CHAPTER

9c

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A Fundamental Approach - Fifth Edition

COMBINED COMPRESSION AND BENDING: COLUMNS

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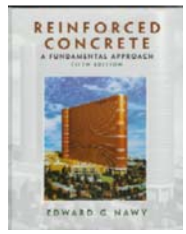

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Strength of Eccentrically Loaded Columns: Axial Load and Bending

- Behavior of Eccentrically Loaded Non-Slender Columns
  – Stress distribution and Whitney's Block for beams can applied in this case.
  – Figure 6 shows a typical rectangular column cross-section with strain, stress, and force distribution diagrams.
  – Notice the additional nominal force P_n at the limit failure state acting at an eccentricity e from the plastic centriod of the section.

Strength of Eccentrically Loaded Columns: Axial Load and Bending

- Behavior of Eccentrically Loaded Non-Slender Columns (cont'd)

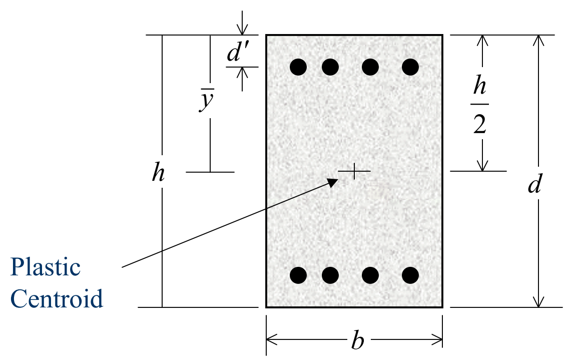


Figure 6a. Stresses and Forces in Columns

Strength of Eccentrically Loaded Columns: Axial Load and Bending

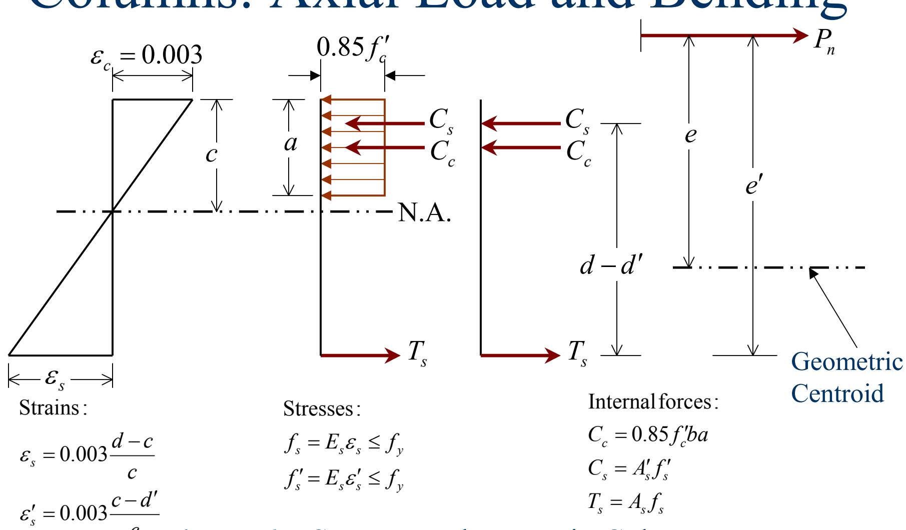


Figure 6b. Stresses and Forces in Columns

Strength of Eccentrically Loaded Columns: Axial Load and Bending

- Behavior of Eccentrically Loaded Non-Slender Columns (cont'd)
  - Definitions of Terms for Figure 6

$c =$ distance to neutral axis

$\bar{y} =$ distance of geometric centroid

$e =$ eccentricity of load to geometric centroid

$e' =$ eccentricity of load to tension steel

$d' =$ effective cover of compression steel

Strength of Eccentrically Loaded Columns: Axial Load and Bending

- Behavior of Eccentrically Loaded Non-Slender Columns (cont'd)
  - The depth of neutral primarily determines the strength of the column.
  - The equilibrium equations for forces and moments (see Figure 6) can be expressed as follows for non-slender (short) columns:

$$P_n(\text{at failure}) = C_c + C_s - T_s \qquad (19)$$

Strength of Eccentrically Loaded Columns: Axial Load and Bending

- Behavior of Eccentrically Loaded Non-Slender Columns (cont'd)
  - Nominal resisting moment M_n, which is equal to $P_n e$, can be obtained by writing the moment equilibrium equation about the plastic centroid.
  - For columns with symmetrical reinforcement, the plastic centroid is the same as the geometric centroid.

Strength of Eccentrically Loaded Columns: Axial Load and Bending

- Behavior of Eccentrically Loaded Non-Slender Columns (cont'd)

$$M_n = P_n e = C_c\left(\bar{y} - \frac{a}{2}\right) + C_s\left(\bar{y} - d'\right) + T_s\left(d - \bar{y}\right) \qquad (20)$$

  - since

$$C_c = 0.85 f'_c ba$$
$$C_s = A'_s f'_s \qquad (21)$$
$$T_s = A_s f_s$$

Strength of Eccentrically Loaded Columns: Axial Load and Bending

- Behavior of Eccentrically Loaded Non-Slender Columns (cont'd)
  - Eqs. 19 and 20 can be rewritten as

$$P_n = 0.85 f_c' ba + A_s' f_s' + A_s f_s \quad (22)$$

$$M_n = P_n e = 0.85 f_c' ba \left(\bar{y} - \frac{a}{2} \right) + A_s' f_s' (\bar{y} - d') + A_s f_s (d - \bar{y}) \quad (23)$$

  - where $\bar{y}$ for rectangular section = $h/2$

Strength of Eccentrically Loaded Columns: Axial Load and Bending

- Behavior of Eccentrically Loaded Non-Slender Columns (cont'd)
  - In Eqs. 22, the depth of the neutral axis c is assumed to be less than the effective depth d of the section, and the steel at the tension face is actual tension.
  - Such a condition changes if the eccentricity e of the axial force P_n is very small.

Strength of Eccentrically Loaded Columns: Axial Load and Bending

- Behavior of Eccentrically Loaded Non-Slender Columns (cont'd)
  - For such small eccentricities, where the total cross-section is in compression, contribution of the tension steel should be added to the contribution of concrete and compression steel.
  - The term $A_s f_s$ in Eqs. 22 and 23 in such a case would have a reverse sign since all the steel is in compression.

Strength of Eccentrically Loaded Columns: Axial Load and Bending

- Behavior of Eccentrically Loaded Non-Slender Columns (cont'd)
  - It is also assumed that $ba - A_s \approx ba$; that is, the volume of concrete displaced by compression steel is negligible.
  - Symmetrical reinforcement is usually used such that $A_s' = A_s$ in order to prevent the possible interchange of the compression reinforcement with the tension reinforcement during bar cage placement.

Strength of Eccentrically Loaded Columns: Axial Load and Bending

- Behavior of Eccentrically Loaded Non-Slender Columns (cont'd)
  - If the compression steel is assumed to have yielded and $A_s = A'_s$, Eqs 22 and 23 can be rewritten as.

$$P_n = 0.85 f'_c ba \tag{24}$$

$$M_n = P_n e = 0.85 f'_c ba\left(\bar{y} - \frac{a}{2}\right) + A'_s f_y\left(\bar{y} - d'\right) + A_s f_y\left(d - \bar{y}\right) \tag{25}$$

or

$$M_n = P_n e' = 0.85 f'_c ba\left(d - \frac{a}{2}\right) + A'_s f_y\left(d - d'\right) \tag{26}$$

Strength of Eccentrically Loaded Columns: Axial Load and Bending

- Behavior of Eccentrically Loaded Non-Slender Columns (cont'd)
  - Additionally, Eqs. 24 and 26 can be combined to obtain a single equation for P_n as

$$M_n = P_n e' = P_n\left(d - \frac{a}{2}\right) + A'_s f_y\left(d - d'\right) \tag{27}$$

  - Also, from Eq. 22

$$a = \beta_1 c = \frac{A_s f_y - A'_s f'_s + P_n}{0.85 f'_c b} \tag{28}$$

Strength of Eccentrically Loaded Columns: Axial Load and Bending

- Behavior of Eccentrically Loaded Non-Slender Columns (cont'd)
  – When the magnitude of f_s' or f_s is less than f_y, the actual stresses can be calculated using the following equations:

$$f_s' = E_s\varepsilon_s' = E_s\frac{0.003(c-d')}{c} \le f_y \qquad (29a)$$

or

$$f_s' = \varepsilon_s' E_s = 0.003E_s\left(1-\frac{d'}{c}\right) \le f_y \qquad (29b)$$

Strength of Eccentrically Loaded Columns: Axial Load and Bending

- Behavior of Eccentrically Loaded Non-Slender Columns (cont'd)
  – For f_s:

$$f_s = E_s\varepsilon_s = E_s\frac{0.003(d_t-c)}{c} \le f_y \qquad (30a)$$

or

$$f_s = \varepsilon_s E_s = 0.003E_s\left(\frac{d_t}{c}-1\right) \le f_y \qquad (30b)$$

Trial and Adjustment Procedure for Analysis (Design) of Columns

- Eqs. 22 and 23 determine the nominal axial load P_n that can be safely applied at an eccentricity e for any eccentrically loaded column. Te following unknowns can be identified:

1. Depth of the equivalent stress block, a.
2. Stress in compression steel, f_s'
3. Stress in tension steel, f_s
4. P_n for the given e, or vice versa

Trial and Adjustment Procedure for Analysis (Design) of Columns

- The stresses f_s' and f_s can be expressed in terms of the neutral axis c as in Eqs. 29 and 30 and thus in terms of a.
- The two remaining unknowns, a and P_n, can be solved using Eqs. 22 and 23.
- However, combining Eqs. 22 and 23 to 28 leads to a cubical equation in terms of the neutral axis depth c. Also, check must be done if f_s' and f_s are less than f_y.

Trial and Adjustment Procedure for Analysis (Design) of Columns

- The Suggested Procedure:
  1. For a given section geometry and eccentricity e, assume a value for the distance c down to the neutral axis. This value is a measure of the compression block depth a since $a = \beta_1 c$.
  2. Using the assumed value of c, calculate the axial load P_n using Eq. 22 and $a = \beta_1 c$.
  3. Calculate f_s' and f_s, respectively, using Eqs. 29 and 10.

Trial and Adjustment Procedure for Analysis (Design) of Columns

- The Suggested Procedure:
  4. Calculate the eccentricity corresponding to the calculated load P_n in Step 3 using Eq. 23. The calculated eccentricity should match the given eccentricity e. If not, repeat the steps until a convergence is accomplished.
  5. If the calculated eccentricity is larger than the given eccentricity, this indicates that the assumed c and corresponding a are less than the actual depth.

Trial and Adjustment Procedure for Analysis (Design) of Columns

- The Suggested Procedure:
  6. In such a case, try another cycle, assuming a larger value of c.

Strain Limits Methods

- Strain Limits Zones
  – The strain limits for compression-controlled sections can be represented by the following strain distributions across the depth of the cross sections with ε_t = 0.002 for Grade 60 Steel, or generally $\varepsilon_t = f_y/E_s$.
  – Figure 8 illustrates the behavior limits presented in Figure 7.

Strain Limits Methods

- Strain Limits Zones

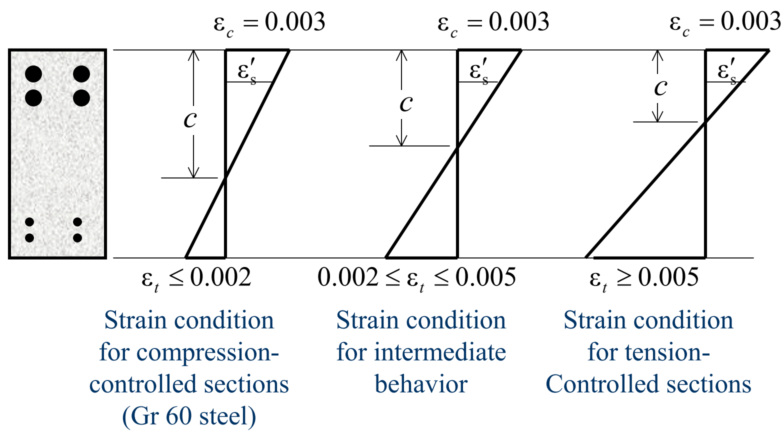


Figure 7. Stresses and Forces in Columns

Strain Limits Methods

- Strain Limits Zones

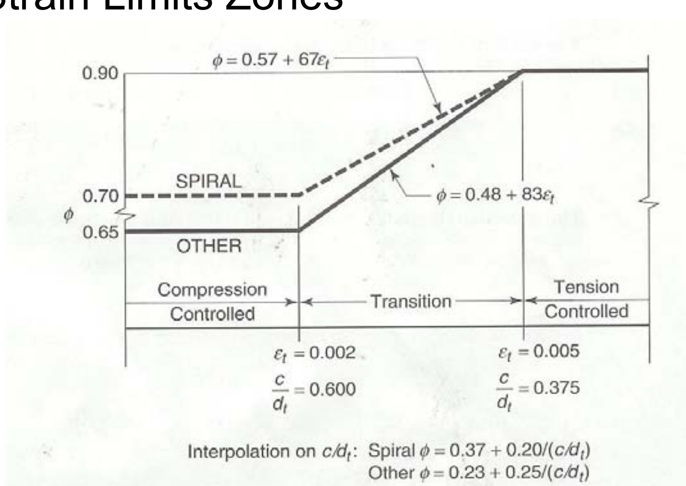


Figure 8. Strain Limit Zones and variation of Strength Reduction Factor ϕ

Strain Limits Methods

- Stress Limits

1) Tension-controlled limit case (ε_t > 0.005)

$$\frac{c}{d_t} = \frac{\varepsilon_c}{\varepsilon_c + \varepsilon_t} = \frac{0.003}{0.003 + 0.005} = 0.375 \quad (31)$$

$$a = c = 0.375\,\beta_1 d_t \quad (32)$$

$$\varepsilon_s' = 0.003\left(1 - \frac{d'}{c}\right) = 0.003\left(1 - 2.67\frac{d'}{d_t}\right) \quad (33)$$

$$f_s' = \varepsilon_s' E_s = 87{,}000\left(1 - 2.67\frac{d'}{d_t}\right) \le f_y \quad (34)$$

Strain Limits Methods

- Stress Limits

1) Compression-controlled limit case (ε_t = 0.002)

$$\frac{c}{d_t} = \frac{\varepsilon_c}{\varepsilon_c + \varepsilon_t} = \frac{0.003}{0.003 + 0.002} = 0.60 \quad (35)$$

$$a = c = 0.60\,\beta_1 d_t \quad (36)$$

$$\varepsilon_s' = 0.003\left(1 - \frac{d'}{0.60 d_t}\right) \quad (37)$$

$$f_s' = \varepsilon_s' E_s = 87{,}000\left(1 - 1.67\frac{d'}{d_t}\right) \le f_y \quad (38)$$

Strain Limits Methods

- Stress Limits
  - Transition zone for limit strain with intermediate behavior
    - This characterizes compression members in which the tensile reinforcement A_s has yielded but the compressive reinforcement A_s' has a stress level less than f_y depending on the geometry of the section.
    - Intermediate ϕ values change linearly with ε_t from ϕ = 0.90 when ε_t > 0.005 to ϕ = 0.65 for tied columns, or ϕ = 0.70 for spiral columns when $\varepsilon_t \leq$ 0.002.

Strain Limits Methods

- Stress Limits
  - Transition zone for limit strain with intermediate behavior (cont'd)
    - It should be noted that for nonprestressed flexural members and for nonprestressed members with axial load less than 0.10 $f_c' A_g$, the net tensile strain ε_t should not be less than 0.004. Hence, in the transition zone of Fig. 8, the minimum strain value in flexural members for determining the ϕ value is 0.004.
    - This limit is necessitated, as a ϕ value can otherwise become so low that additional reinforcement would be needed to give the required nominal moment strength.

Strain Limits Methods

- Summary: Modes of Failure in Columns
  - Based on the magnitude of strain in the tension face reinforcement (Figure 6), the section is subjected to one of the following:
    1. Tension-controlled state, by initial yielding of the reinforcement at the tension side, and strain ε_t greater than 0.005.
    2. Transition state, denoted by initial yielding of the reinforcement at the tension side, but strain with strain ε_t value smaller than 0.005 but greater than 0.002.
    3. Compression-controlled case by initial crushing of the concrete at the compression face.

Strain Limits Methods

- Summary: Modes of Failure in Columns (cont'd)
  - Accordingly, in analysis and design, the following eccentricity limits correspond to the strain limits presented:

$e_t \geq$ limit $e_{0.005}$ ($c = 0.375\ d_t$): tension-controlled

$e_{tt} \leq$ limit $e_{0.005\text{-}0.002}$ ($c = 0.375\ d_t - 0.60\ d_t$): intermediate transition

$e_c \leq$ limit $e_{0.002}$ ($c \geq 0.60\ d_t$): compression-controlled

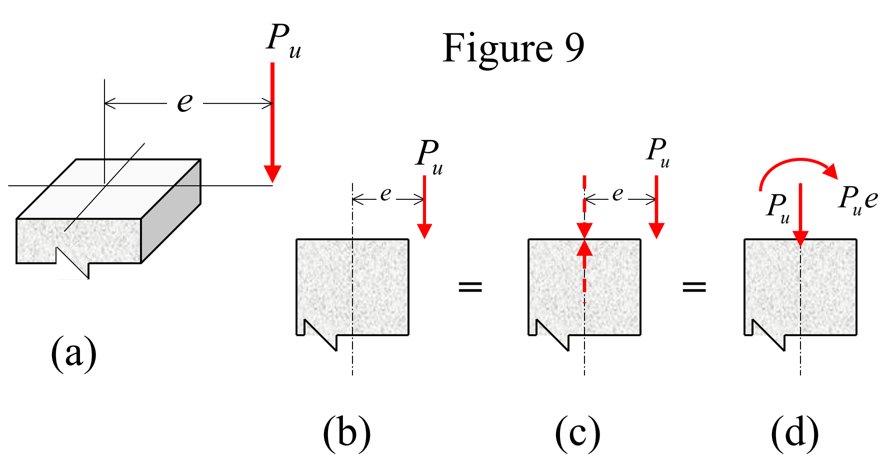
General Load-Moment Relationship

- Axial Load-Moment Combination
  - Assume that P_u is applied to a cross section at an eccentricity *e* from the centroid, as shown in Figs. 9a and 9b.
  - Add equal and opposite forces P_u at the centroid of the cross section, as shown in Fig. 9c.
  - The original eccentric force P_u may now be combined with the upward force P_u to form a couple $P_u e$, that is a pure moment.

General Load-Moment Relationship

- Axial Load-Moment Combination

Figure 9



General Load-Moment Relationship

- Axial Load-Moment Combination
  - This will leave remaining one force, P_u acting downward at the centroid of the cross section.
  - It can be therefore be seen that if a force P_u is applied with an eccentricity e, the situation that results is identical to the case where an axial load of P_u at the centroid and a moment of $P_u e$ are simultaneously applied as shown in Fig. 9d.

General Load-Moment Relationship

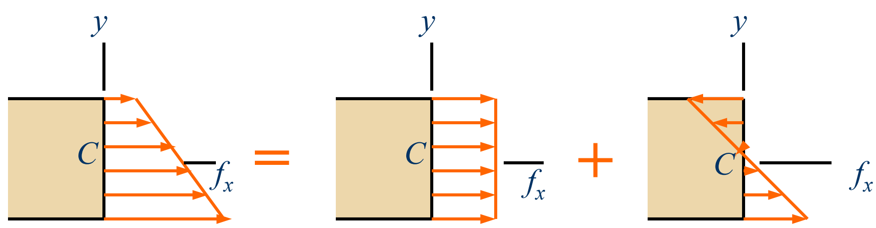
- Axial Load-Moment Combination
  - If *Mu* is defined as the factored moment to be applied on a compression member along with a factored axial load of *Pu* at the centroid, the relationship between the two can expressed as

$$e = \frac{M_u}{P_u} \qquad (39)$$

General Load-Moment Relationship

- Eccentric Axial Loading in A Plane of Symmetry

Figure 10



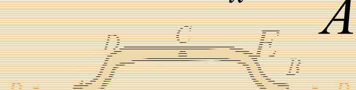
$$f_x = (f_x)_{\text{centric}} + (f_x)_{\text{bending}}$$

General Load-Moment Relationship

- Eccentric Axial Loading in A Plane of Symmetry

  The stress due to eccentric loading on a beam cross section is given by

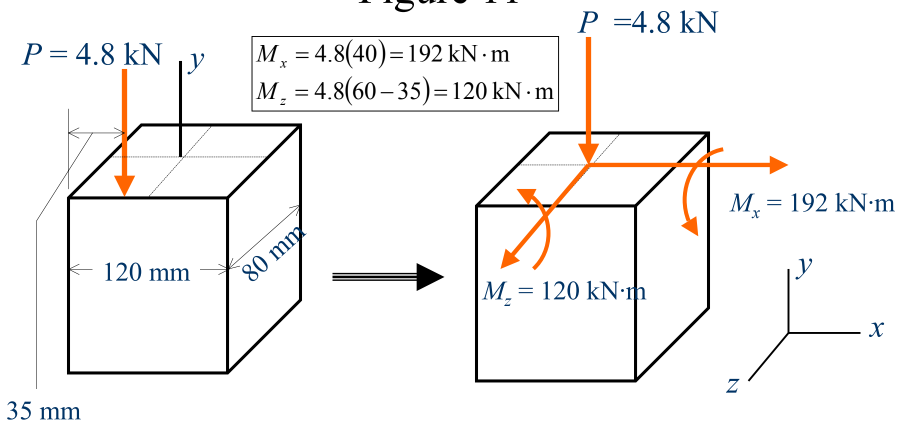
$$\sigma_x = \frac{P}{A} \pm \frac{My}{I} \qquad (40)$$



General Load-Moment Relationship

- Equivalent Force System for Eccentric Loading

Figure 11



General Load-Moment Relationship

- Analysis of Short Columns: Large Eccentricity
  - The first step in the investigation of short columns carrying loads at eccentricity is to determine the strength of given column cross section that carries load at various eccentricities.
  - For this, the design axial load strength ϕP_n is found, where P_n is defined as the nominal axial load strength at a given eccentricity.

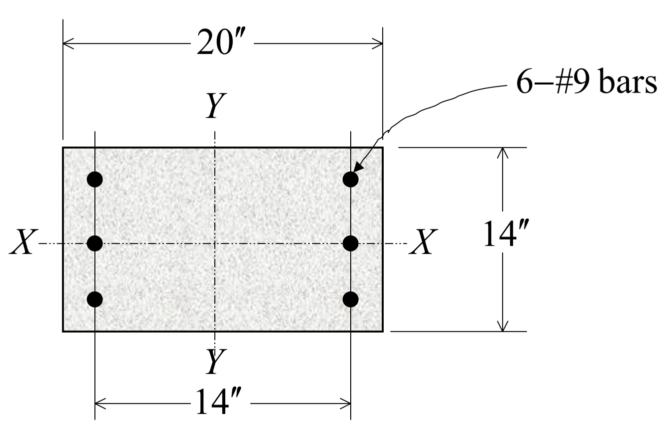
General Load-Moment Relationship

- Example 7

  Find the design axial load strength ϕP_n for the tied column for the following conditions: (a) small eccentricity, (b) pure moment, (c) e = 5 in., and (d) the balanced condition. The column cross section is shown. Assume a short column. Bending about the Y-Y axis. Use f_c' = 4000 psi and f_y = 60,000 psi.

General Load-Moment Relationship

- Example 7 (cont'd)



General Load-Moment Relationship

- Example 7 (cont'd)

(a) Small Eccentricity:

$$A_g = 14(20) = 280 \text{ in}^2$$

$$A_{st} = 6 \text{ in}^2 \text{(area of 6–\#9 bars)}$$

$$\begin{aligned}\phi P_n &= \phi P_{n(\max)} \\ &= 0.80\phi\left[0.85 f_c'\left(A_g - A_{st}\right) + f_y A_{st}\right] \\ &= 0.80(0.70)[0.85(4)(280-6) + (60)(6)] \\ &= 723 \text{ kips}\end{aligned}$$

General Load-Moment Relationship

- Example 7 (cont'd)

(b) Pure Moment:

The analysis of the pure moment condition is similar to the analysis of the case where the eccentricity e is infinite as shown in Figure 12.

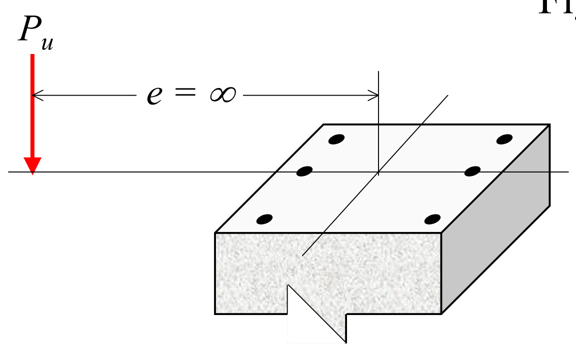
The design moment ϕM_n will be found since P_u and ϕP_n will both be zero.

Assume that A_s is at yield, and then with reference to Figure 13, then

General Load-Moment Relationship

- Example 7 (cont'd)

Figure 12



General Load-Moment Relationship

- Example 7 (cont'd)

  C_1 = concrete compressive force

  C_2 = steel compressive force

  T = steel tensile force

$$\frac{\varepsilon_s'}{c-3} = \frac{0.003}{c} \Rightarrow \varepsilon_s' = 0.003\frac{c-3}{c} \qquad (41)$$

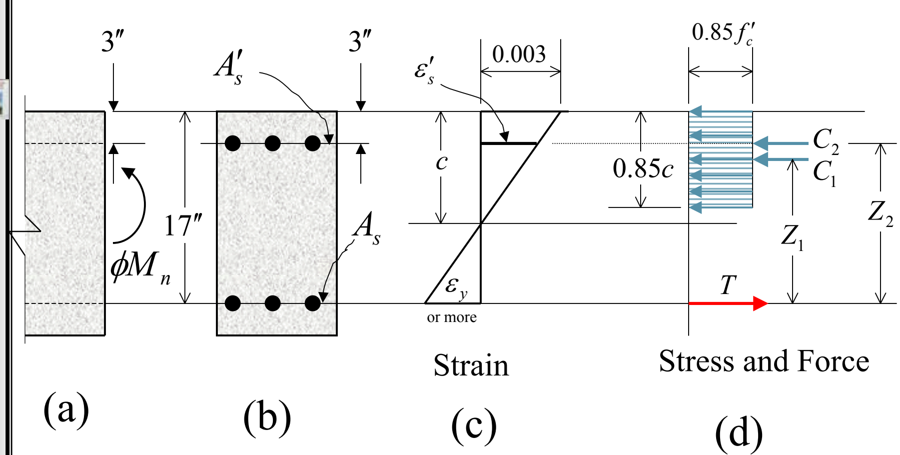
Since

$$f_s' = E_s \varepsilon_s' \qquad (42)$$

General Load-Moment Relationship

- Example 7 (cont'd)

Figure 13



General Load-Moment Relationship

- Example 7 (cont'd)

Substituting $E_s = 29 \times 10^6$ psi and ε_s' given by Eq. 41 into Eq. 42, gives

$$f_s' = 29\times10^6(0.003)\frac{c-3}{c} = 87\frac{c-3}{c} \qquad (43)$$

For equilibrium in Figure 13d,

$$C_1 + C_2 = T \qquad (44)$$

Substituting into above equation, yields

General Load-Moment Relationship

- Example 7 (cont'd)

$$(0.85 f_c)(0.85c)b + f_s' A_s' - 0.85 f_c' A_s' = f_y A_s \qquad (45a)$$

$$(0.85)(4)(0.85c) + 87\frac{c-3}{c}(3) - 0.85(4)(3) = 3(60) \qquad (45b)$$

– The above equation can be solved for c to give

$$c = 3.62 \text{ in.}$$

and thus,

$$f_s' = 87\frac{3.62-3}{3.62} = 14.90 \text{ ksi (compression)} \qquad (46)$$

General Load-Moment Relationship

Table 5. Areas of Multiple of Reinforcing Bars (in^2)

Number	Bar number								
of bars	#3	#4	$5	#6	#7	#8	#9	#10	#11
1	0.11	0.20	0.31	0.44	0.60	0.79	1.00	1.27	1.56
2	0.22	0.40	0.62	0.88	1.20	1.58	2.00	2.54	3.12
3	0.33	0.60	0.93	1.32	1.80	2.37	3.00	3.81	4.68
4	0.44	0.80	1.24	1.76	2.40	3.16	4.00	5.08	6.24
5	0.55	1.00	1.55	2.20	3.00	3.95	5.00	6.35	7.80
6	0.66	1.20	1.86	2.64	3.60	4.74	6.00	7.62	9.36
7	0.77	1.40	2.17	3.08	4.20	5.53	7.00	8.89	10.92
8	0.88	1.60	2.48	3.52	4.80	6.32	8.00	10.16	12.48
9	0.99	1.80	2.79	3.96	5.40	7.11	9.00	11.43	14.04
10	1.10	2.00	3.10	4.40	6.00	7.90	10.00	12.70	15.60

General Load-Moment Relationship

- Example 7 (cont'd)

  Therefore, the forces will be

$$C_1 = 0.85 f_c'(0.85c)b = 0.85(4)(0.85)(3.62)(14) = 146.5 \text{ kips}$$

$$C_2 = f_s' A_s' - 0.85 f_c' A_s' = 14.9(3) - 0.85(4)(3) = 34.5 \text{ kips}$$

  – The internal Moments are

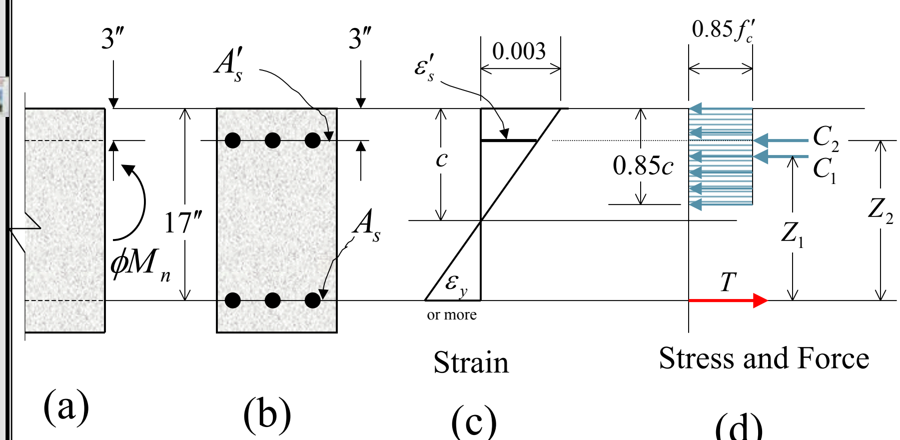
$$M_{n1} = C_1 Z_1 = \frac{146.5}{12}\left[17 - \frac{0.85(3.62)}{2}\right] = 188.8 \text{ ft-kips}$$

$$M_{n2} = C_2 Z_2 = \frac{34.5(14)}{12} = 40.3 \text{ ft-kips}$$

General Load-Moment Relationship

- Example 7 (cont'd)

Figure 13



General Load-Moment Relationship

- Example 7 (cont'd)

Therefore,

$$M_n = M_{n1} + M_{n2} = 188.8 + 40.3 = 229 \text{ ft - kips}$$

and

$$\phi M_n = 0.65(229) = 149 \text{ ft - kips}$$

General Load-Moment Relationship

- Example 7 (cont'd)

(c) The eccentricity e = 5 in:

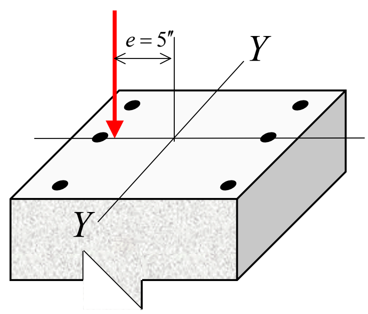
The situation of e = 5 in. is shown in Figure 14

Note that in Part (a), all steel was in *compression* and in Part (b), the steel on the side of the column away from the load was in *tension*. Therefore, there is some value of the eccentricity at which steel will change from tension to compression. Since this is not known, the strain in Figure 15 is assumed.

General Load-Moment Relationship

- Example 7 (cont'd)

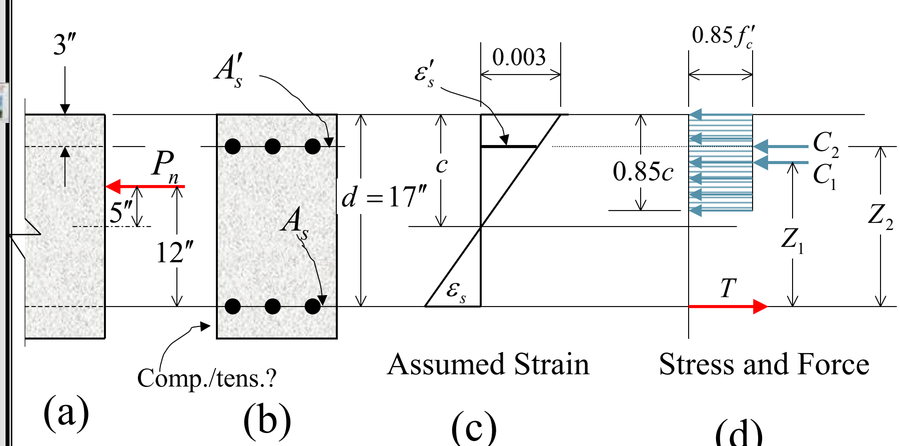
Figure 14



General Load-Moment Relationship

- Example 7 (cont'd)

Figure 15



General Load-Moment Relationship

- Example 7 (cont'd)

The assumptions at ultimate load are

1. Maximum concrete strain = 0.003
2. $\varepsilon_s' > \varepsilon_y$, therefore, $f_s' = f_y$
3. ε_s is tensile
4. $\varepsilon_s < \varepsilon_y$ and thus $f_s < f_y$

These assumptions will be verified later.

The unknown quantities are P_u and c.

The forces will be evaluated as follows:

General Load-Moment Relationship

- Example 7 (cont'd)

$$C_1 = 0.85 f_c' ab = 0.85(4)(0.85c)(14) - 40.46c$$

$$C_2 = f_y A_s' - 0.85 f_c' A_s'$$

$$= 60(3) - 0.85(4)(3) = 169.8 \text{ kips}$$

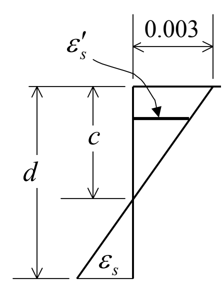
$$T = f_s A_s = \varepsilon_s E_s A_s = 87\left(\frac{d-c}{c}\right) A_s$$

$$= 87\left(\frac{17-c}{c}\right)3 = 261\frac{17-c}{c}$$

From $\sum$ moments = 0 in Fig. 7c:

$$P_n = C_1 + C_2 - T$$

$$= 40.46c + 169.8 - 261\frac{17-c}{c} \qquad (47)$$



$$\frac{\varepsilon_s}{0.003} = \frac{d-c}{c}$$

$$\varepsilon_s = 0.003\frac{d-c}{c}, \text{ and}$$

$$f_s = \varepsilon_s E_s = \left(0.003\frac{d-c}{c}\right) 29\times 10^3 = 87\frac{d-c}{c}$$

General Load-Moment Relationship

- Example 7 (cont'd)

From $\sum$ moments = 0, taking moments about T in Figure 15d:

$$P_n(12) = C_1\left(d - \frac{a}{2}\right) + C_2(14) \qquad (48)$$

$$= \frac{1}{12}\left[40.46c\left(17 - \frac{0.85c}{2}\right) + 169.8(`14)\right]$$

Eqs. 47 and 48 can be solved simultaneously for c to give

$$c = 14.86 \text{ in.}$$

$$P_n = 733 \text{ kips}$$

General Load-Moment Relationship

- Example 7 (cont'd)

Now, the assumptions can be checked:

$$\varepsilon_s' = \left(\frac{14.86 - 3}{14.86}\right)(0.003) = 0.0024$$

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60{,}000}{29\times10^6} = 0.00207 < (\varepsilon_s' = 0.0024) \quad \text{OK}$$

Therefore, $f_s' = f_y$, and based on the location of the neutral axis:

$$f_s = 87\left(\frac{17 - 14.86}{14.86}\right) = 12.53 \text{ ksi} < 60 \text{ ksi} \quad \text{OK}$$

General Load-Moment Relationship

- Example 7 (cont'd)
  - The design moment for an eccentricity of 5 in. can be computed as follows:

$$P_u = \phi P_n = 0.65(733) = 477 \text{ kips}$$

$$\phi M_n = \phi P_n e = \frac{477(5)}{12} = 199 \text{ ft-kips}$$

  - Therefore, the given column has a design load-moment combination strength of 477 kips axial load and 199 ft-kips moment.

General Load-Moment Relationship

- Example 7 (cont'd)

(d) The Balanced Condition Case:

The balanced condition is defined when the concrete reaches a strain of 0.003 at the same time that the tension steel reaches its yield strain, as shown in Fig. 16c.

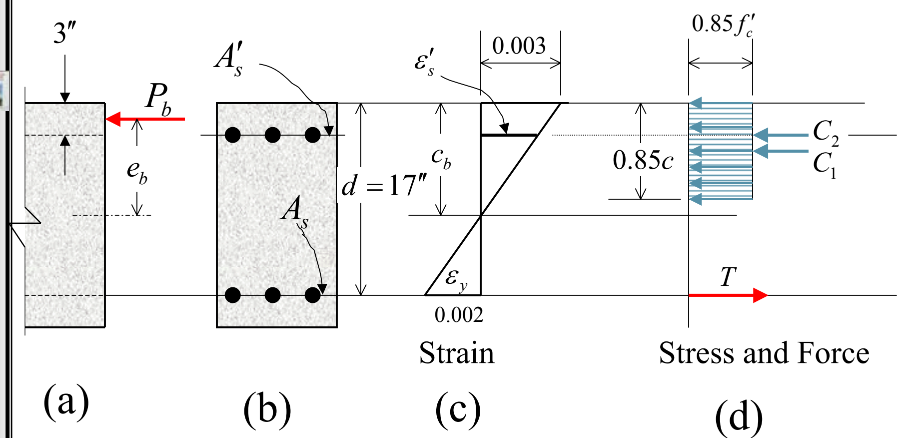
The value of c_b can be calculated from

$$c_b = \frac{87}{87 + f_y} d = \frac{87}{87 + 60}(17) = 10.06 \text{ in.}$$

General Load-Moment Relationship

- Example 7 (cont'd)

Figure 16



General Load-Moment Relationship

- Example 7 (cont'd)

$$\varepsilon_s' = \frac{10.06-3}{10.06}(0.003) = 0.0021 > \varepsilon_y = 0.002$$

Therefore, $f_s' = f_y$ = 60 ksi

The forces can computed as follows:

$$C_1 = 0.85(4)(0.85)(10.06)(14) = 407 \text{ kips}$$

$$C_2 = 60(3) - 0.85(4)(3) = 170 \text{ kips}$$

$$T = 60(3) = 180 \text{ kips}$$

$$P_b = C_1 + C_2 - T = 407 + 170 - 180 = 397 \text{ kips}$$

General Load-Moment Relationship

- Example 7 (cont'd)
  - The value of e_b may be calculated by summing moments about T as follows:

$$P_e(e_b+7)=C_1\left(d-\frac{0.85c_b}{2}\right)+C_2(14)$$

$$397(e_b+7)=407\left[17-\frac{0.85(10.06)2}{2}\right]+170(14)$$

  - From which, e_b = 12.0 in. Therefore, at the balanced condition:

$$\phi P_b=0.65(397)=258\text{ kips}$$

$$\phi M_n=\phi P_b e_b=\frac{258(12)}{12}=258\text{ ft-kips}$$

General Load-Moment Relationship

- Example 7 (cont'd)
  - The results of the four parts can be tabulated (see Table 6) and plotted as shown in Fig. 17.
  - This plot is called an "***interaction diagram***".
  - In the plot, any point on the solid line represents an allowable combination of load and moment.
  - Any point within the solid line represents a load-moment combination that is also allowable, but for which this column is *overdesigned*.

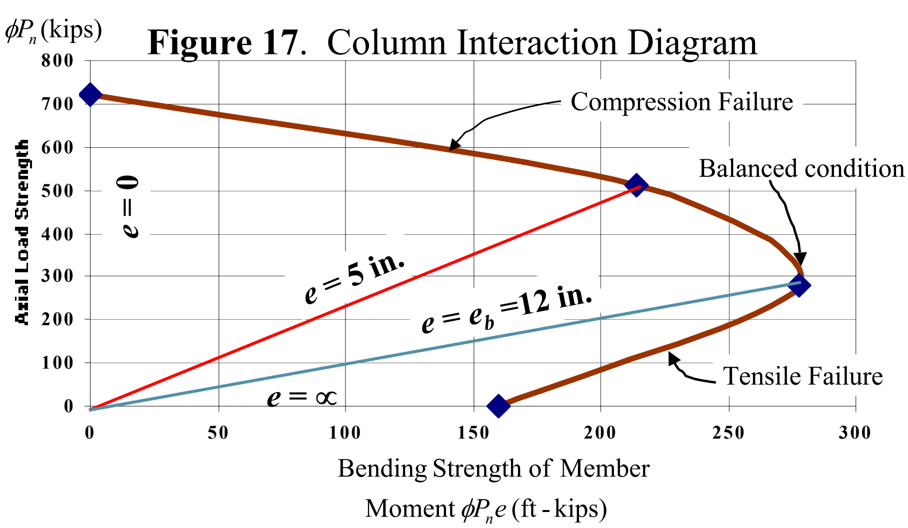
General Load-Moment Relationship

- Example 2 (cont'd)

Table 6

e	Axial load strength (ϕP_n,kips)	Moment strength ($\phi P_n e$, ft- kips)
Small	723	0 (small)
Infinite	0	160
5 in.	513	214
12 in	278	278

General Load-Moment Relationship



General Load-Moment Relationship

- Example 7 (cont'd)
  - Any point outside the solid line represents an unaccepted load-moment combination or a load-moment combination for which this column is *underdesigned*.
  - Radial lines from the origin represent various eccentricities (slope = $\phi P_n / \phi P_n$ or $1/e$).
  - Any eccentricity less than e_b will result in compression controlling the column, and any eccentricity greater than e_b will result in tension controlling the column.

Design of Short Columns: Large Eccentricity

- The design of a column cross section using the previous calculation approach would be a trial-and-error method and would become exceedingly tedious.
- Therefore, design and analysis aids have been developed that shorten the process to a great extent.
- A chart approach has been developed in ACI Publication SP-17 (97), *ACI Design Handbook.*

Design of Short Columns: Large Eccentricity

- The charts take on the general form of Figure 17 but are set up to be more general so that they will remain applicable if various code criteria undergo changes.
- These charts can be used for both analysis and design of columns.
- There are also computer programs available to aid in the design process.

General Case of Columns Reinforced on All Faces: Exact Solution

- In cases where columns are reinforced with bars on all faces and those where the reinforcement in the parallel faces is nosymmetrical, solutions have to be based on using first principles of Eqs. 22 and 23.
- These equations have to be adjusted for this purpose and strain compatibility checks for strain in each reinforcing bar layer have to be performed at all load levels.

General Case of Columns Reinforced on All Faces: Exact Solution

- Figure 18 shows the case of a column reinforced on all four faces.
- Assume that

G_{sc} = center of gravity of steel compressing force.

G_{st} = center of gravity of steel tensile force.

F_{sc} = resultant steel compressive force = $\sum A'_s f_{sc}$

– F_{st} = resultant steel tensile force = $\sum A_s f_{st}$

General Case of Columns Reinforced on All Faces: Exact Solution

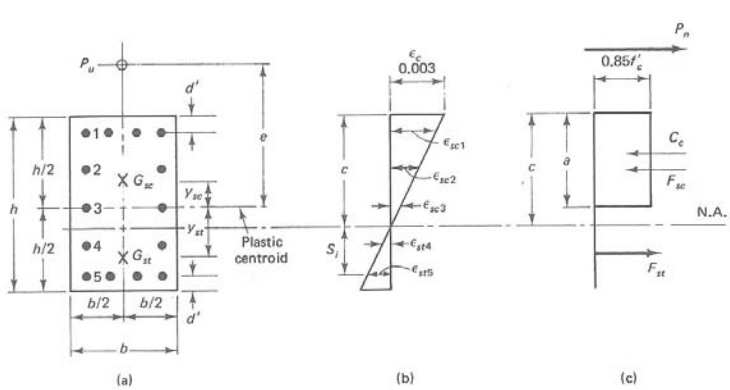


Figure 18. Column Reinforced with Steel on all Faces: (a) Cross-Section; (b) Strain; (c) Forces

General Case of Columns Reinforced on All Faces: Exact Solution

- Equilibrium of the internal and external forces and moments requires that

$$P_n = 0.85 f_c' b \beta_1 c + F_{sc} - F_{sr} \quad (49)$$

$$P_n e = 0.85 f_c' b \beta_1 c \left(\frac{h}{2} - \frac{1}{2}\beta_1 c \right) + F_{sc} y_{sc} + F_{st} y_{st} \quad (50)$$

General Case of Columns Reinforced on All Faces: Exact Solution

- The strain values in each bar layer are determined by the linear strain distribution in Figure 18 to ensure strain compatibility.
- The stress in each reinforcing bar is obtained using the expression

$$f_{si} = E_s \varepsilon_{si} = E_s \varepsilon_c \frac{S_i}{c} = 87{,}000 \frac{S_i}{c} \quad (51)$$

$$f_{si} \le f_y$$

Circular Columns

- The angle θ subtended by the compressive block chord shown in Figure 19b is

Case 1:

$$a \le \frac{h}{2}, \theta < 90^{\circ}$$

$$\theta = \cos^{-1}\left(\frac{h/2 - a}{h/2}\right) \qquad (52a)$$

Case 2:

$$a > \frac{h}{2}, \theta > 90^{\circ}$$

$$\theta = \cos^{-1}\left(\frac{h/2 - a}{h/2}\right) \quad \text{and} \quad \varphi = \cos^{-1}\left(\frac{a - h/2}{h/2}\right) \qquad (52b)$$

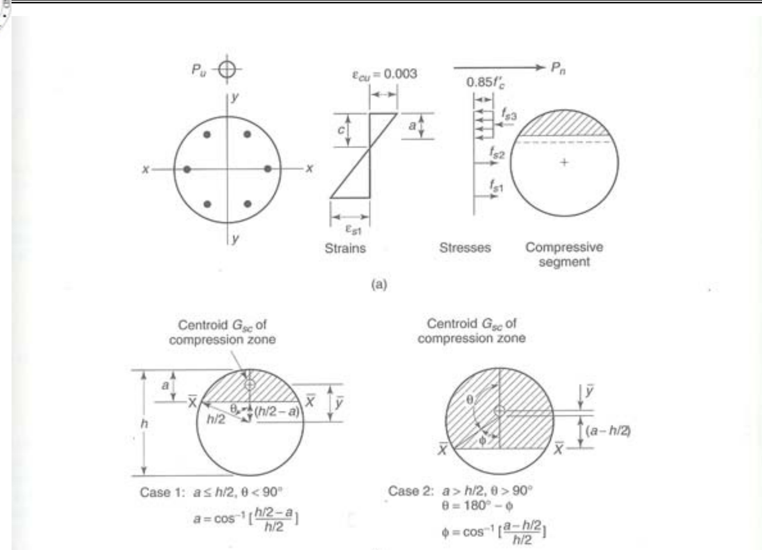


Figure 19. Circular Columns: (a) Strain, Stress and Compression Block Segment; (b) Compression Segment Chord *x*-*x* Geometry

Circular Columns

- The area of the compressive segment of the circular column in Figure 19b is

$$\overline{A}_c = h^2\left(\frac{\theta_{rad} - \sin\theta\cos\theta}{4}\right) \qquad (53a)$$

- The moment of area of the compressive segment about the center of the column is

$$\overline{A}_c\overline{y} = h^3\left(\frac{\sin^3\theta}{12}\right) \qquad (53b)$$

$\overline{y}$ = distance of the centroid of compressive block to section centroid

Circular Columns

$$d_i = \frac{h}{2} - \frac{\gamma h_i}{2}\left(\cos\theta_{\text{bar}}\right) \qquad (54a)$$

where $\gamma = \left(h - 2d'\right)/h$

$$f'_{si} = 87{,}000\left(1 - \frac{d_i}{c}\right) \leq f_y \qquad (54b)$$

where f'_{si} = stress in bars within compressive zone

$$f_{si} = 87{,}000\left(\frac{d_i}{c} - 1\right) \leq f_y \qquad (54c)$$

where f_{si} = stress in bars within tension zone below N.A.

Circular Columns

- Expressions for nominal axial force P_n and nominal bending moment M_n for circular columns

$$P_n = 0.85 f_c' \overline{A}_c + \sum f_{si} A_{si} \qquad (55a)$$

$$M_n = 0.85 f_c' \overline{A}_c \overline{y} + \sum f_{si} A_{si} \left(\frac{h}{2} - d_i' \right) \qquad (55c)$$

Note: moment is taken about the circular column center.

Whitney's Approximate Solution

- Rectangular Concrete Columns
  - Assumptions:
    1. Reinforcement is symmetrically placed in single layers parallel to axis of bending in rectangular sections.
    2. Compression steel has yielded.
    3. Concrete displaced by the compression steel is negligible compared to the total concrete area in compression.
    4. The depth of the stress block is assumed to be 0.54d.
    5. The interaction curve in the compression zone is a straight line.

Whitney's Approximate Solution

- Rectangular Concrete Columns (cont'd)
  - If compression controls, the equation for rectangular sections can be written as

$$P_n = \frac{A_s' f_y}{\dfrac{e}{d - d'} + 0.5} + \frac{bhf_c'}{\dfrac{3he}{d^2} + 1.18} \qquad (56)$$

Whitney's Approximate Solution

- Circular Concrete Columns
  - Transform the circular column to an idealized equivalent rectangular column as shown in Figure 20.
  - For compression failure, the equivalent rectangular column would have
    1. the thickness in the direction of bending equal to $0.8h$, where h is the outside diameter of the circular column (Figure 20b).
    2. the width of the idealized rectangular column to be obtained from the same gross area A_g of the circular column such that $b = A_g/0.8h$.

- Circular Concrete Columns (cont'd)

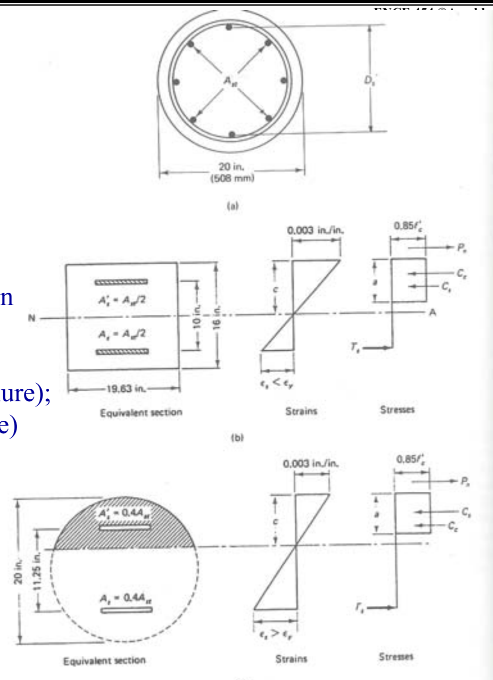


Figure 20. Equivalent column section (a) given circular section (A_{st}, total reinforcement area; (b) equivalent rectangular section (compression failure); (c) equivalent column (tension failure)

Whitney's Approximate Solution

- Circular Concrete Columns (cont'd)
  3. the total area of reinforcement A_{st} to be equally divided in two parallel layers and placed at a distance of $2D_s/3$ in the direction of bending, where D_s is the diameter of the cage measured center to center of the outer vertical bars.
  - For tension failure, use the actual column for evaluating C_c, but place 40% of the total steel A_{st} in parallel at a distance $0.75D_s$ as shown In Figure 20.
  - Once the dimensions are established, the analysis can be similar to rectangular column.

Whitney's Approximate Solution

- Circular Concrete Columns (cont'd)
  - For Tension Failure:

$$P_n = 0.85 f_c' h^2 \left[\sqrt{\left(\frac{0.85e}{h} - 0.38 \right)^2 + \frac{\rho_g m D_s}{2.5h}} - \left(\frac{0.85e}{h} - 0.38 \right) \right] \qquad (57)$$

  - For Compression Failure:

$$P_n = \frac{A_{st} f_y}{\frac{3e}{D_s} + 1.0} + \frac{A_g f_c'}{\frac{9.6he}{(0.8h + 0.67D_s)^2} + 1.18} \qquad (58)$$

Whitney's Approximate Solution

- Circular Concrete Columns (cont'd)
  - Definitions of Variables for Eqs. 57 and 58

h = diameter of section

D_s = diameter of the reinforcement cage center to center of the outer vertical bars

e = eccentricity to plastic centroid of section

$\rho_g = \frac{A_{st}}{A_g} = \frac{\text{gross steel area}}{\text{gross concrete area}}$

$m = \frac{f_y}{0.85 f_c'}$

Whitney's Approximate Solution

- Example 8

Obtain an equivalent rectangular cross section for the circular column shown in figure 20a. Assume that D_s = 15 in.

$$\text{thickness of rect. section} = 0.8 \times 20 = 16 \text{ in.}$$

$$\text{width of rect. section} = \frac{A_g}{0.8h} = \frac{\pi(20)^2}{4}\frac{1}{0.8(20)} = 19.63 \text{ in.}$$

$$d - d' = \frac{2D_s}{3} = \frac{2(15)}{3} = 10 \text{ in.}$$

$$A_s = A_s' = \frac{A_{st}}{2}$$

Whitney's Approximate Solution

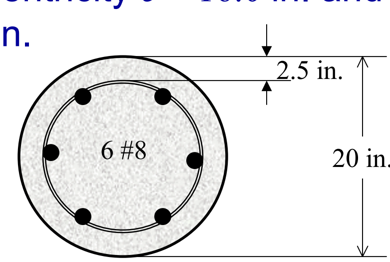
- Example 9

A concrete circular column 20 in. in diameter is reinforced with six No. 8 equally spaced bars as show in the figure. Using Whitney's approximate approach, compute the nominal axial load P_n for (a) eccentricity $e = 16.0$ in. and (b) eccentricity e = 5.0 in.

Given:

$$f_c' = 4000 \text{ psi}$$

$$f_y = 60{,}000 \text{ psi}$$



Whitney's Approximate Solution

- Example 9 (cont'd)

a) e = 16.0 in. with axial load 6 in. outside the circular section. It can be assumed that the section is tension-controlled.

$$D_s = 20 - 2(2.5) = 15 \text{ in.}$$

$$\rho_g = \frac{A_{st}}{A_g} = \frac{\text{area of 6 No.8}}{A_g} = \frac{4.74}{\frac{\pi h^2}{4}} = \frac{4.74}{\frac{\pi (20)^2}{4}} = 0.015$$

Whitney's Approximate Solution

Table 5. Areas of Multiple of Reinforcing Bars (in^2)

Number of bars	Bar number								
	#3	#4	$5	#6	#7	#8	#9	#10	#11
1	0.11	0.20	0.31	0.44	0.60	0.79	1.00	1.27	1.56
2	0.22	0.40	0.62	0.88	1.20	1.58	2.00	2.54	3.12
3	0.33	0.60	0.93	1.32	1.80	2.37	3.00	3.81	4.68
4	0.44	0.80	1.24	1.76	2.40	3.16	4.00	5.08	6.24
5	0.55	1.00	1.55	2.20	3.00	3.95	5.00	6.35	7.80
6	0.66	1.20	1.86	2.64	3.60	4.74	6.00	7.62	9.36
7	0.77	1.40	2.17	3.08	4.20	5.53	7.00	8.89	10.92
8	0.88	1.60	2.48	3.52	4.80	6.32	8.00	10.16	12.48
9	0.99	1.80	2.79	3.96	5.40	7.11	9.00	11.43	14.04
10	1.10	2.00	3.10	4.40	6.00	7.90	10.00	12.70	15.60

Whitney's Approximate Solution

- Example 9 (cont'd)

Using Eq. 57, yields

$$m = \frac{f_y}{0.85 f_c'} = \frac{60{,}000}{0.85(4000)} = 17.65$$

$$P_n = 0.85 f_c' h^2 \left[\sqrt{\left(\frac{0.85e}{h} - 0.38 \right)^2 + \frac{\rho_g m D_s}{2.5h}} - \left(\frac{0.85e}{h} - 0.38 \right) \right]$$

$$P_n = 0.85(4)(20)^2 \left[\sqrt{\left(\frac{0.85(16)}{20} - 0.38 \right)^2 + \frac{0.015(17.65)(15)}{2.5(20)}} - \left(\frac{0.85(16)}{20} - 0.38 \right) \right]$$

$$= 151.79 \text{ kips}$$

Whitney's Approximate Solution

- Example 9 (cont'd)

b) e = 5.0 in. with axial load inside the circular section. It can be assumed that the section is compression-controlled.

total steel area $A_{st} = 4.74 \text{ in}^2$, $A_g = \frac{\pi(20)^2}{4} = 314.2 \text{ in}^2$

Using Eq. 58, gives

$$P_n = \frac{A_{st} f_y}{\frac{3e}{D_s} + 1.0} + \frac{A_g f_c'}{\frac{9.6he}{(0.8h + 0.67D_s)^2} + 1.18}$$

$$P_n = \frac{4.74(60)}{\frac{3(5)}{15} + 1} + \frac{314.2(4)}{\frac{9.6(20)(5)}{(0.8 \times 20 + 0.67 \times 15)^2} + 1.18} = 626.58 \text{ kips}$$