Introduction

- Walls are generally used to provide lateral support for:
  - An earth fill
  - Embankment, or
  - Some other material to support vertical loads.

- Some of the more common types of walls are shown in Figure 1.

- One main purpose for these walls is to maintain a difference in elevation of the ground surface on each side of the wall.
Introduction

- The earth whose ground surface is at the higher elevation is commonly called the **backfill**, and the wall is said to retain this backfill.
- All walls in Figure 1 have applications in either building or bridge projects.
- They do not necessarily behave in an identical manner under load, but still serve the same basic function.
Introduction

- The basic function of these walls is to provide lateral support for a mass of earth or other material that is at a higher elevation behind the wall than the earth or other material in front of the wall.
- Hence they all may be broadly termed **retaining structures** or **retaining walls**.
- Some retaining walls may support vertical loads in addition to lateral loads from the retained materials.

Gravity Walls

- The gravity wall (Figure 1a) depends mostly on its own weight for stability.
- It is usually made of plain concrete and is used for walls up to approximately 10 ft in height.
- The semigravity wall is a modification of the gravity wall in which small amounts of reinforcing steel are introduced.
- This, in effect, reduces the massiveness of the wall.
Introduction

- **Cantilever Walls**
  - The cantilever wall (Figure 1b and Figure 2) is the most common type of retaining structure and generally is used for walls in the range from 10 to 25 ft in height.
  - It is so named because its individual parts (toe, heel, and stem) behave as, and design as, cantilever beams.
  - Aside from its stability, the capacity of the wall is a function of the strength of its individual parts.
Introduction

- Counterfort Walls
  - The counterfort wall (Figure 1c) may be economical when the wall height is in excess of 25 ft.
  - The counterforts are spaced at intervals and act as tension members to support the stem.
  - The stem is then designed as a continuous member spanning horizontally between counterforts.
Introduction

- **Battress Walls**
  - The buttress wall (Figure 1d) is similar to the counterfort wall except that the buttress are located on the side of the stem opposite to the retained material and act as compression member to support the stem.
  - The counterfort wall is more commonly used because it has a clean, uncluttered exposed face and allows for more efficient use of space in front of the wall.

Introduction

- **Basement Foundation Wall**
  - The basement foundation wall (Figure 1e) may act as cantilever retaining wall.
  - The first floor may provide an additional horizontal reaction similar to the basement floor slab, however, thereby making the wall act as a vertical beam.
  - This wall would then be designed as a simply supported member spanning between the first floor and the basement floor slab.
Introduction

- **Bridge Abutment**
  - The bridge abutment (Figure 1f) is similar in some respects to the basement wall.
  - The bridge superstructure induces horizontal as well as vertical loads, thus altering the normal cantilever behavior.

- **Bearing Walls**
  - The bearing wall (Figure 1g) may exit with or without lateral loads.
  - A bearing wall may be defined as a wall that supports any vertical load in addition to its own weight.
  - Depending on the magnitudes of the vertical and lateral loads, the wall must have to be designed for combined bending and axial compression.
Lateral Forces on Retaining Walls

■ Design of Retaining Walls
  – The design of retaining wall must account for all applied loads.
  – The load that presents the greatest problem and its primary concern is the lateral earth pressure induced by the retained soil.
  – The comprehensive earth pressure theories evolving from the original Coulomb and Rankine theories can be found in almost any textbook on soil mechanics.

■ The Magnitude and Direction of Pressure
  – The magnitude and direction of the pressure as well as the pressure distribution exerted by a soil backfill upon a wall are highly indeterminate, due to many variables.
  – These variables include, but are not limited to,
    • the type of backfill used,
    • The drainage of the backfill material,
    • The level of the water table,
    • The slope of the backfill material,
    • Added loads applied on the backfill,
    • The degree of soil compaction, and
    • Movement of the wall caused by the action of backfill.
Lateral Forces on Retaining Walls

- Water Drainage
  - An important consideration is that water must be prevented from accumulating in the backfill material.
  - Walls are rarely designed to retained saturated material, which means that proper drainage must be provided.
  - It is generally agreed that the best backfill material behind a retaining structure is a well-drained, cohesion-less material.

Lateral Forces on Retaining Walls

- Lateral Pressure
  - The lateral pressure can exit and develop in three different categories:
    - Active state;
    - At rest state; and
    - Passive state.
  - If a wall is absolutely rigid, earth pressure at rest will develop.
  - If the wall should deflect or move a very small amount away from the backfill, active earth pressure will develop.
Lateral Forces on Retaining Walls

- Lateral Pressure (cont’d)
  - In effect, this active earth pressure reduces the lateral earth pressure occurring in the at-rest state.
  - Should the wall be forced to move toward the backfill, for some reason, passive earth pressure will develop and increase the lateral earth pressure appreciably above that occurring in the at-rest state.
  - As indicated, the magnitude of earth pressure at rest lies somewhere between active and passive earth pressures.

- Lateral Pressure (cont’d)
  - Under normal conditions, earth pressure at rest is of such a magnitude that the wall deflects slightly, thus relieving itself of the at-rest pressure.
  - The active pressure results.
  - For this reason, retaining walls are generally designed for active earth pressure due to the retained soil.
Lateral Forces on Retaining Walls

Lateral Pressure (cont’d)

– Because of the involved nature of a rigorous analysis of an earth backfill and the variability of the material and conditions, assumptions and approximations are made with respect to the nature of lateral pressure on a retaining structure.

– It is common practice to assume a linear active and passive earth pressure distribution.

– The pressure intensity is assumed to increase with depth as a function of the weight of the soil in a manner similar that of fluid.

Level Backfill

– If a level backfill (of well-drained cohesionless soil) is considered, then the assumed pressure diagram is shown in Figure 3.

– The unit pressure intensity \( p_y \) in any plane a distance \( y \) down from the top is

\[
p_y = K_a w_e y
\]  

(1)
Lateral Forces on Retaining Walls

Figure 3. Analysis of Forces Acting on Walls: Level Backfill

Level Backfill (cont’d)

Therefore, the total active earth pressure acting on a 1-ft width of wall may be calculated as the product of the average pressure on the total wall height \( h_w \) and the area on which this pressure acts:

\[
H_a = \frac{1}{2} p_y \times \text{area} = \frac{1}{2} (K_a w_e h_w)(h_w \times 1)
\]

\[
= \frac{1}{2} K_a w_e h_w^2
\]
Lateral Forces on Retaining Walls

- Level Backfill (cont’d)
  - Where $K_a$, the coefficient of active earth pressure, has been established by both Rankine and Coulomb to be

  $$K_a = \frac{1 - \sin \phi}{1 + \sin \phi} = \tan^2 \left(45^\circ - \frac{\phi}{2}\right)$$

  - $w_e$ = unit weight of earth (lb/ft³)
  - $\phi$ = angle of internal friction (soil on soil)

- $K_a$ usually varies from 0.27 to 0.40.
- The term $K_a w_e$ in Eq. 2 is generally called an equivalent fluid weight, since the resulting pressure is identical to that which would occur in a fluid of that weight (units are lb/ft³).
- In a similar manner, the total passive earth pressure may be established as

  $$H_p = \frac{1}{2} K_a w_e (h')^2$$
Lateral Forces on Retaining Walls

- **Level Backfill (cont’d)**
  - Where \( h' \) is the height of earth and \( K_p \) is the coefficient of passive earth pressure:

  \[
  K_p = \frac{1 + \sin \phi}{1 - \sin \phi} = \tan^2 \left( 45^\circ + \frac{\phi}{2} \right) = \frac{1}{K_a}
  \]  

  (5)

  - Note that \( K_p \) usually varies from 2.5 to 4.0
  - The total force in each case is assumed to act at one-third the height of the triangular pressure distribution, as shown in Figure 3.

- **Sloping Backfill**
  - If a sloping backfill is considered, then the assumed active pressure distribution is shown in Figure 4, where \( H_s = \frac{1}{2} K_p W h_b^2 \) and \( h_b \) is the height of the backfill at the back of the footing, and \( K_a \) is the coefficient of active earth pressure. Thus

  \[
  K_a = \cos \theta \left[ \cos \theta - \sqrt{\cos^2 \theta - \cos^2 \phi} \right] / \left[ \cos \theta + \sqrt{\cos^2 \theta - \cos^2 \phi} \right]
  \]  

  (6)

  - \( \theta \) = slope angle of backfill
  - \( \phi \) = angle of internal friction (soil on soil)
Lateral Forces on Retaining Walls

Figure 4. Analysis of Forces Acting on Walls: Sloping Backfill

Sloping Backfill (cont’d)

- For walls approximately 20 ft in height or less, it is recommended that the horizontal force component $H_H$ simply be assumed equal $H_s$ and be assumed to act at $h_s/3$ above the bottom of the footing, as shown in Figure 4.
- The effect of the vertical force component $H_V$ is neglected. This is a conservative approach.
- Assuming a well-drained, cohesionless soil backfill that has a unit weight of 110 lb/ft$^3$ and $\phi = 33^o$ 40’, values of equivalent fluid weight for sloping backfill can be found in Table 1.
Table 1. $K_d w_e$ values for Sloping Backfill

<table>
<thead>
<tr>
<th>$\theta$ (deg)</th>
<th>$K_d w_e$ (lb/ft$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>32</td>
</tr>
<tr>
<td>10</td>
<td>33</td>
</tr>
<tr>
<td>20</td>
<td>38</td>
</tr>
<tr>
<td>30</td>
<td>54</td>
</tr>
</tbody>
</table>

Level Backfill with Surcharge

- Loads are often imposed on the backfill surface behind a retaining wall.
- They may either live loads or dead loads.
- These loads are generally termed a surcharge and theoretically may be transformed into equivalent height of earth.
- A uniform surcharge over the adjacent area adds the same effect as an additional (equivalent) height of earth.
Lateral Forces on Retaining Walls

- Level Backfill with Surcharge (cont’d)
  - This equivalent height of earth \( h_{su} \) may be obtained by

\[
h_{su} = \frac{w_s}{w_e}
\]

- In effect, this adds a rectangle of pressure behind the wall with a total lateral surcharge force assumed acting at its midheight, as shown in Figure 5.
- Surcharge loads far enough from the wall cause no additional pressure acting on the wall.
Lateral Forces on Retaining Walls

**Example 1**

Compute the active earth pressure horizontal force on the wall shown for the following conditions:

- \( \phi = 25^\circ \)
- \( \theta = 0^\circ \)
- \( w_s = 400 \text{ psf} \)
- \( w_e = 100 \text{ lb/ft}^3 \)
- \( h_w = 15 \text{ ft} \)
Lateral Forces on Retaining Walls

Example 1 (cont'd)

\[ h_{sw} = \frac{w_s}{w_c} = \frac{400}{100} = 4 \text{ ft} \]

\[ K_a = \frac{1 - \sin \phi}{1 + \sin \phi} = \frac{1 - \sin 25}{1 + \sin 25} = 0.406 \]

\[ H_a = \frac{1}{2} K_a w_c h_w^2 = \frac{1}{2} (0.406)(0.100)(15)^2 = 4.57 \text{ k/ft of wall} \]

\[ H_{sw} = K_a w_c h_w h_w = 0.406(0.100)(4)(15) = 2.44 \text{ k/ft of wall} \]

Total horizontal force = 4.57 + 2.44 = 7.01 k/ft of wall

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Example 2

Find the passive earth pressure force in front of the wall for Example 1 if \( h' = 4 \text{ ft} \).

\[ K_p = \frac{1 + \sin \phi}{1 - \sin \phi} = \frac{1 + \sin 25}{1 - \sin 25} = 2.46 \]

\[ H_p = \frac{1}{2} K_p w_c (h')^2 \]

\[ = \frac{1}{2} (2.46)(0.100)(4)^2 = 1.97 \text{ k/ft of wall} \]
A retaining wall must be stable as a whole, and it must have sufficient strength to resist the forces acting on it.

Four possible modes of failure will be considered:

1. **Overturing About the Toe**
   - Point $O$ as shown in Figure 6, could occur due to lateral loads. The stabilizing moment must be sufficiently in excess of the overturning moment so that an adequate factor of safety against overturning is provided.
Design of Reinforced Concrete Cantilever Retaining Walls

2. Sliding on the Base of the Footing
   • Surface $OM$ in Figure 6, could also occur due to lateral loads.
   • The resisting force is based on an assumed coefficient of friction of concrete on earth.
   • The factor of safety against sliding should never be less than 1.5 and should preferably be 2.0 or more.

3. Excessive Soil Pressure
   • Excessive soil pressure under the footing will lead to undesirable settlements and possible rotation of the wall. Actual soil pressures should not be allowed to exceed specified maximum pressures, which depend on the characteristics of the underlying soil.

4. The Structural Failure of Components
   • The structural failure of component parts of the wall such as stem, toe, and heel, each acting as a cantilever beam, could occur. These must be designed to have sufficient strength.
Design of Reinforced Concrete Cantilever Retaining Walls

**General Design Procedure**

1. Establish the general shape of the wall based on the desired height and function.
2. Establish the site soil conditions, loads, and other design parameters. This includes the determination of allowable soil pressure, earth-fill properties for active and passive pressure calculations, amount of surcharge, and desired factors of safety.

3. Establish the tentative proportions of the wall.
4. Analyze the stability of the wall. Check factors of safety against overturning and sliding and compare actual soil pressure with allowable soil pressure.
5. Assuming that all previous steps are satisfactory, design the component parts of the cantilever wall, stem, toe, and heel.
Using a procedure similar to that used for one-way slabs, the analysis and design of cantilever retaining walls is based on a 12-in. (1 ft)-wide strip measured along the length of the wall.

The tentative proportions of a cantilever retaining wall may be obtained from the following rules of thumb (Figure 6):

1. Footing width $L$: Use $\frac{1}{2} h_w$ to $\frac{2}{3} h_w$.

**Figure 6.** Cantilever Retaining Wall Proportions
Design of Reinforced Concrete Cantilever Retaining Walls

2. Footing thickness $h$: Use $1/10 \, h_w$.
3. Stem thickness $G$ (at the top of footing): Use $1/12 \, h_w$.
4. Toe width $A$: Use $\frac{1}{4} \, L$ to $\frac{1}{3} \, L$.
5. Use a minimum wall batter of $\frac{1}{4}$ in./ft to improve the efficiency of the stem as bending member.
6. The top of the stem thickness $D$ should not be less than 10 in.

Design of Reinforced Concrete Cantilever Retaining Walls

- The give rules of thumb will usually result in walls that can be reasonably be designed.
- Depending on the specific conditions, however, dimensions may have to be adjusted somewhat to accommodate such design criteria as minimum $A_s$, maximum $\rho$, shear strength, anchorage, and development.
Design of Reinforced Concrete Cantilever Retaining Walls

- One common alternative design approach is to assume a footing thickness and then immediately design the stem thickness for an assumed steel ratio.
- Once the stem thickness is established, the wall stability can be checked.
- Whichever procedure is used, adjustment of dimensions during the design is not uncommon.

Example 3

For the wall shown, determine the factors of safety against overturning and sliding and determine the soil pressures under the footing. Use $K_a = 0.3$ and $w_e = 100$ lb/ft$^3$. The coefficient of friction $f = 0.50$.

To be discussed and solved in class.
Example 3 (cont’d)

Design of Reinforced Concrete Cantilever Retaining Walls

\[ h_w = 10' - 0'' \]

\[ 6' - 0'' \]

\[ 3' - 0'' \]

\[ 1' - 0'' \]

Example 4

Design a retaining wall for the conditions shown in the figure. Use \( f'_c = 3000 \text{ psi} \) and \( f_y = 60,000 \text{ psi} \).

To be discussed and solved in class.
Design of Reinforced Concrete Cantilever Retaining Walls

Example 4 (cont’d)

Other design data are given as follows:

- $w_e = 100 \text{ lb/ft}^3$
- Allowable soil pressure = 4000 psf
- Equivalent fluid weight, $K_a w_e = 30 \text{ lb/ft}^3$
- Surcharge = 400 psf
- Factor of safety (overturning) = 2.0
- Factor of safety (sliding) = 1.4
- Coefficient of friction $f = 0.50$