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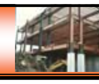
A Fundamental Approach - Fifth Edition



COMBINED COMPRESSION AND BENDING: COLUMNS

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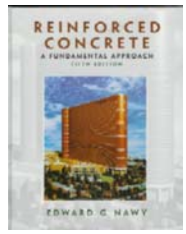

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ENCE 454 – Design of Concrete Structures
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Introduction

- Axial Compression
  - Columns are defined as members that carry loads in compression.
  - Usually they carry bending moments as well, about one or both axes of the cross section.
  - The bending action may produce tensile forces over a part of the cross section.
  - Despite of the tensile forces or stresses that may be produced, columns are Generally referred to as "*compression members*" because the compression forces or stresses dominate their behavior.

Introduction

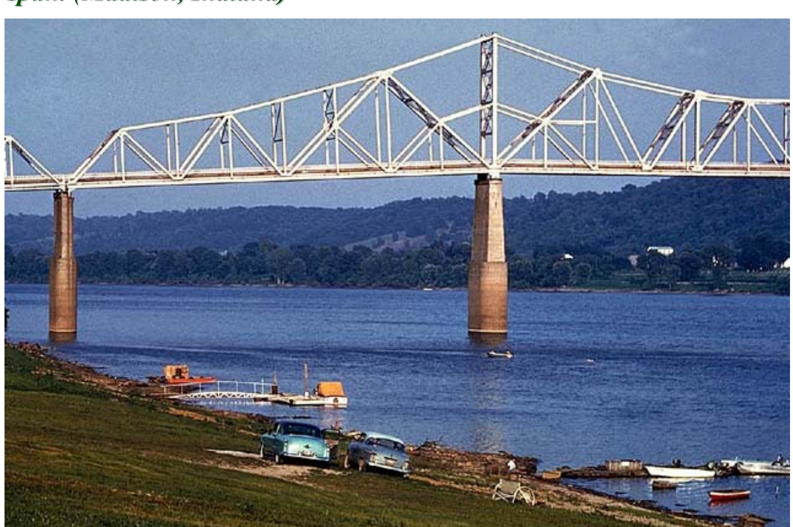
- Axial Compression (cont'd)
  - In addition to the most common type of compression members (vertical elements in structures), compression members include:
    - Arch ribs
    - Rigid frame members inclined or otherwise
    - Compression elements in trusses
    - shells

Introduction

Introduction

Reinforced Concrete Columns

Ohio River Bridge. Typical cantilever and suspended span bridge, showing the truss geometry in the end span and cantilevered portion of the main span. (Madison, Indiana)



Introduction

Introduction

Introduction

- Failure of Columns
  - Failure of columns could occur as a result of material failure by
    - Initial yielding of the steel at the tension face;
    - Initial crushing of the concrete at the compression face; or
    - Loss of lateral stability (buckling)
  - If a column fails due to initial material failure, it is then considered short or non-slender column.

Introduction

- Failure of Columns (cont'd)
  - As the length of the column increases, the probability that failure will occur by buckling also increases.
  - The slenderness ratio kl_u/r is a measure of the type of column. According to ACI, if

$$\frac{kl_u}{r} \leq 22$$

then the column is considered a short column

kl_u = effective length
k =factor that depends on end condition of column and condition of bracing
l_u = unsupported length of column
r = radius of gyration = $\sqrt{I/A}$

Introduction

- Buckling and Axial Compression
  - Buckling is a mode of failure generally resulting from structural instability due to compressive action on the structural member or element involved.
  - Examples
    - Overloaded metal building columns.
    - Compressive members in bridges.
    - Roof trusses.
    - Hull of submarine

Introduction

- Buckling and Axial Compression

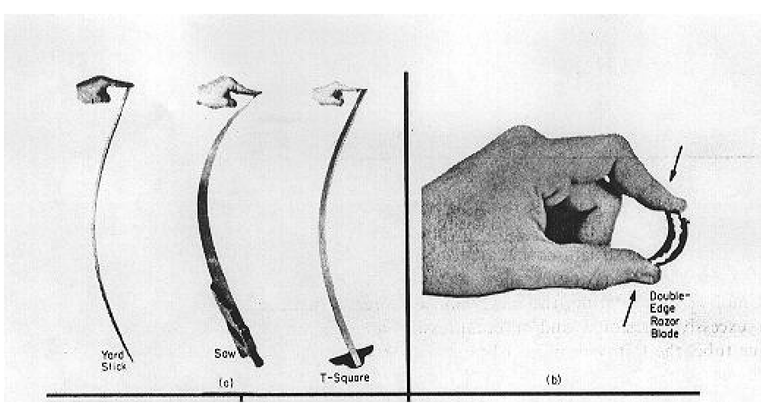


Figure 1a

Introduction

- Buckling and Axial Compression

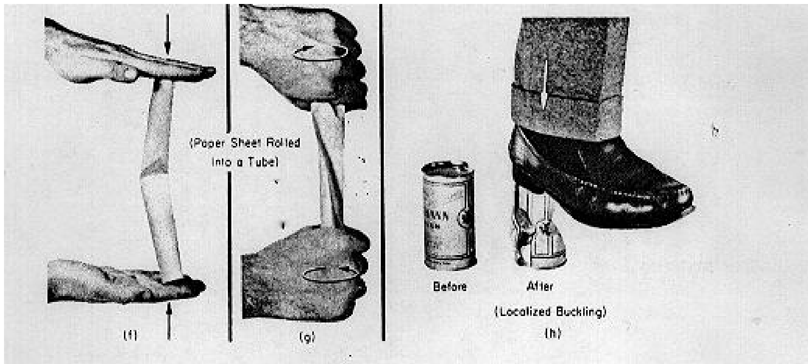


Figure 1b

Introduction

- Buckling and Axial Compression
  - Definition

    "Buckling can be defined as the sudden large deformation of structure due to a slight increase of an existing load under which the structure had exhibited little, if any, deformation before the load was increased."

Introduction

- Buckling and Axial Compression

Figure 2. Buckling Failure of Reinforced Concrete Columns

Introduction

- Critical Buckling Load, P_{cr}

  The critical buckling load (Euler Buckling) for a long column is given by

$$P_{cr} = \frac{\pi^2 EI}{L^2} \qquad (1)$$

where

E = modulus of elasticity of the material

I = moment of inertia of the cross section

L = length of column

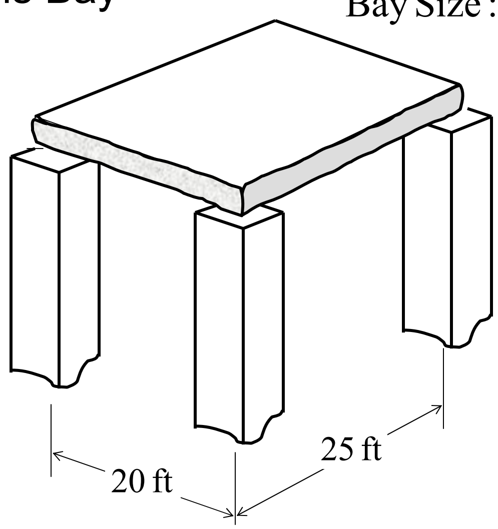
Introduction

- Columns Bay
  - The spacing of columns in plan establishes what is called a ***Bay***.
  - For example, if the columns are 20 ft on center in one direction and 25 ft in the other direction, the bay size is 20 ft × 25 ft.
  - Larger bay sizes increase the user's flexibility in space planning.

Introduction

- Columns Bay

$$\text{Bay Size}: 20 \text{ ft} \times 25 \text{ ft}$$



Introduction

- Column load transfer from beams and slabs

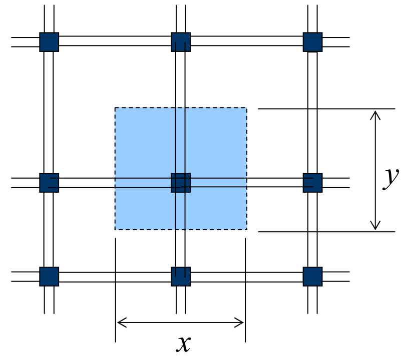
1) Tributary area method:

Half distance to adjacent columns

Load on column = area ฅ floor load

Floor load = DL + LL

DL = slab thickness ฅ conc. unit wt.



Example: x = 16.0 ft, y = 13.0 ft, LL = 62.4 lb/ft^2, slab thickness = 4.0 in.

Floor load = 4.0 (150)/12 + 62.4 = 112.4 lb/ft^2

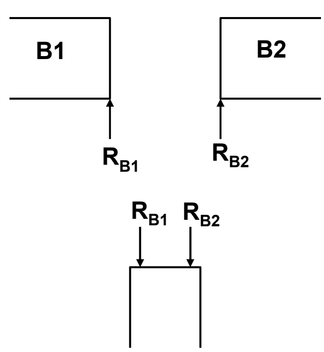
Load on column = (16.0)(13.0)(112.4) = 23.4 kips =10,800 kg

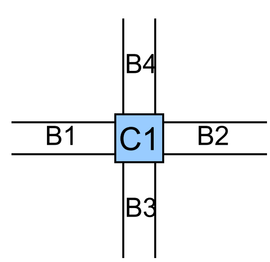
Introduction

- Column load transfer from beams and slabs

2) Beams reaction method:

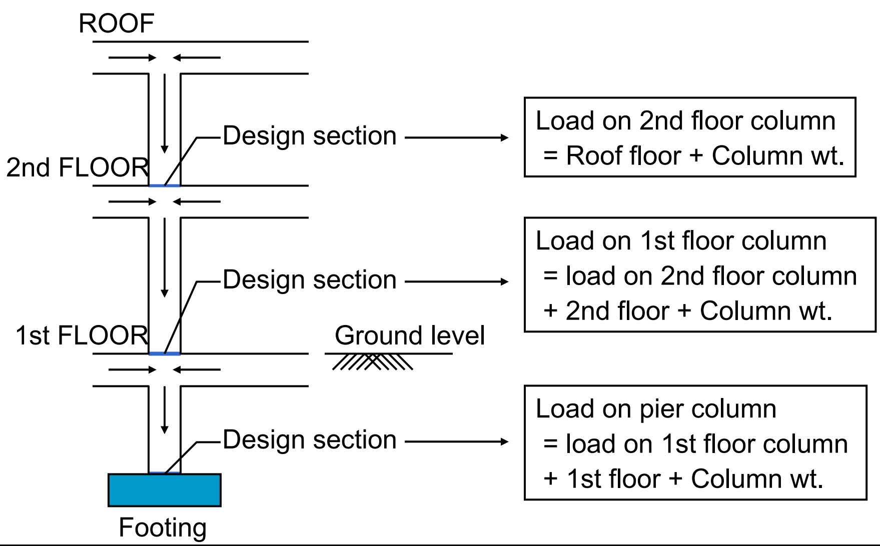
Collect loads from adjacent beam ends





Introduction

- Load summation on column section for design

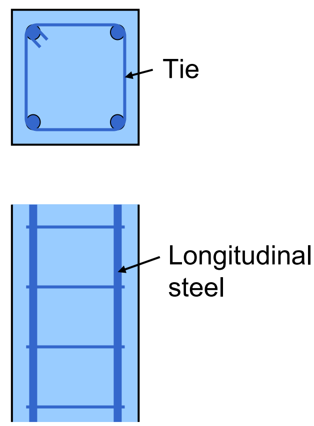


Types of Columns

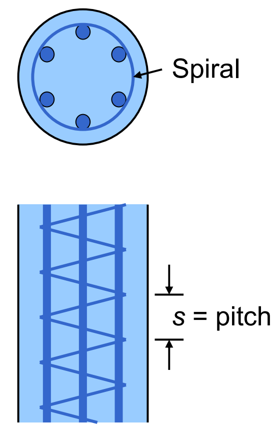
- Types of Reinforced Concrete Columns
  – Members reinforced with longitudinal bars and lateral ties.
  – Members reinforced with longitudinal bars and continuous spirals.
  – Composite compression members reinforced longitudinally with structural steel shapes, pipe, or tubing, with or without additional longitudinal bars, and various types of lateral reinforcement.

Types of Columns

- Types of Reinforced Concrete Columns



Tied column



Spirally reinforced column

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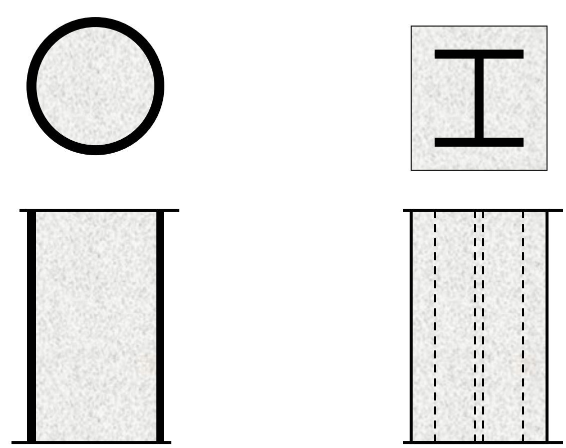
Types of Columns

- Types of Reinforced Concrete Columns



Composite columns

Types of Columns

- Types of Columns in Terms of Their Strengths

1. **Short or Non-Slender Columns**

   A column is said to be short when its length is such that lateral buckling need not be considered. Most of concrete columns fall into this category

2. **Slender Columns**

   When the length of the column is such that buckling need to be considered, the column is referred to as slender column. It is recognized that as the length increases, the usable strength of a given cross section is decreased because of buckling problem

Types of Columns

- Types of Columns in Terms of the Position of the Load on the Cross Section

1. Concentrically Loaded Columns

   Concentrically loaded columns (see Figure 3)carry no moment. In practice, however, all columns have to be designed for some unforeseen eccentricity.

2. Eccentricity Loaded Columns

   Eccentricity loaded columns are subjected to moment in addition to the axial force. The moment can be converted to a load P and eccentricity e (see Figure 4)

Types of Columns

- Types of Columns in Terms of the Position of the Load on the Cross Section

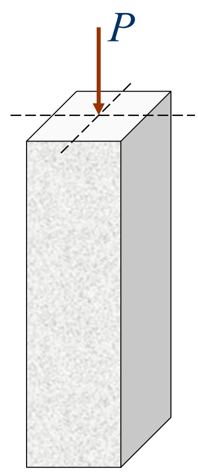


Figure 3. Concentrically Loaded Column

Types of Columns

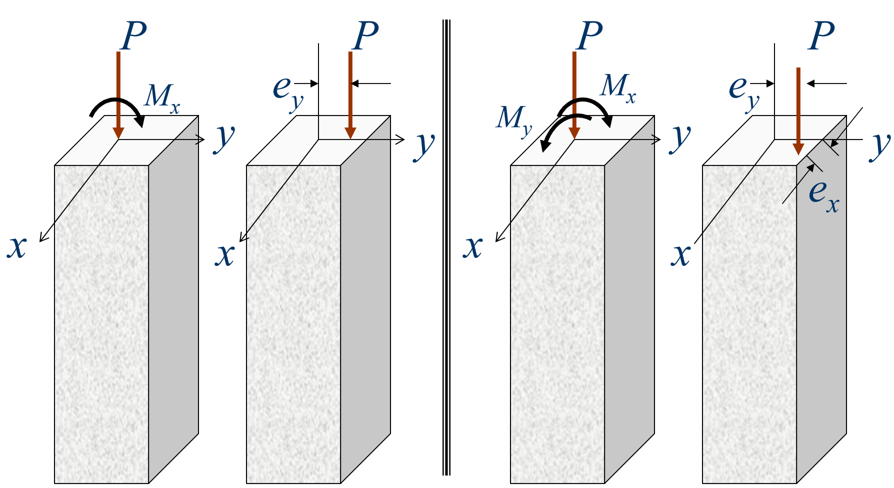


Figure 4a. Axial load plus uniaxial moment

Figure 4b. Axial load plus biaxial moment

Strength of Short or Slender Columns

- If a compression member is loaded parallel to its axis by a load *P* without eccentricity, the load *P* theoretically induces a uniform compressive stress over the cross-sectional area.
- If the compressive load is applied a small distance e away from the longitudinal axis, however, there is a tendency for the column to bend due to the moment $M = Pe$.

Strength of Short or Slender Columns

- Concentric Axial Loading in a Plane of Symmetry
  – When the line of action of the axial load *P* passes through the centriod of the cross section, it can be assumed that the distribution of normal stress is uniform throughout the section, i.e, $f=P/A$.
  – Such a loading is said to be centric, as shown in Figure 3.

Strength of Short or Slender Columns

- Eccentric Axial Loading in a Plane of Symmetry
  - When the line of action of the concentrated load P does not pass through the centroid of the cross section, the distribution of normal stress is no longer uniform.
  - Such loading is said to eccentric, as shown in Figure 4.

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Strength of Short or Slender Columns

- Eccentric Axial Loading in a Plane of Symmetry
  - The stress due to eccentric loading on a beam cross section is given by

$$f = \frac{P}{A} \pm \frac{M_x y}{I_x} \pm \frac{M_y x}{I_y} \qquad (2)$$

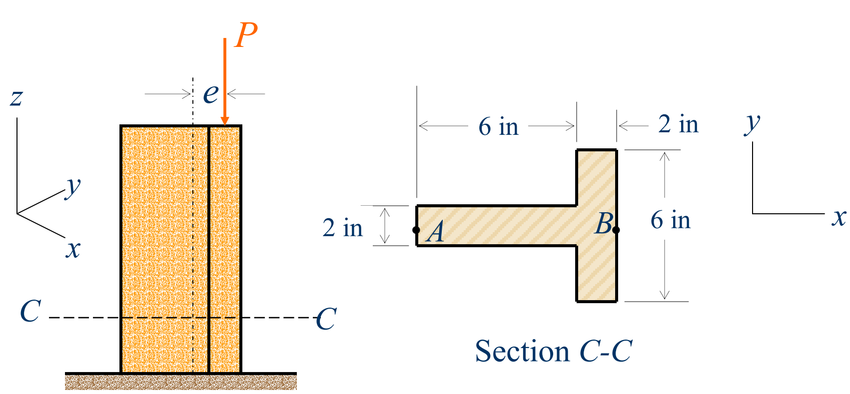
Strength of Short or Slender Columns

- Example 1

The T-section shown in the figure is used as a short post to support a compressive load *P* of 150 kips. The load is applied on centerline of the stem at a distance *e* = 2 in. from the centroid of the cross section. Determine the normal stresses at points *A* and *B* on a transverse plane *C-C* near the base of the post.

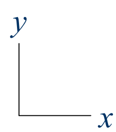
Strength of Short or Slender Columns

- Example 1 (cont'd)



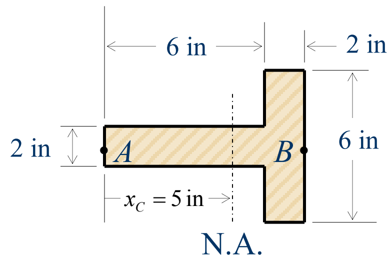
Strength of Short or Slender Columns

- Example 1 (cont'd)
  – Computing the cross-sectional properties:



$$\text{Area} = A = 2[6\times 2] = 24\ \text{in}^2$$

$$x_C = \frac{3(6\times 2)+(6+1)(6\times 2)}{24} = 5\ \text{in. from point } A$$



$$I_y = \frac{2(5)^3}{3} + \frac{6(3)^3}{3} - \frac{4(1)^3}{3} = 136\ \text{in}^4$$

Strength of Short or Slender Columns

- Example 1 (cont'd)

$$f = \frac{P}{A} \pm \frac{M_x y}{I_x} \pm \frac{M_y x}{I_y}$$

Equivalent force system:

$$P = 150\ \text{kip acts through centroid}$$

$$M = Pe = (150)(2)\times 12 = 3{,}600\ \text{kip}\cdot\text{in}$$

Computations of normal stresses:

$$f_A = -\frac{P}{A} + \frac{M_y x}{I_y} = -\frac{150}{24} + \frac{300(5)}{136} = 4.78\ \text{ksi (T)}$$

$$f_B = -\frac{P}{A} - \frac{M_y x}{I_y} = -\frac{150}{24} - \frac{300(3)}{136} = -12.87\ \text{ksi (C)}$$

Strength of Non-Slender Concentrically Loaded Columns

- Background
  - The concrete column that is loaded with a compressive axial load *P* at zero eccentricity is probably nonexistent, and even the axial/small eccentricity combination is relatively rare.
  - Nevertheless, the case of columns that are loaded with compressive axial loads at small eccentricity e is considered first. In this case we define the situation in which the induced small moments are of little significance.

Strength of Non-Slender Concentrically Loaded Columns

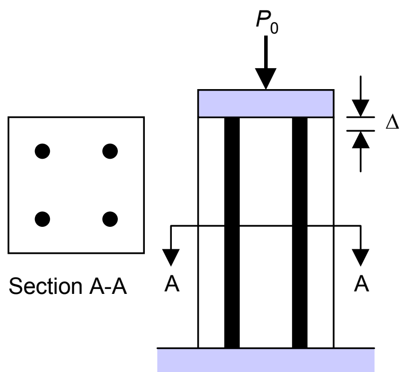
- Notations for Columns Loaded with Small Eccentricities

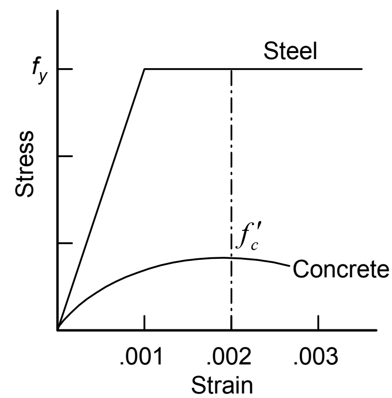
A_g = gross area of the column section (in^2)
A_{st} = total area of longitudinal reinforcement (in^2)
P_0 = nominal or theoretical axial load at zero eccentricity
P_n = nominal or theoretical axial load at given eccentricity
P_u = factored applied axial load at given eccentricity
ρ_g = ratio of total longitudinal reinforcement area to cross-sectional area of column:

$$\rho_g \text{ or } \rho_t = \frac{A_{st}}{A_g} \qquad (3)$$

Strength of Non-Slender Concentrically Loaded Columns

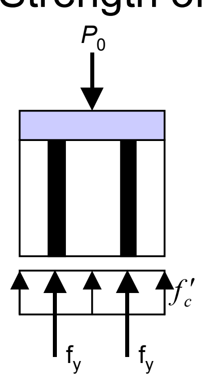
- Strength of Short Axially Loaded Columns





Strength of Non-Slender Concentrically Loaded Columns

- Strength of Short Axially Loaded Columns



$F_s = A_{st} f_y$

$F_c = (A_g - A_{st}) f'_c$

[$\Sigma F_y = 0$]

$$P_0 = f'_c\left(A_g - A_{st}\right) + f_y A_{st}$$

From experiment (e.g., ACI):

$$P_0 = 0.85 f'_c\left(A_g - A_{st}\right) + f_y A_{st}$$

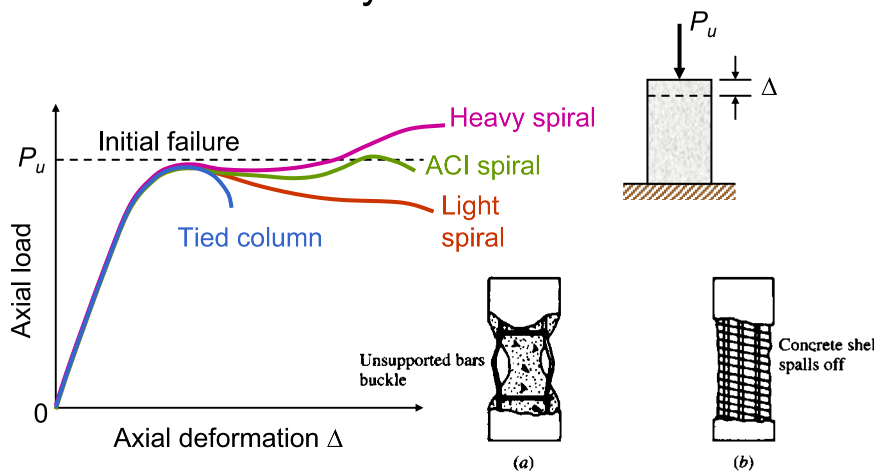
where

A_g = Gross area of column section

A_{st} = Longitudinal steel area

Strength of Non-Slender Concentrically Loaded Columns

- Column Failure by Axial Load



Strength of Non-Slender Concentrically Loaded Columns

- ACI Code Requirements for Column Strength

$$\phi P_n \geq P_u \qquad (4)$$

Spirally reinforced column:

$$\phi P_{n(\max)} = 0.85\phi\left[0.85 f_c'\left(A_g - A_{st}\right) + f_y A_{st}\right], \qquad \phi = 0.70 \qquad (5)$$

Tied column:

$$\phi P_{n(\max)} = 0.80\phi\left[0.85 f_c'\left(A_g - A_{st}\right) + f_y A_{st}\right], \qquad \phi = 0.65 \qquad (6)$$

Strength of Non-Slender Concentrically Loaded Columns

Table 1. Resistance or Strength Reduction Factors

Structural Element	Factor ϕ
Beam or slab; bending or flexure	0.90
Columns with ties	0.65
Columns with spirals	0.70
Columns carrying very small axial load (refer to Chapter 9 for more details)	0.65 – 0.9 or 0.70 – 0.9
Beam: shear and torsion	0.75

Strength of Non-Slender Concentrically Loaded Columns

- ACI Code Requirements for Column Strength (cont'd)
  - Normally, for design purposes, $(A_g - A_{st})$ can be assumed to be equal to A_g without great loss in accuracy.
  - Accordingly, Eqs. 5 and 6, respectively, give

$$A_g = \frac{P_n}{0.68 f_c' + 0.8 \rho_t f_y} \quad (7a)$$

$$\rho_t = \frac{A_{st}}{A_g}$$

$$A_g = \frac{P_n}{0.78 f_c' + 0.85 \rho_t f_y} \quad (7b)$$

Strength of Non-Slender Concentrically Loaded Columns

- ACI Code Requirements for Column Strength (cont'd)
  - For first trial section, with appreciable eccentricity, the designer can try the following equations for assuming gross section area A_g:

$$A_g \geq \frac{P_n}{0.45\left(f'_c + f_y \rho_t\right)} \quad \text{for tied columns} \qquad (8a)$$

$$A_g \geq \frac{P_n}{0.55\left(f'_c + f_y \rho_t\right)} \quad \text{for spirally reinforced columns} \qquad (8b)$$

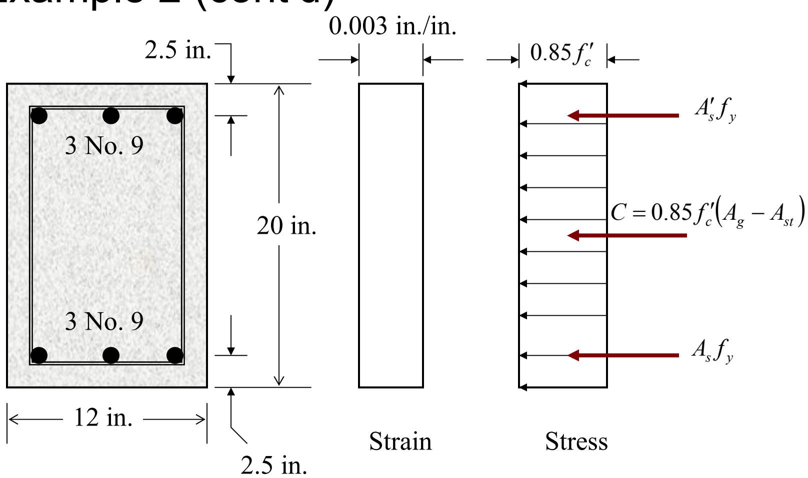
Strength of Non-Slender Concentrically Loaded Columns

- Example 2

A non-slender (short column) column is subjected to axial load only. It has the geometry shown and is reinforced with three No. 9 bars on each of the two faces parallel to the x axis of bending. Calculate the maximum nominal axial load strength $P_{n(\max)}$. Assume that f_y = 60,000 psi and f'_c = 4000 psi.

Strength of Non-Slender Concentrically Loaded Columns

- Example 2 (cont'd)



Strength of Non-Slender Concentrically Loaded Columns

- Example 2 (cont'd)

$$A_s = A'_s = 3\ \text{in}^2$$

Therefore, $A_{st} = 6\ \text{in}^2$. Eq. 6 gives

$$P_{n(\max)} = 0.80[0.85(4000)[(12\times 20 - 6)] + 60{,}000(6)] = 924{,}480\ \text{lb}$$

If $A_g - A_{st}$ is taken to equal A_g, then Eq. 6 results in

$$P_{n(\max)} = 0.80[0.85(4000)(12\times 20) + 60{,}000(6)] = 940{,}800\ \text{lb}$$

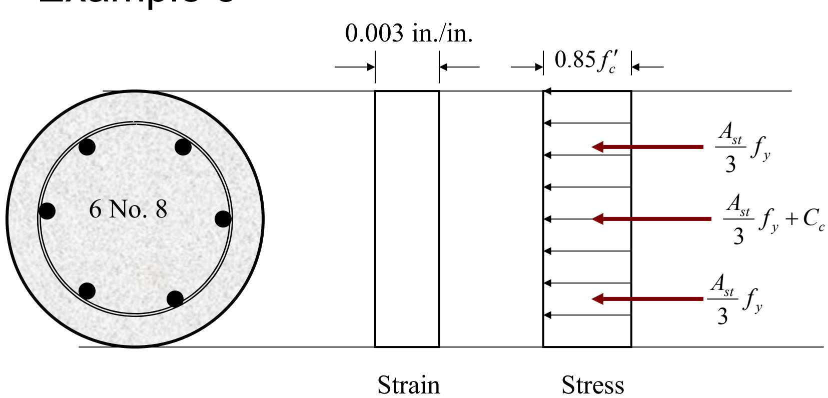
Strength of Non-Slender Concentrically Loaded Columns

- Example 3 (cont'd)

  A 20-in.-diameter, non-slender, spirally reinforced circular column is symmetrically reinforced with six No. 8 bars as shown. Calculate the strength $P_{n(\max)}$ of this column if subjected to axial load only. Use f_c' = 4000 psi and f_y = 60,000 psi.

Strength of Non-Slender Concentrically Loaded Columns

- Example 3



Strength of Non-Slender Concentrically Loaded Columns

- Example 3 (cont'd)

$$A_{st} = 4.74 \text{ in}^2 \text{ (6 No. 8 bars)}$$

$$A_g = \frac{\pi}{4}(20)^2 = 314 \text{ in}^2$$

Therefore, Eq. 5 gives

$$P_{n(\max)} = 0.85[0.85(4000)[(314 - 4.74)] + 60{,}000(4.74)] = 1{,}135{,}501 \text{ lb}$$

If $A_g - A_{st}$ is taken to equal A_g, then Eq. 5 results in

$$P_{n(\max)} = 0.85[0.85(4000)(314) + 60{,}000(4.74)] = 1{,}149{,}200 \text{ lb}$$

Strength of Non-Slender Concentrically Loaded Columns

- ACI Code Limits on percentage of reinforcement

$$0.01 \le \left[\rho_g = \frac{A_{st}}{A_g}\right] \le 0.08 \qquad (9)$$

Lower limit: To prevent failure mode of plain concrete

Upper limit: To maintain proper clearances between bars

Code Requirements Concerning Column Details

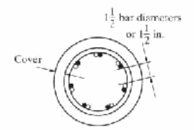
- Minimum Number of Bars
  - The minimum number of longitudinal bars is
    - four within rectangular or circular ties
    - Three within triangular ties
    - Six for bars enclosed by spirals
- Clear distance between Bars
  - The clear distance between longitudinal bars must not be less than 1.5 times the nominal bar diameter nor 1 ½ in.

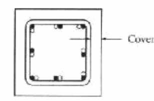
Code Requirements Concerning Column Details

- Clear distance between Bars (cont'd)
  - Table 2 (Table 9 Handout) may be used to determine the maximum number of bars allowed in one row around the periphery of circular or square columns.
- Cover
  - Cover shall be 1 ½ in. minimum over primary reinforcement, ties or spirals.

Code Requirements Concerning Column Details

Table 2. Preferred Maximum Number of Column Bars in One Row





Recommended spiral or tie bar number	Core size (in.) = column size − 2 × cover	Circular area (in.²)	Bar number #5	#6	#7	#8	#9	#10	#11	Square area (in.²)	Bar number #5	#6	#7	#8	#9	#10	#11
3[a]	9	63.6	8	7	7	6	—	—	—	81	8	8	8	8	4	4	4
	10	78.5	10	9	8	7	6	—	—	100	12	8	8	8	8	4	4
	11	95.0	11	10	9	8	7	6	—	121	12	12	8	8	8	8	4
	12	113.1	12	11	10	9	8	7	6	144	12	12	12	8	8	8	8
	13	132.7	13	12	11	10	8	7	6	169	16	12	12	12	8	8	8
	14	153.9	14	13	12	11	9	8	7	196	16	16	12	12	12	8	8
	15	176.7	15	14	13	12	10	9	8	225	16	16	16	12	12	12	8
4	16	201.1	16	15	14	12	11	9	8	256	20	16	16	16	12	12	8
	17	227.0	18	16	15	13	12	10	9	289	20	20	16	16	12	12	8
	18	254.5	19	17	15	14	12	11	10	324	20	20	16	16	16	12	12
	19	283.5	20	18	16	15	13	11	10	361	24	20	20	16	16	12	12
	20	314.2	21	19	17	16	14	12	11	400	24	24	20	20	16	12	12
	21	346.4	22	20	18	17	15	13	11	441	28	24	20	20	16	16	12
	22	380.1	23	21	19	18	15	14	12	484	28	24	24	20	20	16	12
5	23	415.5	24	22	21	19	16	14	13	529	28	28	24	24	20	16	16
	24	452.4	25	23	21	20	17	15	13	576	32	28	24	24	20	16	16
	25	490.9	26	24	22	20	18	16	14	625	32	28	28	24	20	20	16
	26	530.9	28	25	23	21	19	16	14	676	32	32	28	24	24	20	16
	27	572.6	29	26	24	22	19	17	15	729	36	32	28	28	24	20	16

[a] No. 4 tie for No. 11 or larger longitudinal reinforcement.

Code Requirements Concerning Column Details

- Tie Requirements
  - According to Section 7.10.5 of ACI Code, the minimum is
    - No. 3 for longitudinal bars No. 10 and smaller
    - Otherwise, minimum tie size is No. 4 (see Table 2 for a suggested tie size)
  - The center-to-center spacing of ties must not exceed the smaller of 16 longitudinal bar diameter, 48 tie-bar diameter, or the least column dimension.

Code Requirements Concerning Column Details

- Spiral Requirements
  - According to Section 7.10.4 of ACI Code, the minimum spiral size is 3/8 in. in diameter for cast-in-place construction (5/8 is usually maximum).
  - Clear space between spirals must not exceed 3 in. or be less than 1 in.

Code Requirements Concerning Column Details

- Spiral Requirements (cont'd)
  - The spiral steel ratio ρ_s must not be less than the value given by

$$\rho_{s(\min)} = 0.45\left(\frac{A_g}{A_c} - 1\right)\frac{f_c'}{f_y} \qquad (10)$$

where

$$\rho_s = \frac{\text{volume of spiral steel in one turn}}{\text{volume of column core in height } (s)}$$

s = center-to-center spacing of spiral (in.), also called pitch
A_g = gross cross-sectional area of the column (in^2)
A_c = cross-sectional area of the core (in^2) (out-to-out of spiral)
f_y = spiral steel yield point (psi) ≤ 60,000 psi
= compressive strength of concrete (psi)

Code Requirements Concerning Column Details

- Spiral Requirements (cont'd)
  - An Approximate Formula for Spiral Steel Ratio
    - A formula in terms of the physical properties of the column cross section can be derived from the definition of ρ_s.
    - In reference to Fig. 5, the overall core diameter (out-to-out of spiral) is denoted as D_c, and the spiral diameter (center-to-center) as D_s.
    - The cross-sectional area of the spiral bar or wire is given the symbol A_{sp}.

Code Requirements Concerning Column Details

- Spiral Requirements (cont'd)

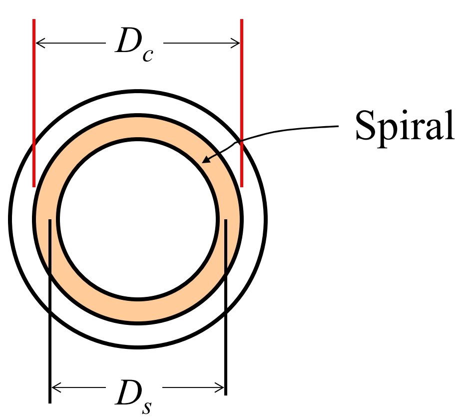


Figure 5. Definition of D_c and D_s

Code Requirements Concerning Column Details

- Spiral Requirements (cont'd)
  - From the definition of ρ_s, an expression may written as follows:

$$\text{actual } \rho_s = \frac{A_{sp}\pi D_s}{\left(\pi D_c^2/4\right)(s)} \qquad (11)$$

  - If the small difference between D_c and D_s is neglected, then in terms of D_c, the actual spiral steel ratio is given by

$$\text{actual } \rho_s = \frac{4A_{sp}}{D_c s} \qquad (12)$$