REINFORCED CONCRETE
A Fundamental Approach - Fifth Edition

CHAPTER 6b

SHEAR AND DIAGONAL TENSION IN BEAMS

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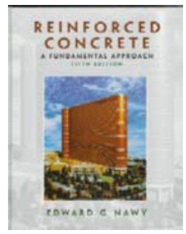
SPRING 2004



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Web Reinforcement Design

Procedure for shear

- The design of stirrups for shear reinforcement involves the determination of stirrup size and spacing pattern.
- A general procedure is as follows:

1. Determine the critical section, shear values based on clear span, and draw a shear diagram for the factored shear force V_u.
2. Check whether

$$V_u \le \phi\left(V_c + 8\sqrt{f'_c}\, b_w d\right)$$

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Procedure for shear

Where b_w is the web width or diameter of the circular section. If this condition is not met, the cross section has to be enlarged.

3. Use minimum shear reinforcement A_v if V_u larger than one-half ϕV_c with the following exceptions:
   (a) Concrete joist construction
   (b) Slabs and Footings
   (c) Small shallow beams of depth not exceeding 10 in. or 2 ½ times flange thickness

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Procedure for shear

$$\min A_v = 0.75\sqrt{f'_c}\,\frac{b_w s}{f_y} \quad \text{or} \quad A_v = \frac{50 b_w s}{f_y}$$ whichever is larger

Good construction practice dictates that some stirrups always be used to facilitate proper handling of the reinforcement cage.

4. If $V_u > \phi V_c$, shear reinforcement must be provided such as that $V_u \leq \phi(V_c + V_s)$, where

$$V_s = \begin{cases} \dfrac{A_v f_y d}{s} & \text{for vertical stirrups} \\ \dfrac{A_v f_y d}{s}(\sin\alpha + \cos\alpha) & \text{for inclined stirrups} \end{cases}$$

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Procedure for shear

If $\alpha = 45^0$, then

$$V_s = \frac{1.414 A_v f_y d}{s}$$

5. Maximum spacing s must be

$$s = \frac{d}{2} \leq 24 \text{ in.}$$

except that if $V_s > 4\sqrt{f_c'} b_w d$, then the spacing should be

$$s \leq \frac{d}{4} \leq 12 \text{ in.}$$

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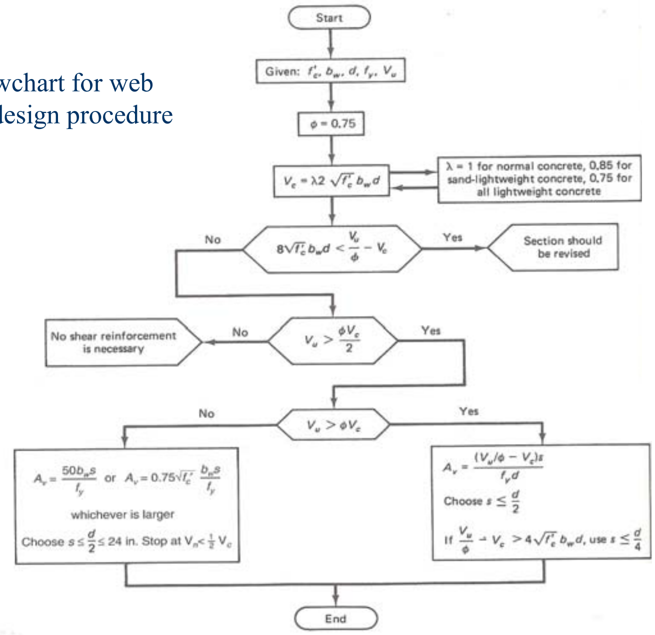
Procedure for shear

- The flowchart of Figure 14 contains the sequence of calculations needed for the design of vertical stirrups.
- Simple corresponding modifications of this chart can be made so that the chart can be used in the design of inclined web reinforcement steel.

Figure 14. Flowchart for web reinforcement design procedure



Web Reinforcement Design Procedure for shear

- Example 3

  A continuous reinforced concrete is 15 in. wide and 33 in. deep, and has an effective depth of 31 in. The beam is subjected to two concentrated live loads of 62.5 kips each as shown, and a distributed live load of 0.32 k/ft. The distributed load does not include the weight of the beam. Design the web reinforcement if f'_c = 4000 psi and f_y = 60,000 psi. Assume normal weight concrete.

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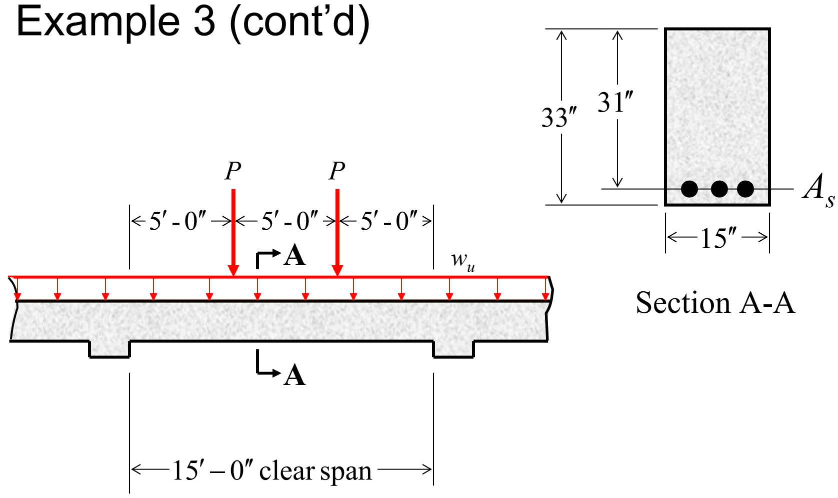
Procedure for shear

- Example 3 (cont'd)



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Procedure for shear

- Example 3 (cont'd)
  - Calculate the factored loads:
    - Estimate self-weight of the beam

$$w_{sw} = \frac{15 \times 33}{144}(150) = 515.625\,\frac{\text{lb}}{\text{ft}} = 0.52\,\frac{\text{kips}}{\text{ft}}$$

    - Factored distributed load:

$$w_u = 1.2(0.52 + 0.32) = 1.0\,\frac{\text{Kips}}{\text{ft}}$$

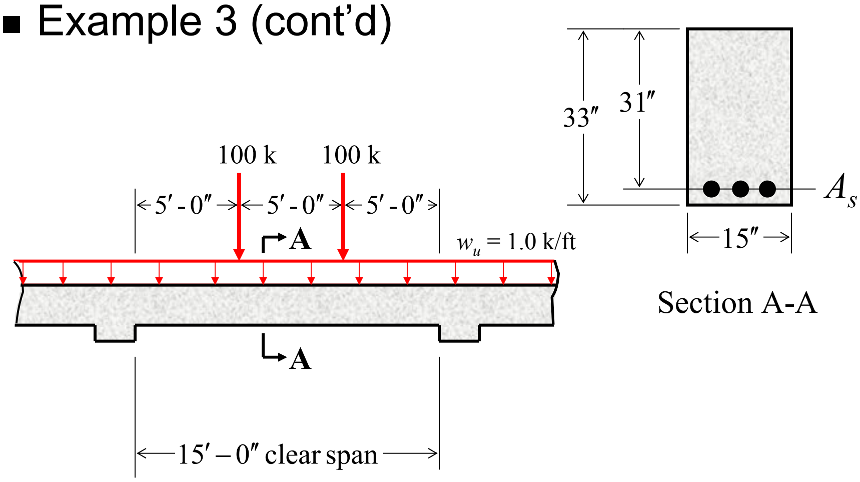
    - Factored concentrated load

$$P_u = 1.6(62.5) = 100\text{ kips}$$

Web Reinforcement Design

Procedure for shear

- Example 3 (cont'd)



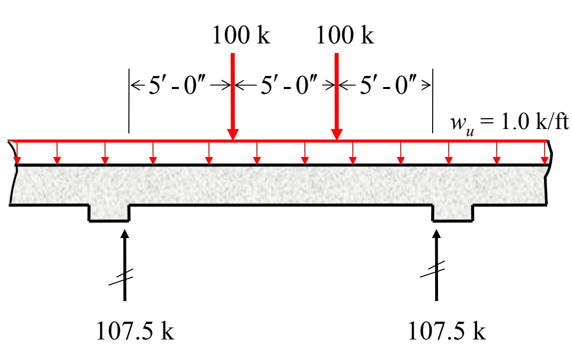
Web Reinforcement Design

Procedure for shear

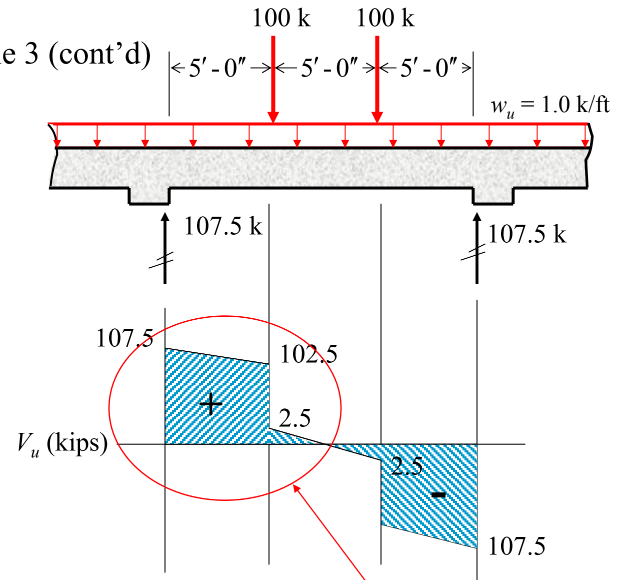
- Example 3 (cont'd)

$$R_1 = R_2 = \frac{2(100)+1(15)}{2} = 107.5\text{ k}$$

  - Establish the shear force diagram for V_u:

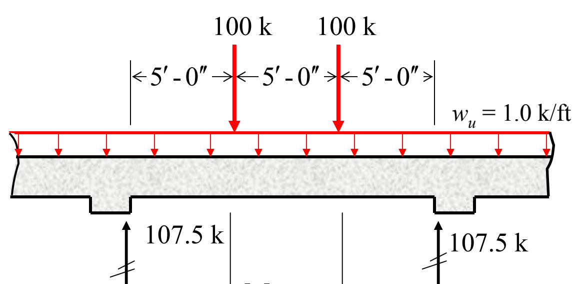


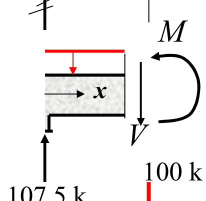
Example 3 (cont'd)



See Figure 15 for enlargement

Example 2 (cont'd)

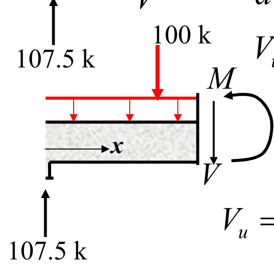




$$V_u = 107.5 - x(1) \qquad \text{for } 0 \le x \le 5$$

$$d = 31'' = 2.58'$$

$$V_u^* = V_u(2.58) = 107.5 - 2.58(1) = 104.9 \text{ k}$$



$$V_u = 107.5 - 100 - x(1) \qquad \text{for } 5 \le x \le 10$$

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Procedure for shear

- Example 3 (cont'd)
  - Because of the symmetry, we will focus on the left half of the shear diagram as shown in Fig. 2.
  - Shear capacity:

$$V_u \leq \phi\left(V_c + 8\sqrt{f_c'}b_w d\right)$$

$$V_c = 2\lambda\sqrt{f_c'}b_w d = \frac{2(1)\sqrt{4,000}(15)(31)}{1000} = 58.82 \text{ kips}$$

$$8\sqrt{f_c'}b_w d = \frac{8\sqrt{4000}(15)(31)}{1000} = 235.27 \text{ kips}$$

$$\phi\left(V_c + 8\sqrt{f_c'}b_w d\right) = 0.75(58.82 + 235.27) = 220.57 \text{ kips}$$

Web Reinforcement Design

Procedure for shear

- Example 3 (cont'd)
  - Shear Capacity (cont'd)

$$V_u \leq \phi\left(V_c + 8\sqrt{f_c'}b_w d\right)$$

$$\left(V_u^* = 104.9\right) < 220.57 \quad \text{cross-section OK}$$

$$\left(V_u^* = 104.9\right) > \left(\frac{1}{2}\phi V_c = 22.06\right) \quad \text{stirrups are required}$$

$$\phi V_c = 0.75(58.82) = 44.12 \text{ kips}$$

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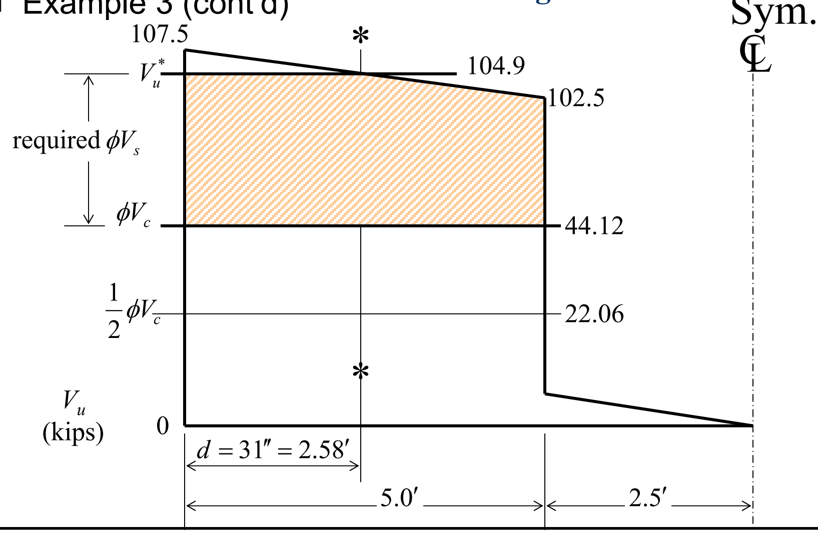
Procedure for shear

- Example 3 (cont'd)

Figure 15



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Procedure for shear

- Example 3 (cont'd)
  – Stirrups are required to the point where

$$V_u = \frac{1}{2}\phi V_c = 22.06 \text{ kips}$$

  – From Figure 15, this point is located at the first concentrated load and it is at distance 5 ft from the face of the support.
  – Determine the "required ϕV_s" on the V_u diagram:

$$\text{required } \phi V_s = \max V_u - \phi V_c - wx$$

$$= 107.5 - 44.12 - (1)x$$

$$\text{required } \phi V_s = 63.38 - x \qquad \text{for } 2.58 \le x \le 5$$

Web Reinforcement Design

Procedure for shear

$$\text{Eq.17}: V_s = \frac{V_u - \phi V_c}{\phi}$$

- Example 3 (cont'd)
  - Try No. 3 vertical stirrups (A_v = 0.22 in^2):

$$\text{required } s^* = \frac{\phi A_v f_y d}{\text{required } \phi V_s^*} = \frac{0.75(0.22)(60)(31)}{104.9 - 44.12} = 5.05 \text{ in.}$$

use 5 in.

  - Establish ACI Code maximum spacing requirements:

$$4\sqrt{f_c'}b_w d = \frac{4\sqrt{4000}(15)(31)}{1000} = 117.6 \text{ kips}$$

$$V_s^* = \frac{\phi V_s^*}{\phi} = \frac{104.9 - 44.12}{0.75} = 81.04 \text{ kips}$$

Web Reinforcement Design

Procedure for shear

- Example 3 (cont'd)

Table 1. Areas of Multiple of Reinforcing Bars (in^2)

Number of bars	Bar number								
	#3	#4	$5	#6	#7	#8	#9	#10	#11
1	0.11	0.20	0.31	0.44	0.60	0.79	1.00	1.27	1.56
2	0.22	0.40	0.62	0.88	1.20	1.58	2.00	2.54	3.12
3	0.33	0.60	0.93	1.32	1.80	2.37	3.00	3.81	4.68
4	0.44	0.80	1.24	1.76	2.40	3.16	4.00	5.08	6.24
5	0.55	1.00	1.55	2.20	3.00	3.95	5.00	6.35	7.80
6	0.66	1.20	1.86	2.64	3.60	4.74	6.00	7.62	9.36
7	0.77	1.40	2.17	3.08	4.20	5.53	7.00	8.89	10.92
8	0.88	1.60	2.48	3.52	4.80	6.32	8.00	10.16	12.48
9	0.99	1.80	2.79	3.96	5.40	7.11	9.00	11.43	14.04
10	1.10	2.00	3.10	4.40	6.00	7.90	10.00	12.70	15.60

Web Reinforcement Design

Procedure for shear

- Example 3 (cont'd)
  - Since 81.04 kips < 117.6 kips, the maximum spacing shall be the smallest of the following values (see Eq. 18):

$$s_{\max} = \max\left[\frac{A_v f_y}{0.75\sqrt{f'_c}b_w}, \frac{A_v f_y}{50 b_w}\right] = \max\left(\frac{0.22(60{,}000)}{0.75\sqrt{4000}(15)}, \frac{0.22(60{,}000)}{50(15)}\right) = 18.6 \text{ in.}$$

$$s_{\max} = \frac{d}{2} = \frac{31}{2} = 15.5 \text{ in.}$$ ← controls

$$s_{\max} = 24 \text{ in.}$$

Therefore, use a maximum spacing of 15 in.

Web Reinforcement Design

Procedure for shear

- Example 3 (cont'd)
  - Determine the spacing requirements between the critical section and the first concentrated load:

$$\text{required } s = \frac{\phi A_v f_y d}{\text{required } \phi V_s} = \frac{0.75(0.22)(60)(31)}{63.38 - x}$$

  - The results of applying above equation for values of x ranges from 3 to 5 are tabulated as shown

x (ft)	Required s (in)
3	5.08
4	5.17
5	5.26

Web Reinforcement Design

Procedure for shear

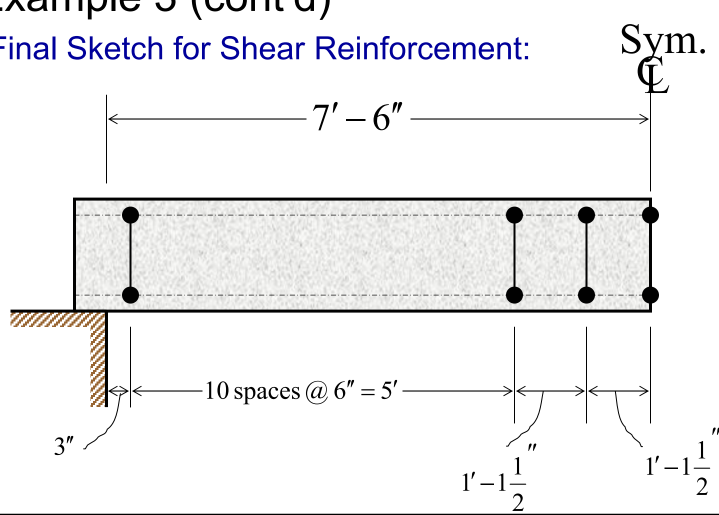
- Example 3 (cont'd)
  - Since no stirrups are required in the distance between the concentrated loads, it is clear that the maximum spacing of 15 in. need not be used in that distance.
  - A spacing of 5 in. will be used between the face of the support and the concentrated load.
  - The center part of the beam will be reinforced with stirrups at a spacing slightly less than the maximum spacing of 15 in.

Web Reinforcement Design

Procedure for shear

- Example 3 (cont'd)

Final Sketch for Shear Reinforcement:

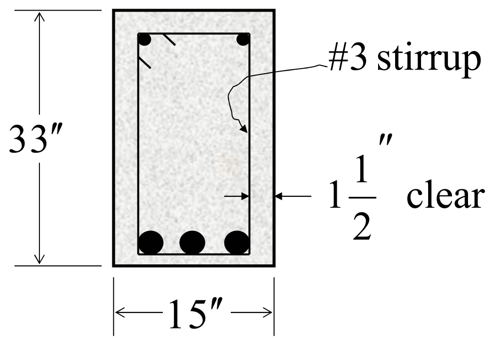


Web Reinforcement Design

Procedure for shear

- Example 3 (cont'd)

Final Sketch for Shear Reinforcement:



Section A-A

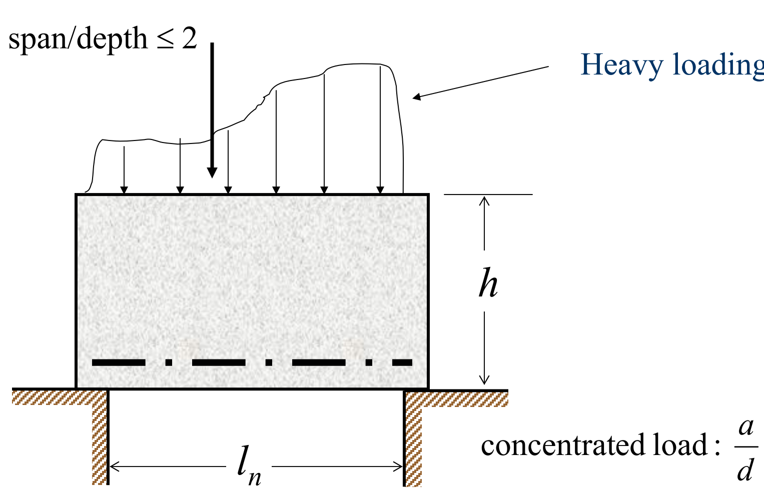
Deep Beams

- Deep beams are structural elements loaded as beams but having a large depth/thickness ratio.
- For deep beams, the shear span/depth ratio is not to exceed 2 for concentrated load and 4 for distributed load, where the shear span is the clear span of the beam for distributed load.

Deep Beams

- Because of the geometry of deep beams, they behave in a non-linear analysis as two-dimensional rather one-dimensional members and are subjected to a two-dimensional state of stress.
- As a result, plane sections before bending do not necessarily remain plane after bending.
- Strain distribution no longer considered linear.

Deep Beams



concentrated load : $\frac{a}{d} < 2$

distributed load : $\frac{l_n}{d} < 4$

Deep Beams

- Design Criteria
  - Deep beam have a higher nominal shear resistance V_c than do normal beams.
  - While the critical section for calculating the factored shear force V_u is taken at a distance d from the face of the support in normal beams, the shear plane in the deep beam is considerably steeper in inclination and closer to the support.

Deep Beams

- Design Criteria (cont'd)
  - If x is the distance of the failure plane from the face of the support, the expression for distance become

$$\begin{array}{ll} \text{uniform load:} & x = 0.15 l_n \\ \text{concentrated load:} & x = 0.50 a \end{array}$$

  - In either case, the distance x should not exceed the effective depth d of the beam.

Deep Beams

- Design Criteria (cont'd)
  - The factored shear force V_u has to satisfy the condition

$$V_u \leq \phi\left(10\sqrt{f_c'}b_w d\right)$$

or

$$V_n \leq 10\sqrt{f_c'}b_w d$$

  - If not, the section has to be enlarged.
  - The strength reduction factor ϕ = 0.75 for deep beams (the same as for normal beams).

Deep Beams

- Design Criteria (cont'd)
  - The nominal shear resistance force V_c of plain concrete can be taken as

$$V_c = \left(3.5 - 2.5\frac{M_u}{V_u d}\right)\left(1.9\sqrt{f_c'} + 2500\rho\frac{V_u d}{M_u}\right)b_w d \leq 6\sqrt{f_c'}b_w d$$

  - Where

$$1.0 < 3.5 - 2.5\left(\frac{M_u}{V_u d}\right) \leq 2.5$$

Deep Beams

- Design Criteria (cont'd)
  - If $V_u > \phi V_c$, shear reinforcement has to be provided such that $V_u \leq \phi(V_c + V_s)$, where V_s is the force resisted by the shear reinforcement:

$$V_s = \left[\frac{A_v}{s_v}\left(\frac{1 + l_n/d}{12}\right) + \frac{A_{vh}}{s_h}\left(\frac{11 - l_n/d}{12}\right)\right] f_y d$$

A_v = total area of vertical stirrups spaced at s_v in the horizontal direction at both faces of the beam

A_{vh} = total area of horizontal stirrups spaced at s_h in the vertical direction at both faces of the beam

Deep Beams

- Design Criteria (cont'd)
  - Maximum and Minimum Requirements

$$\max s_v \leq \frac{d}{5} \quad \text{or } 12 \text{ in.}$$

$$\max s_h \leq \frac{d}{5} \quad \text{or } 12 \text{ in.}$$

$$\min A_{vh} = 0.0015 b s_h$$

$$\min A_v = 0.0025 b s_v$$

  - The shear reinforcement required at the critical section must be provided throughout the entire length of the beam.