cash flows corresponding to a borrower and a lender.



## Interest Formulae

- Interest formulae play a central role in the economic evaluation of engineering alternatives.
- Types of Interest
  - A payment that is due at the end of a time period in return for using a borrowed amount for this period is called *simple interest*.
  - For fractions of a time period, the interest should be multiplied by the fraction.



## Interest Formulae

- Types of Interest (cont'd)
  - Simple interest is calculated by the following formula:

$$I = P n i \quad (3)$$

$P$  = the principal in dollars or other currency

$i$  = the interest rate expressed as a fraction per unit time

$n$  = the number of years or time periods that is consistent in units with the interest rate.



## Interest Formulae

- Types of Interest (cont'd)
  - The compound interest can be computed as

$$I = P((i + 1)^n - 1) \quad (4)$$

- Compound interest is a type of interest that results from computing interest on an interest payment due at the end of a time period.



## Interest Formulae

- Types of Interest (cont'd)
  - If an interest payment is due at the end of a time period that has not been paid, this interest payment is treated as an additional borrowed amount over the next time period producing additional interest amount called compound interest.



## Interest Formulae

### ■ **Example 2:** Simple Interest

A contractor borrows \$50,000 to finance the purchase of a truck at a simple interest rate of 8% per annum. At the end of two years the interest owed would be

$$I = (\$50,000) (0.08) (2) = \$8,000$$



## Interest Formulae

### ■ **Example 3:** Simple Interest Over Multiple Years

- A loan of \$1,000 is made at an interest of 12% for 5 years. The interest is due at the end of each year with the principal is due at the end of the fifth year.
- In this case, the principal ( $P$ ) is \$1000.00, the interest rate ( $i$ ) is 0.12, and the number of years or periods ( $n$ ) is 5.
- Table 1 shows the payment schedule based on using Eq. 3.
- The amount at the start of each year is the same since according to the terms of the loan, interest due is payable at the end of the year



## Interest Formulae

### ■ Example 3 (cont'd):

$$I = P n i$$

**Table 1.** Resulting Payment Schedule for Example 3

Year	Amount at Start of Year (\$)	Interest at End of Year (\$)	Owed Amount at End of Year (\$)	Payment (\$)
1	1000.00	120.00	1120.00	120.00
2	1000.00	120.00	1120.00	120.00
3	1000.00	120.00	1120.00	120.00
4	1000.00	120.00	1120.00	120.00
5	1000.00	120.00	1120.00	1120.00



## Interest Formulae

### ■ **Example 4:** Compound Interest

- A loan of \$1,000 is made at an interest of 12% compounded annually for 5 years. The interest and the principal are due at the end of the fifth year.
- In this case, the principal ( $P$ ) is \$1000.00, the interest rate ( $i$ ) is 0.12, and the number of years or periods ( $n$ ) is 5.
- Table 2 shows the resulting payment schedule.



## Interest Formulae

### ■ Example 4 (cont'd):

**Table 2.** Resulting Payment Schedule for Example 4

Year	Amount at Start of Year (\$)	Interest at End of Year (\$)	Owed Amount at End of Year (\$)	Payment (\$)
1	1000.00	120.00	1120.00	0.00
2	1120.00	134.40	1254.40	0.00
3	1254.40	150.53	1404.93	0.00
4	1404.93	168.59	1573.52	0.00
5	1573.52	188.82	1762.34	1762.34



## Interest Formulae

### ■ Discrete Compounding and Discrete Payments

- Interest formulae cover variations of computing various interest types and payment schedules for a loan.
- The interest formulae are provided in the form of factors.
- For example, Eq. 3 includes the factor  $(ni)$ , which is used as a multiplier to obtain  $I$  from  $P$ .



## Interest Formulae

- Discrete Compounding and Discrete Payments (cont'd)
  - Seven factors are provided herein as follows:
    1. Single-payment, *compound-amount factor*;
    2. Single-payment, *present-worth factor*;
    3. Equal-payment-series, *compound-amount factor*;
    4. Equal-payment-series, *sinking-fund factor*;
    5. Equal-payment-series, *capital-recovery factor*;
    6. Equal-payment-series, *present-worth factor*; and
    7. Uniform-gradient-series factor.



## Interest Formulae

- Discrete Compounding and Discrete Payments (cont'd)
  - Single-Payment, Compound-Amount Factor
    - The single-payment compound-amount factor is used to compute a future payment ( $F$ ) for a borrowed amount at the present ( $P$ ) for  $n$  year at an interest of  $i$ .
    - The future sum is calculated by applying the following formula:

$$F = P(1 + i)^n \quad (5)$$



## Interest Formulae

- **Example 5:** Single-Payment Compound-Amount Factor
  - A loan of \$1,000 is made at an interest of 12% compounded annually for 4 years. The interest is due at the end of each year with the principal is due at the end of the fourth year.
  - The principal ( $P$ ) is \$1000, the interest rate ( $i$ ) is 0.12, and the number of years or periods ( $n$ ) is 4.



## Interest Formulae

- **Example 5:** Single-Payment Compound-Amount Factor (cont'd)
  - Therefore,

$$F = 1000(1 + 0.12)^4 = 1,573.50 \quad (6)$$

- Figure 4 shows the cash flow for the single present amount, i.e.,  $P = \$1,000$ , and the single future amount, i.e.,  $F = \$1,573.50$ .

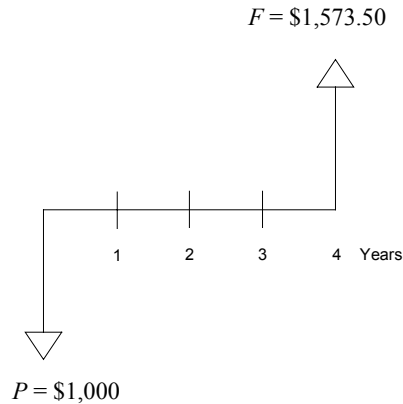






## Interest Formulae

### ■ Example 5 (cont'd)



**Figure 4.** Cash Flow for Single-Payment Compound-Amount from the Perspective of a Lender



## Interest Formulae

### ■ Discrete Compounding and Discrete Payments (cont'd)

#### – Single-Payment Present-Worth Factor

- The single-payment, present-worth factor provides the present amount ( $P$ ) for a future payment ( $F$ ) for  $n$  periods at an interest rate  $i$  as follows:

$$P = \frac{F}{(1+i)^n} \quad (7)$$

- The factor  $1/(1+i)^n$  is known as the single-payment present-worth factor, and may be used to find the present worth  $P$  of a future amount  $F$ .



## Interest Formulae

### ■ **Example 6:** Single-Payment Present-Worth Factor for Construction Equipment

- A construction company wants to set aside enough money today in an interest-bearing account in order to have \$100,000 four years from now for the purchase of a replacement piece of equipment.
- If the company can receive 12% interest on its investment, the single payment present-worth factor is calculated as follows:

$$P = \frac{100,000}{(1 + 0.12)^4} = 63,550 \quad (8)$$



## Interest Formulae

### ■ **Example 9:** Calculating the Interest Rate for Savings

- A construction company wants to set aside \$1,000 today in an interest-bearing account in order to have \$1,200 four years from now.
- The required interest rate must satisfy the following condition:

$$F = 1,000(1 + i)^4 = 1,200 \quad (11)$$

$$i = \sqrt[4]{\frac{1,200}{1,000}} - 1 = 0.046635 \quad (12)$$

- The interest rate  $i$  needed is 0.046635 or approximately 4.7%.



## Interest Formulae

### ■ Example 10: Calculating the Number of Years

- A construction company wants to set aside \$1,000 today at an annual interest rate of 10% in order to have \$1,200.
- The required number of years necessary to yield this amount can be computed based on the following condition:

$$F = 1,000(1 + 0.1)^n = 1,200 \quad (13)$$

$$(1 + 0.1)^n = \frac{1,200}{1,000} \text{ or } n = \frac{\ln(1.2)}{\ln(1.1)} = 1.9129285 \quad (14)$$

$n$  = approximately 2 years



## Interest Formulae

### ■ Discrete Compounding and Discrete Payments (cont'd)

#### – Equal-Payment-Series, Compound-Amount Factor

- The total compound amount is simply the sum of the compound amounts for years 1 through  $n$ .
- This summation is a geometric series as follows:

$$F = A + A(1+i) + A(1+i)^2 + \dots + A(1+i)^{n-1} \quad (15)$$

- With some mathematical manipulation, it can be expressed as

$$F = A \frac{(1+i)^n - 1}{i} \quad (16)$$



## Interest Formulae

### ■ **Example 11:** Equal Payment-Series Compound Amount Factor for Total Savings

- A contractor makes four equal annual deposits of \$100 each into a bank account paying 12% interest per year. The first deposit will be made one year from today.
- The money that can be withdrawn from the bank account immediately after the fourth deposit is

$$F = \$100 \left( \frac{(1+0.12)^4 - 1}{0.12} \right) = \$477.9 \quad (17)$$



## Interest Formulae

### ■ Discrete Compounding and Discrete Payments (cont'd)

#### – Equal-Payment-Series, Sinking-Fund Factor

- For annual interest rate  $i$  over  $n$  years, the equal end-of-year amount to accomplish a financial goal having a future amount of  $F$  at the end of the  $n^{\text{th}}$  year can be computed from Eq. 16 as

$$A = F \left( \frac{i}{(1+i)^n - 1} \right) \quad (18)$$

- $A$  = required end-of-year payments to accumulate a future amount  $F$



## Interest Formulae

- **Example 12:** Equal-Payment Series Sinking-Fund Factor for Future Savings
  - A student is planning to have personal savings totaling \$1,000 four years from now.
  - If the annual interest rate will average 12% over the next four years, the equal end-of-year amount to accomplish this goal is calculated using the following formula:

$$A = \$1,000 \left( \frac{0.12}{(1+0.12)^4 - 1} \right) = \$209.2 \quad (19)$$



## Interest Formulae

- Discrete Compounding and Discrete Payments (cont'd)
  - Equal-Payment-Series, Capital-Recovery Factor
    - Figure 6 summarizes the flow of disbursements and receipts from the depositor's point of view.
    - The formula for this case is given by

$$A = P \left( \frac{i(1+i)^n}{(1+i)^n - 1} \right) \quad (21)$$



## Interest Formulae

- Discrete Compounding and Discrete Payments (cont'd)
  - Equal-Payment-Series, Capital-Recovery Factor

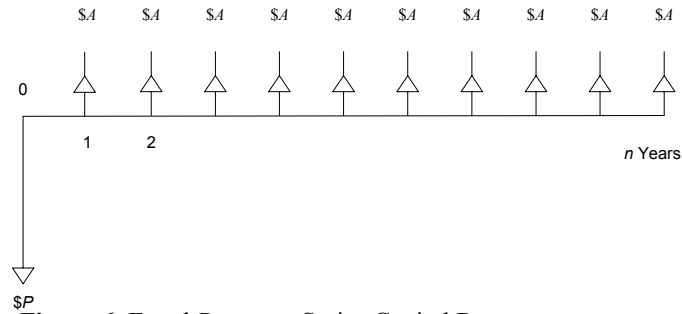


Figure 6. Equal-Payment-Series Capital Recovery



## Interest Formulae

- **Example 13:** Equal-Payment-Series Capital-Recovery Factor for a Loan
  - A contractor borrows \$1,000 and agrees to repay them in four years at an interest rate of 12% per year.
  - The payment in four equal end-of-year payments is calculated by applying Equation 21 as follows:

$$A = 1,000 \left( \frac{0.12(1 + 0.12)^4}{(1 + 0.12)^4 - 1} \right) = \$329.2 \quad (22)$$



## Interest Formulae

- Discrete Compounding and Discrete Payments (cont'd)
  - Equal-Payment-Series Present-Worth Factor
    - The present worth  $P$  of an equal-payment series  $A$  over  $n$  periods at an interest rate  $i$  is

$$P = A \left( \frac{(1+i)^n - 1}{i(1+i)^n} \right) \quad (23)$$



## Interest Formulae

- **Example 14:** Equal-Payment-Series Present-Worth Factor for Investing in a Machine
  - If a certain machine undergoes a major overhaul now, its output can be increased by 5%, which translates into additional cash flow of \$100 at the end of each year for four years.
  - If the annual interest rate is 12%, the amount that could be invested in order to overhaul this machine is calculated by applying Eq. 23 as follows:

$$P = \$100 \left( \frac{(1+0.12)^4 - 1}{0.12(1+0.12)^4} \right) = \$303.7 \quad (24)$$



## Interest Formulae

- **Example 15:** Present Worth of Annuity Factor for Bridge Replacement
  - In Example 6.8 of the Textbook, a town was planning to replace an existing bridge that costs \$5000 annually in operation and maintenance and has a remaining useful life of 20 years.
  - The new bridge will cost \$500,000 to build and an additional \$2000 for annual operation and maintenance.
  - The new bridge will have a useful life of 50 years, thus extending the life by 30 years.



## Interest Formulae

- Example 15 (cont'd):
  - If the interest rate is 8%, the present worth of annuity factor for 20 years, according to Eq. 23, is

$$\frac{(1+i)^n - 1}{i(1+i)^n} = \frac{(1+0.08)^{20} - 1}{0.08(1+0.08)^{20}} = 9.818 \quad (25)$$

- And for 30 years is

$$\frac{(1+i)^n - 1}{i(1+i)^n} = \frac{(1+0.08)^{30} - 1}{0.08(1+0.08)^{30}} = 11.258 \quad (26)$$





## Interest Formulae

### ■ **Example 16:** Capital Recovery Factor for Bridge Replacement

- Examples 6-8 (Text) and 15 presented the case of a town replacing an existing bridge.
- For the interest rate of 8%, the capital recovery factor (to compute equal payments) for 50 years of the cost of the new bridge (\$500,000) according to Eq. 21 is

$$\frac{i(1+i)^n}{(1+i)^n - 1} = \frac{0.08(1+0.08)^{50}}{(1+0.08)^{50} - 1} = 0.08174 \quad (27)$$



## Interest Formulae

### ■ Example 16 (cont'd):

- The annual cost of the new bridge can be taken as the total cost of the bridge multiplied by the capital recovery factor producing the following amount:

$$\text{Annual cost of new bridge} = \$500,000 (0.08174) = \$40,900 \quad (28)$$



## Interest Formulae

### ■ Discrete Compounding and Discrete Payments (cont'd)

#### – Uniform-Gradient-Series Factor

- Sometimes periodic payments do not occur in equal amounts and may increase or decrease by constant amounts (e.g., \$100, \$120, \$140, \$160, \$180, and \$200).
- The uniform-gradient-series factor  $G$  is a value of 0 at the end of year 1,  $G$  at the end of year 2,  $2G$  at the end of year 3, and so on to  $(n - 1)G$  at the end of year  $n$ .



## Interest Formulae

### ■ Discrete Compounding and Discrete Payments (cont'd)

#### – Uniform-Gradient-Series Factor (cont'd)

- An equivalent equal-payment  $A$  can be computed using the following expression:

$$A = G \left( \frac{1}{i} - \frac{n}{(1+i)^n - 1} \right) \quad (29)$$



## Interest Formulae

### ■ **Example 17:** Uniform Gradient-Series Factor for Payments

- If the uniform gradient amount is \$100 and the interest rate is 12%, the uniform annual equivalent value at the end of the fourth year is calculated by applying Equation 29 as follows:

$$A = \$100 \left( \frac{1}{0.12} - \frac{4}{(1+0.12)^4 - 1} \right) = \$135.9 \quad (30)$$



## Interest Formulae

### ■ **Example 18:** Computation of Bridge Replacement Benefits

- Examples 8, 15, and 16 presented the case of a town replacing an existing bridge.
- The existing bridge costs \$ 5,000 annually in operation and maintenance and has a remaining useful life of 20 years.
- The new bridge will cost \$ 500,000 to build and an additional \$ 2,000 for annual operation and maintenance.





## Interest Formulae

- Example 18 (cont'd):
  - The new bridge will have a useful life of 50 years, and thus the extension of the bridge life will be 30 years.
  - The applicable interest rate is 8%.
  - This example demonstrates the computation of the annual benefit gained from replacing the bridge.
  - The benefits of the new bridge include the additional function availability for an additional 30 years, and reducing the operation maintenance costs by \$2,000 per year during the next 20 years.



## Interest Formulae

- Example 18 (cont'd):
  - This example does not analyze the costs of replacing the bridge, i.e., the focus is only on the benefits.
  - The benefits in the 20th year, credited to the bridge life extension is equal to the annual cost of the new bridge, calculated in Equation 6-28, multiplied by the present worth of annuity factor for 30 years, calculated in Equation 26. Therefore, the benefit is

$$\text{Benefits in the 20th year} = \$40,900 (11.258) = \$460,500 \quad (31)$$



## Interest Formulae

### ■ Example 18 (cont'd):

- The present worth in the 1<sup>st</sup> year of bridge extension is equal to the benefits in the 20<sup>th</sup> year, calculated in Equation 31, multiplied by the single payment present worth factor for 20 years, calculated in Equation 10.
- Therefore, the present worth is

$$\text{Present worth in the 1st year} = \$ 460,500 (0.2145) = \$98,800 \quad (32)$$



## Interest Formulae

### ■ Example 18 (cont'd):

- The annual savings in operation and maintenance costs between the 1<sup>st</sup> and the 20<sup>th</sup> years are equal to the difference in the operation and maintenance costs of the existing bridge and the new bridge.
- Therefore, the annual saving is

$$\begin{aligned} \text{Annual saving in operation and maintenance costs} &= & (33) \\ & \$5,000 - \$2,000 = \$3,000 \end{aligned}$$



## Interest Formulae

- Example 18 (cont'd):
  - The present worth in the 1<sup>st</sup> year of operation and maintenance savings is equal to the annual savings in operation and maintenance costs between the 1<sup>st</sup> and the 20<sup>th</sup> years, calculated in Equation 33, multiplied by the present worth of annuity factor for 20 years, calculated in Equation 25.
  - Therefore, its present worth is

$$\text{Present worth in 1st year} = \$3,000 (9.818) = \$29,500 \quad (34)$$



## Interest Formulae

- Example 18 (cont'd):
  - The present worth of total credit is the sum of the present worth in the 1<sup>st</sup> year of bridge extension, calculated in Eq. 32, and the present worth in the 1<sup>st</sup> year of operation and maintenance savings, calculated in Eq. 34.
  - Therefore, the present worth of total credit is given by

$$\text{Present worth of total credit} = \$98,800 + \$29,500 = \$128,300 \quad (35)$$



## Interest Formulae

- Example 18 (cont'd):
  - Finally, the average annual credit or benefit spread over 50 years is equal to the present worth of total credit, calculated in Eq. 35, multiplied by the capital recovery factor, calculated in Eq. 27.
  - Therefore, the average annual credit, or benefit, is

$$\text{Average annual credit, or benefit} = \$128,300 (0.08174) = \$10,500 \quad (36)$$



## Interest Formulae

- Compounding Frequency and Continuous Compounding

- Compounding Frequency

- The effective interest rate ( $i$ ) for any time interval ( $l$ ), that can be different from the compounding period, is given by:

$$i = \left(1 + \frac{r}{m}\right)^{l(m)} - 1 \quad (37a)$$

- Clearly if  $l(m) = 1$ , then  $i = r/m$ .





## Interest Formulae

### – Compounding Frequency (cont'd)

- The product  $l(m)$  is called  $c$ , which corresponds to the number of compounding periods in the time interval  $l$ .
- It should be noted that  $c$  should be  $> 1$ .
- For the special case of  $l = 1$ , the effective interest rate ( $i$ ) for a year is given by

$$i = \left(1 + \frac{r}{m}\right)^m - 1 \quad (37b)$$



## Interest Formulae

### – Continuous Compounding

- The limiting case for the effective rate is when compounding is performed infinite times in a year.
- Using  $l = 1$ , the following limit produces the continuously compounded interest rate,  $i_a$ :

$$i_a = \lim_{m \rightarrow \infty} \left(1 + \frac{r}{m}\right)^m - 1 \quad (38a)$$

- This limit produces the following effective interest rate:

$$i_a = e^r - 1 \quad (38b)$$

- The concept of continuous compounding is illustrated in Table 4.





# Interest Formulae

## – Continuous Compounding (cont'd)

**Table 4.** Example Illustrating the Concept of Continuous Compounding

Compounding Frequency	Number of Periods	Effective Interest Rate per Period (%)	Effective Annual Interest Rate (%)
Annually	1.0	18	18
Semiannually	2.0	9	18.81
Quarterly	4.0	4.5	19.25186
Monthly	12.0	1.5	19.56182
Weekly	52.0	0.3462	19.68453
Daily	365.0	0.0493	19.71642
Continuously	$\infty$	0	19.72174