CHAPTER

5c

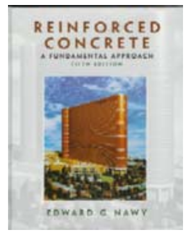
REINFORCED CONCRETE

A Fundamental Approach - Fifth Edition

FLEXURE IN BEAMS

A. J. Clark School of Engineering •Department of Civil and Environmental Engineering

SPRING 2004



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ENCE 454 – Design of Concrete Structures

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Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

- Rectangular Beam Design for Moment (Tension Only)
  - In a general sense, the design procedure for a rectangular cross section of a reinforced beam basically requires the determination of three quantities.
  - The compressive strength of concrete and the yield strength f_y of steel are usually prescribed.

Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

- Rectangular Beam Design for Moment (Tension Only)
  - The three quantities that need to be determined in a design problem for rectangular reinforced concrete beam are:
    - Beam Width, b
    - Beam Depth, d
    - Steel Area, A_s.

Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

- Rectangular Beam Design for Moment (Tension Only)
  - Theoretically, a wide shallow beam may have the same ϕM_n as a narrow deep beam.
  - However, practical considerations and code requirements will affect the final selection of these three quantities.
  - There is no easy way to determine the best cross section, since economy depends on much more than simply the volume of concrete and amount of steel.

Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

- Simplified Design Formulas
  - The simplified design formulas, namely Eqs. 12 to 14, developed by the internal couple method can be used for the design of singly reinforced concrete beams.
  - However, one can always start from the Whitney cross-section figure to design and analyze these beams.
  - These equations, shown on the next slide for convenience, are used to help the designer.

Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

- Simplified Design Formulas
  - or

$$M_n = \left[\omega r f_c'(1-0.59\omega)d^3\right] \qquad (12)$$

  - where ω = ρf_y / f_c\`. Eq. 12 sometimes is expressed as

$$M_n = Rbd^2 \qquad (13)$$

  - where

$$R = \omega f_c'(1-0.59\omega) \qquad (14)$$

Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

- Simplified Design Formulas
  - Eqs. 12 t o14 can be useful for the developments of design charts and design tables.
  - For example, tables or plots of the coefficient of resistance *R* versus the reinforcement ratio $\rho = A_s/bd$ for singly reinforced beams can be constructed for different combinations of the compressive concrete strength f_c' and the yield strength of steel f_y, as shown in Figure 18 and Table 2 (sample).

$$R = f_c'\omega(1 - 0.59\,\omega)$$

$$\omega = \rho\frac{f_y}{f_c'}$$

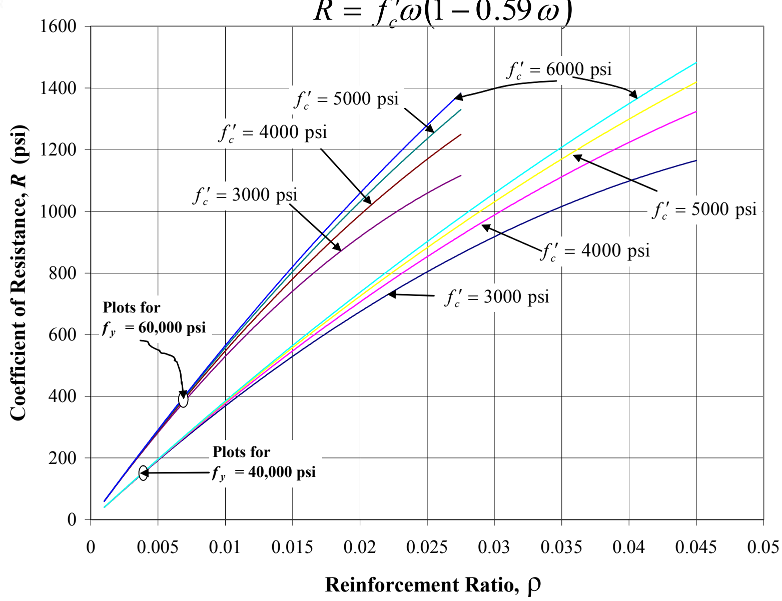


Figure 18. Coefficient of Resistance *R* Versus Reinforcement Ratio ρ

Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

Table 2. Sample Coefficient of Resistance Vs. Steel Ratio ρ

$f'_c = 3000$ psi $f_y = 40{,}000$ psi

ρ	***R*** **(psi)**
0.0010	39.69
0.0011	43.62
0.0012	47.55
0.0013	51.47
0.0014	55.39
0.0015	59.29
0.0016	63.20
0.0017	67.09
0.0018	70.98
0.0019	74.87
0.0020	78.75
0.0021	82.62

$f'_c = 4000$ psi $f_y = 60{,}000$ psi

ρ	***R*** **(psi)**
0.0010	59.47
0.0011	65.36
0.0012	71.24
0.0013	77.11
0.0014	82.96
0.0015	88.81
0.0016	94.64
0.0017	100.47
0.0018	106.28
0.0019	112.09
0.0020	117.88
0.0021	123.67

Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

- Simplified Design Formulas
  - The general analysis expression for M_n may be written as

$$M_n = Rbd^2 \qquad \text{(in.-lb)} \qquad (32a)$$

or

$$M_n = \frac{Rbd^2}{12} \qquad \text{(ft-lb)} \qquad (32b)$$

NOTE: Values of R versus ρ for various combinations of f_y and f'_c are tabulated in psi in Tables A-1 to A-8 (handout)

Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

- Note that Eq. 32 can also be used to simplify the analysis of a reinforced beam having a rectangular cross section.
- The following beam can be analyzed based on a lengthy procedure. However, now this beam will be analyzed based on Eq. 32.

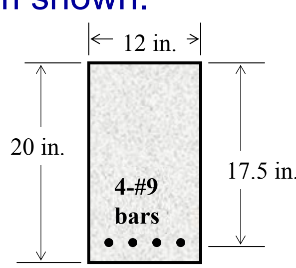
Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

- Example 7

Find the nominal flexural strength and design strength of the beam shown.

$f'_c = 4{,}000$ psi

$f_y = 60{,}000$psi



Four No. 9 bars provide $A_s = 4.00$ in^2

$$\rho = \frac{A_s}{bd} = \frac{4.00}{12(17.5)} = 0.0190$$

$$(\rho = 0.0190) > \left(\rho_{\min} = \max\left[\frac{3\sqrt{4000}}{60{,}000}, \frac{200}{60{,}000} \right] = 0.0033 \right)$$

OK

Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

Table 3
Design Constants

f'_c(psi)	$\max\left(\frac{3\sqrt{f'_c}}{f_y}, \frac{200}{f_y}\right)$	ρ_b
f_y = 40,000 psi		
3000	0.0050	0.03712
4000	0.0050	0.04949
5000	0.0053	0.05823
6000	0.0058	0.06551
f_y = 50,000 psi		
3000	0.0040	0.02753
4000	0.0040	0.03671
5000	0.0042	0.04318
6000	0.0046	0.04858
f_y = 60,000 psi		
3000	0.0033	0.02138
4000	0.0033	0.02851
5000	0.0035	0.03354
6000	0.0039	0.03773
f_y = 75,000 psi		
3000	0.0027	0.01552
4000	0.0027	0.02069
5000	0.0028	0.02435
6000	0.0031	0.02739

Values used in the example.

Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

- Example 7 (cont'd)
  - Also check the strain limit zone for tension controlled:

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{4(60{,}000)}{0.85(4000)(12)} = 5.8824 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{5.8824}{0.85} = 6.92 \text{ in.}$$

$$\frac{c}{d_t} = \frac{6.92}{17.5} = 0.396 > 0.375 \text{ but } < 0.600 \text{ (Fig.14)}$$

So section is in the transition zone, and therefore, a lower ϕ-factor should be applied. For most economical section, increase the depth. For this section:

$$\phi = 0.23 + \frac{0.25}{c/d_t} = 0.23 + \frac{0.25}{0.396} = 0.86$$

Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

- Example 7 (cont'd)

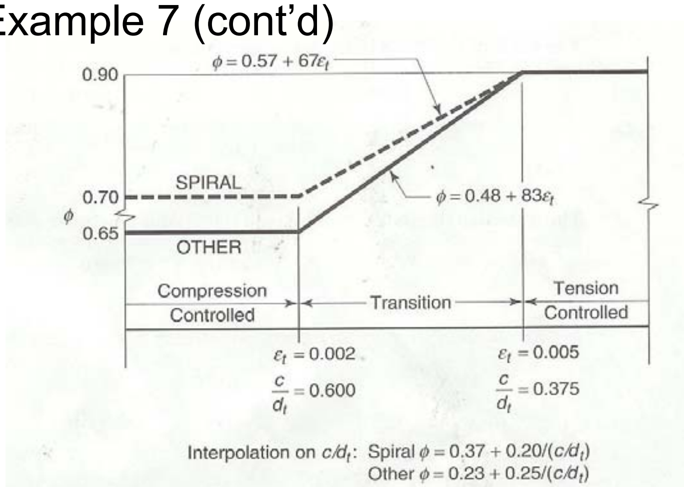


Figure 14. Strain Limit Zones and variation of Strength Reduction Factor ϕ

Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

- Example 7 (cont'd)

  - From Table 4 (Table A-6 , Handout), with f_y = 60,000 psi, f_c' = 4,000 psi, and ρ = 0.0190, the value of R = 948.88 ksi is found .
  - Using Eq. 32b, the nominal and design strengths are respectively

$$M_n = \frac{Rbd^2}{12} = \frac{(948.88)(12)(17.5)^2}{12\times1000} = 290.6 \text{ ft - kips}$$

$$M_u = \phi M_n = 0.86(290.6) = 250 \text{ ft - kips}$$

Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

- Example 7 (cont'd)

Table 4
Part of Table A-6
of Handout

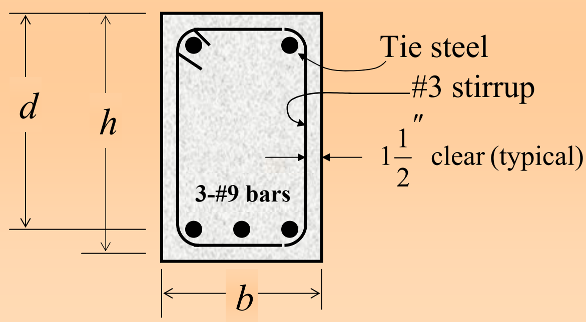
ρ	***R*** **(psi)**	ρ	***R*** **(psi)**
0.0145	758.69	0.0190	948.88
0.0146	763.15	0.0191	952.87
0.0147	767.60	0.0192	956.84
0.0148	772.04	0.0193	960.80
0.0149	776.47	0.0194	964.75

Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

- ACI Code Requirements for Concrete Protection for Reinforcement
  - For beams, girders, and columns not exposed to weather or in contact with the ground, the minimum concrete cover on any steel is 1.5 in (38 mm).
  - For slabs, it is 0.75 in.
  - Clear space between bars in a single layer shall not be less than the bar diameter, but not less 1 in.

Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

- Stirrups
  - Stirrups are special form of reinforcement that primarily resist shear forces that will be discussed later.



Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

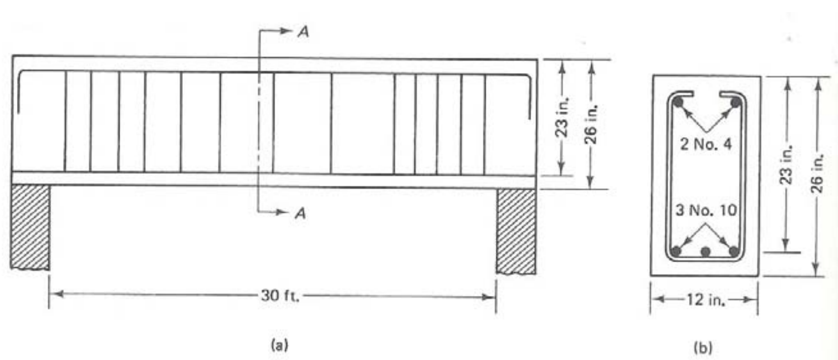


Figure 19. Details of Reinforcement: (a) section elevation (b) midspan section

Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

- A. Cross Section (b and h) Known; Find the Required A_s:

1. Convert the service loads or moments to design M_u (including the beam weight).
2. Based on knowing h, estimate d by using the relationship $d = h - 3$ in. (conservative for bars in a single layer). Calculate the required R from

$$R = \frac{M_n}{bd^2} \qquad (33)$$

Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

3. From Tables A-1 through A-8 of your Handout, find the required steel ratio ρ.
4. Compute the required A_s:

$$A_s = \rho bd \qquad (34)$$

   Check $A_{s,\min}$ by using Table 3 or ACI equations.
5. Select the bars. Check to see if the bars can fit into the beam in one layer (preferable). Check the actual effective depth and compare with the assumed effective depth. If the actual effective depth is slightly in excess of

Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

the assumed effective depth, the design will be slightly conservative (on the safe side). If the actual effective depth is less than the assumed effective depth, the design is on the unconservative side and should be revised.

6. Sketch the design showing the details of the cross section and the reinforcement exact location, and the stirrups, including the tie bars.

Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

- B. Design for Cross Section and Required A_s (free design):

1. Calculate the external factored moment. To obtain the beam self-weight, an assumption has to be made for the value of d. The minimum thickness for deflection specified in the ACI Code can be used as a guide. Assume a b/d ratio r between 0.3 and 0.6 and calculate $b = rd$. A first trial assumption $b \approx d/2$ is recommended.
2. (a) Select a value of moment factor R based

Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

on $\varepsilon_t = 0.005$ or higher, or $c/d_t \leq 0.375$. Assume that $b \approx d/2$, calculate d for $M_n = Rbd^2$ and proceed to analyze the section.

(b) Alternatively, choose d on the basis of minimum deflection requirement. Choose a width b as in (a). Assume a moment arm $jd = 0.85d$ to $0.90d$. Compute A_s as a first trial, then analyze the section using $b = d/2$.

- Assume the neutral axis depth ratio $c/d_t < 0.375$.
- Get A_s from $C = T$, then check ε_t value > 0.005.

Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

- For designers who prefer charts, Eq. 32 can be used for the first trial in design.
- The value of *R* can be obtained from Figure 18 or Tables A1- A8 (handout).
- The steps for design of the beam cross section presented earlier follows the flowchart of the following figure (Figure 20).

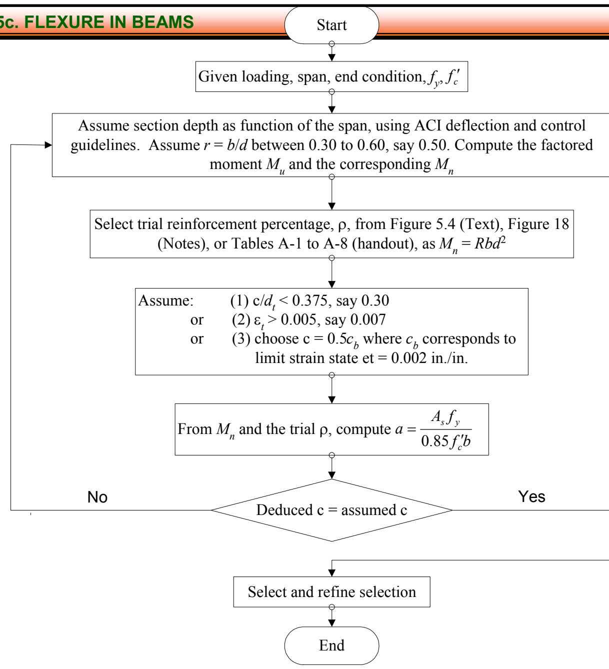


Figure 20. Flowchart sequence of operations for flexural design of singly reinforced beams

Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

- Example 8

Design a rectangular reinforced concrete beam to carry a service dead load moment of 50 ft-kips (which includes the moment due to the weight of the beam) and a service live load moment of 100 ft-kips. Architectural considerations require the beam width to be 10 in. and the total depth h to be 25 in. Use f'_c = 3,000 psi and f_y = 60,000 psi.

Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

- Example 8 (cont'd)

Following Procedure A outlined earlier,

1. The total design moment is

$$M_u = 1.2M_D + 1.6M_L$$
$$= 1.2(50) + 1.6(100) = 220 \text{ ft-kips}$$

2. Estimate d:

$$d = h - 3 = 25 - 3 = 22 \text{ in.}$$

$$\text{required } R = \frac{M_n}{bd^2} = \frac{(M_u/\phi)}{bd^2} = \frac{(220/0.9)}{(10)(22)^2} \times 12 \times 1000 = 606.06 \text{ psi}$$

Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

- Example 8 (cont'd)

3. From Table 5 (Table A-5 Handout), for R = 606.06 psi and by interpolation,

$$\rho = 0.01172$$

4. Required $A_s = \rho bd = 0.01172\ (10)\ (22) = 2.58 \text{ in}^2$
   Check $A_{s,\min}$. From Table 3,

$$A_{s,\min} = 0.0033bd = 0.0033(10)(22) = 0.73 \text{ in}^2$$

Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

- Example 8 (cont'd)
  - By interpolation:

605.37	0.0117
606.06	ρ
609.71	0.0118

Therefore,

$$\frac{606.06 - 605.37}{609.71 - 605.37} = \frac{\rho - 0.0117}{0.0118 - 0.0117}$$

$$\rho = \underline{0.01172}$$

Table 5 (Table A-5 Handout)

ρ	*R* (psi)
0.0114	592.26
0.0115	596.65
0.0116	601.02
0.0117	605.37
0.0118	609.71
0.0119	614.04
0.0120	618.35
0.0121	622.65
0.0122	626.94
0.0123	631.21
0.0124	635.46

Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

- Example 8 (cont'd)

Table 3
Design Constants

Values used in the example.

f'_c (psi)	$\max\left(\frac{3\sqrt{f'_c}}{f_y}, \frac{200}{f_y}\right)$	ρ_b
f_y = 40,000 psi		
3000	0.0050	0.03712
4000	0.0050	0.04949
5000	0.0053	0.05823
6000	0.0058	0.06551
f_y = 50,000 psi		
3000	0.0040	0.02753
4000	0.0040	0.03671
5000	0.0042	0.04318
6000	0.0046	0.04858
f_y = 60,000 psi		
3000	0.0033	0.02138
4000	0.0033	0.02851
5000	0.0035	0.03354
6000	0.0039	0.03773
f_y = 75,000 psi		
3000	0.0027	0.01552
4000	0.0027	0.02069
5000	0.0028	0.02435
6000	0.0031	0.02739

Check strains:

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{2.58(60)}{0.85(3)(10)} = 6.07'' \Rightarrow c = \frac{a}{\beta_1} = \frac{6.07}{0.85} = 7.14''$$

$$\varepsilon_t = 0.003\left(\frac{d-c}{c}\right) = 0.003\left(\frac{22-7.14}{7.14}\right) = 0.0062 > 0.005 \text{ OK}$$

Hence, the section is tension-controlled, and $\phi = 0.9$

Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

- Example 8 (cont'd)

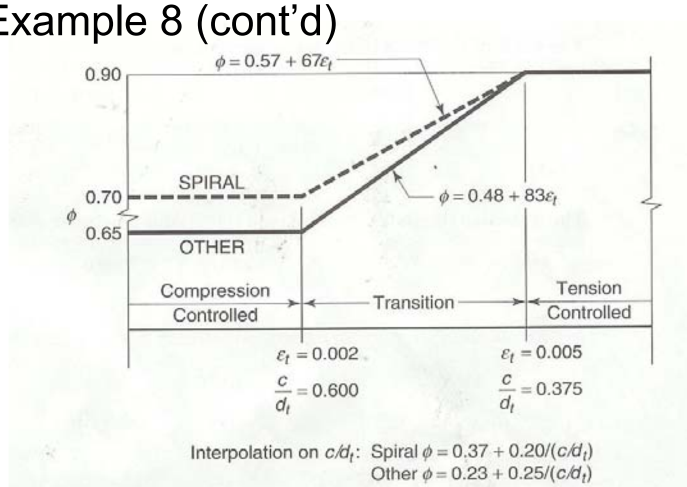


Figure 14. Strain Limit Zones and variation of Strength Reduction Factor ϕ

Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

- Example 8 (cont'd)

5. Select the bars;

   In essence, the the bar or combination of bars that provide 2.58 in^2 of steel area will be satisfactory. From Table 6

   6 No. 6 bars: $A_s = 2.64$ in^2

   9 No. 5 bars: $A_s = 2.79$ in^2

   3 No. 9 bars: $A_s = 3.00$ in^2

   4 No. 8 bars: $A_s = 3.16$ in^2

Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

- Example 8 (cont'd)

Table 6. Areas of Multiple of Reinforcing Bars (in^2)

Number of bars	Bar number								
	#3	#4	#5	#6	#7	#8	#9	#10	#11
1	0.11	0.20	0.31	0.44	0.60	0.79	1.00	1.27	1.56
2	0.22	0.40	0.62	0.88	1.20	1.58	2.00	2.54	3.12
3	0.33	0.60	0.93	1.32	1.80	2.37	3.00	3.81	4.68
4	0.44	0.80	1.24	1.76	2.40	3.16	4.00	5.08	6.24
5	0.55	1.00	1.55	2.20	3.00	3.95	5.00	6.35	7.80
6	0.66	1.20	1.86	2.64	3.60	4.74	6.00	7.62	9.36
7	0.77	1.40	2.17	3.08	4.20	5.53	7.00	8.89	10.92
8	0.88	1.60	2.48	3.52	4.80	6.32	8.00	10.16	12.48
9	0.99	1.80	2.79	3.96	5.40	7.11	9.00	11.43	14.04
10	1.10	2.00	3.10	4.40	6.00	7.90	10.00	12.70	15.60

Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

- Example 8 (cont'd)

The width of beam required for 3 No. 9 bars is 9.5 in. (see Table 7), which is satisfactory. Note that beam width $b = 10$ in.

Check the actual effective depth d:

Actual $d = h -$ cover $-$ stirrup $- d_b/2$

#9 bar. See Table 8

#3 bar for stirrup. See Table 8 for Diameter of bar.

$$25 - 1.5 - 0.38 - \frac{1.128}{2} = 22.6 \text{ in.}$$

The actual effective depth is slightly higher than the estimated one (22 in.). This will put the beam on The safe side (conservative).

Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

- Example 8 (cont'd)

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Table 7. Minimum Required Beam Width, b (in.)

Number of bars	Bar number							
	# 3 and #4	$5	#6	#7	#8	#9	#10	#11
2	6.0	6.0	6.5	6.5	7.0	7.5	8.0	8.0
3	7.5	8.0	8.0	8.5	9.0	9.5	10.5	11.0
4	9.0	9.5	10.0	10.5	11.0	12.0	13.0	14.0
5	10.5	11.0	11.5	12.5	13.0	14.0	15.5	16.5
6	12.0	12.5	13.5	14.0	15.0	16.5	18.0	19.5
7	13.5	14.5	15.0	16.0	17.0	18.5	20.5	22.5
8	15.0	16.0	17.0	18.0	19.0	21.0	23.0	25.0
9	16.5	17.5	18.5	20.0	21.0	23.0	25.5	28.0
10	18.0	19.0	20.5	21.5	23.0	25.5	28.0	31.0

Note that beam width b = 10 in.

Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

- Example 8 (cont'd)

Table 8. Reinforced Steel Properties

Bar number	3	4	5	6	7	8	9	10	11	14	18
Unit weight per foot (lb)	0.376	0.668	1.043	1.502	2.044	2.670	3.400	4.303	5.313	7.650	13.60
Diameter (in.)	0.375	0.500	0.625	0.750	0.875	1.000	1.128	1.270	1.410	1.693	2.257
Area (in^2)	0.11	0.20	0.31	0.44	0.60	0.79	1.00	1.27	1.56	2.25	4.00

Recheck strains:

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{3.0(60)}{0.85(3)(10)} = 7.06'' \Rightarrow c = \frac{a}{\beta_1} = \frac{7.06}{0.85} = 8.31''$$

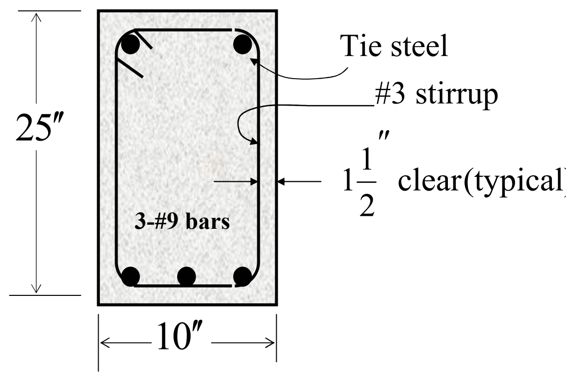
$$\varepsilon_t = 0.003\left(\frac{d-c}{c}\right) = 0.003\left(\frac{22.6-8.31}{8.31}\right) = 0.0052 > 0.005 \text{ OK}$$

Calculate M_u and compare with required:

$$M_u = \phi M_n = \phi A_s f_y\left(d - \frac{a}{2}\right) = 0.9(3)(60)\left(22.6 - \frac{7.06}{2}\right)/12 = 257 \text{ ft-kips} > 220 \text{ ft-kips OK}$$

Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

- Example 8 (cont'd)
  - Final Design Sketch



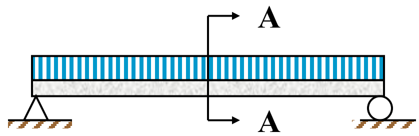
Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

- Example 9

  Design a simply supported rectangular reinforced beam with tension steel only to carry a service load of 0.9 kip/ft and service live load of 2.0 kips/ft. (the dead load does not include the weight of the beam.) The span is 18 ft. Assume No. 3 stirrups. Use f_c' = 4,000 psi and f_y = 60,000 psi

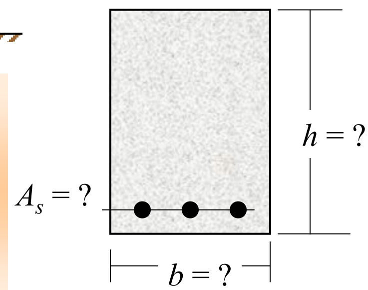
Trial-and-Adjustment Procedures for the Design of Singly Reinforced Beams

- Example 9 (cont'd)



In this problem we have to determine h, b, and A_s. This is called "free design".

This problem can solved according to the outlines of Procedure B presented earlier. This example will be discussed in class for complete solution.

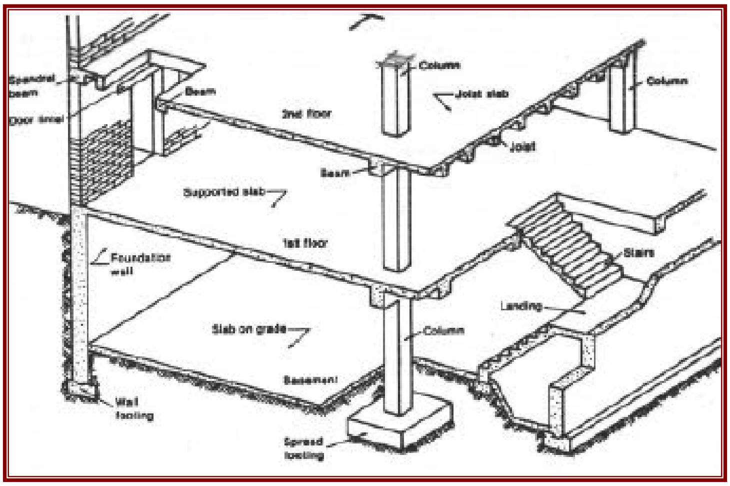


Introduction to Slabs

- Slabs are considered specialized type of bending members.
- They are used both in structural steel and reinforced concrete construction.
- Types of Slabs:
  - One-way Slab
  - Two-way Slab
  - Flat Slab

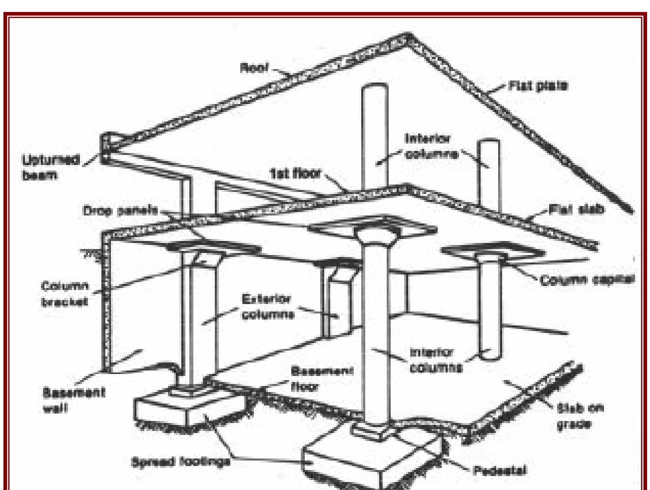
Introduction to Slabs

- Typical Structure (1)



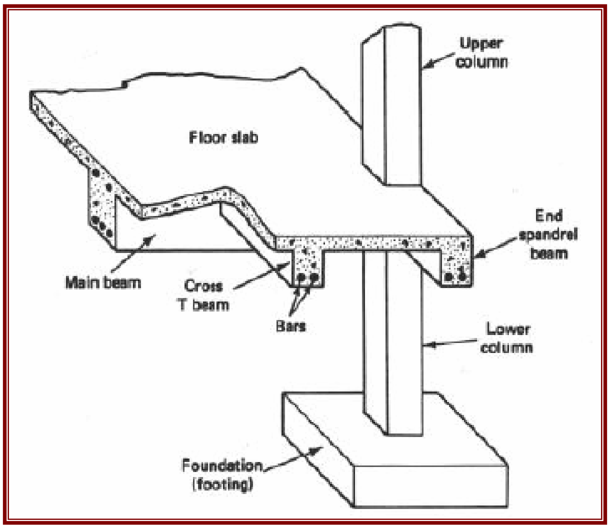
Introduction to Slabs

- Typical Structure (2)



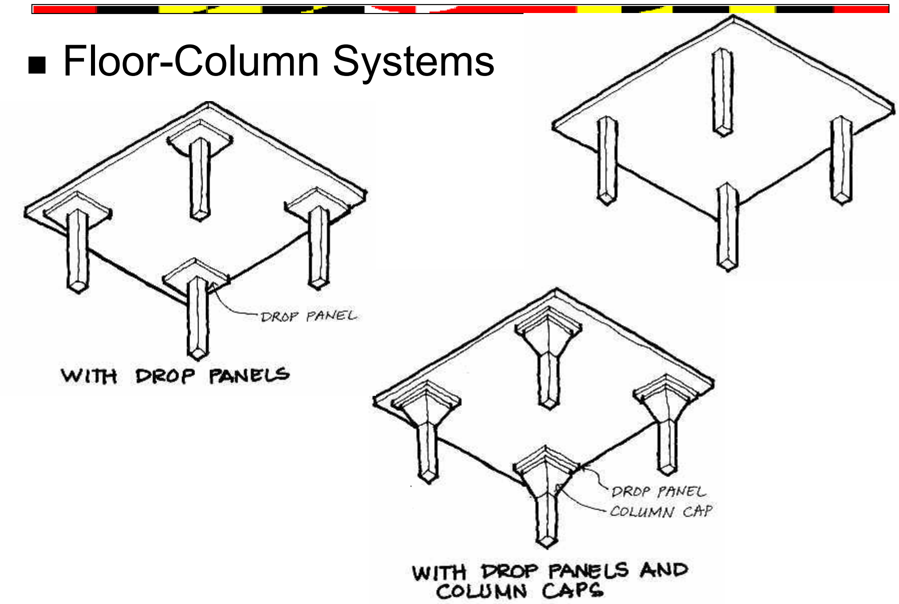
Introduction to Slabs

- Floor-Column Systems



Introduction to Slabs

- Floor-Column Systems



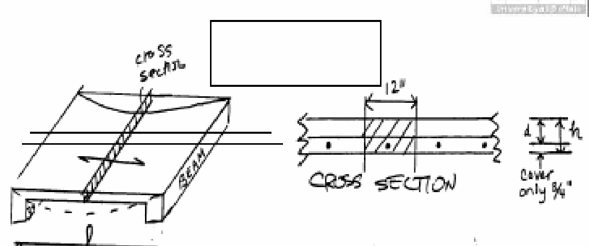
Introduction to Slabs

- One-Way Slab
  - A one-way slab can be defined as a structural reinforced concrete slab supported on two opposite sides so that the bending occurs in one direction only, that is, perpendicular to the supported edges.
  - One-way slabs are concrete floor panels for which the ratio of the long span to the short span equals or exceed a value of 2.0.
  - If this ratio is less than 2.0, then the floor panel becomes a two-way slab (chapter 11).

Introduction to Slabs

- One-Way Slab

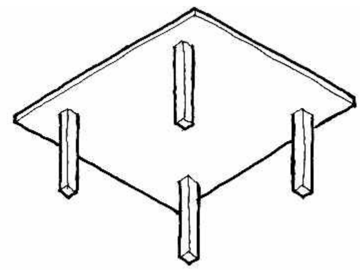
Design of One Way Slabs



Design of one way slabs is like design of parallel 12" beams.

Introduction to Slabs

- Flat Slab
  - A specific type of two-way slab is categorized as a flat slab. A flat slab may be defined as a concrete slab reinforced in two or more directions, generally without beams or girders to transfer the loads to the supporting members.



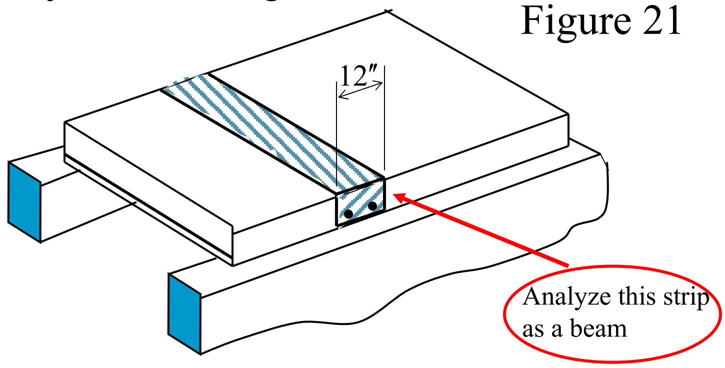
One-Way Slab: Design and Analysis for Flexure

- A one-way slab is assumed to be a rectangular beam with width b = 12 in., as shown in Figure 21.
- When loaded with uniformly distributed load, the slabs deflects so that it has curvature, and therefore bending moment, in only one direction (Figure. 21).

One-Way Slab: Design and Analysis for Flexure

- One-Way Slab Design

Figure 21



The procedure for finding ϕM_n for one-way slab is almost identical to that of a beam with rectangular cross section.

One-Way Slab: Design and Analysis for Flexure

- ACI Code Requirements for Slabs
  - Minimum Steel Area, *As*,min:
    - For grade 40 or 50 steel:

$$A_s = 0.0020bh \qquad (35a)$$

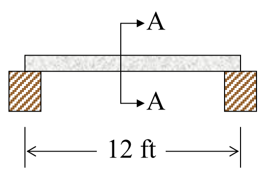
    - For grade 60 steel:

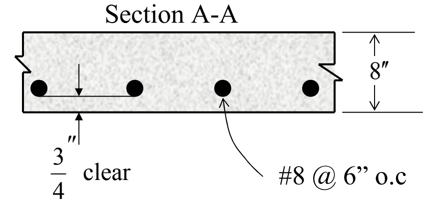
$$A_s = 0.0018bh \qquad (35b)$$

  - Concrete protection:
    - Concrete protection for reinforcement in slabs must be not less than **0.75** in.
    - For surfaces exposed to weather and ground, min. protection is **2** in (#6 to #18) and **1.5** in (#5)

One-Way Slab: Design and Analysis for Flexure

- Example 10
  - The one-way slab shown spans 12 ft from center of the support to the center of support. Calculate ϕM_n and determine the service live load (psf) that the slab may carry. Use f_c' = 3,000 psi and f_y = 40,000 psi.





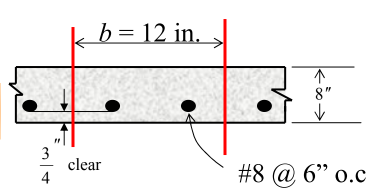
One-Way Slab: Design and Analysis for Flexure

- Example 10 (cont’d)
  - Analyze a 12-in wide strip of slab:
  - For f_c' = 3,000 psi and f_y = 40,000 psi

$$A_{s,\min} = 0.0020(12)(8) = 0.19\ \text{in}^2$$

$$A_s = 2(0.79) = 1.58\ \text{in}^2 > 0.19\ \text{in}^2 \quad \text{OK}$$

$$d = 8 - 0.75 - 0.5 = 6.75\ \text{in.}$$



$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{1.58(40)}{0.85(3)(12)} = 2.07\ \text{in.} \Rightarrow c = \frac{2.07}{0.85} = 2.44\ \text{in.}$$

$$\frac{c}{d_t} = \frac{2.44}{6.75} = 0.361 < 0.375\ \text{(tension controlled)} \quad \text{OK}$$

So use $\phi = 0.9$

One-Way Slab: Design and Analysis for Flexure

Table 1. ASTM Standard - English Reinforcing Bars

Bar Designation	Diameter in	Area in^2	Weight lb/ft
#3 [#10]	0.375	0.11	0.376
#4 [#13]	0.500	0.20	0.668
#5 [#16]	0.625	0.31	1.043
#6 [#19]	0.750	0.44	1.502
#7 [#22]	0.875	0.60	2.044
#8 [#25]	1.000	0.79	2.670
#9 [#29]	1.128	1.00	3.400
#10 [#32]	1.270	1.27	4.303
#11 [#36]	1.410	1.56	5.313
#14 [#43]	1.693	2.25	7.650
#18 [#57]	2.257	4.00	13.60

Note: Metric designations are in brackets

One-Way Slab: Design and Analysis for Flexure

- Example 10 (cont'd)

$$jd = d - \frac{a}{2} = 6.75 - \frac{2.07}{2} = 5.72 \text{ in.}$$

$$M_n = A_s f_y (jd) = \frac{1.58(40)(5.72)}{12} = 30.13 \text{ ft-kips}$$

Therefore,

$$\phi M_n = 0.9(30.13) = 27.1 \text{ ft-kips}$$

$$M_u = \phi M_n = 27.1 = \frac{w_u L^2}{8}$$

$$w_u = \frac{27.1(8)}{L^2} = \frac{27.1(8)}{(12)^2} = 1.51 \text{ k/ft}$$

One-Way Slab: Design and Analysis for Flexure

ACI Code

$$\phi R_n \geq 1.2D + 1.6L$$

- Example 10 (cont'd)

$$w_u = 1.2w_D + 1.6w_L$$

$$w_D = \text{weight of slab} = \frac{8(12)}{144}(0.150) = 0.10 \text{ k/ft}$$

$$1.51 = 1.2(0.10) + 1.6w_L$$

$$1.6w_L = 1.51 - 1.2(0.10)$$

Hence,

$$w_L = \frac{1.51 - 1.2(0.1)}{1.6} = 0.869 \text{ k/ft} = 869 \text{ psf}$$