CHAPTER

10b

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REINFORCED CONCRETE

A Fundamental Approach - Fifth Edition

BOND DEVELOPMENT OF REINFORCING BARS

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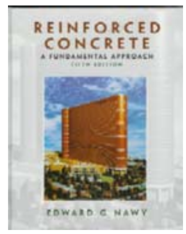

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ENCE 454 – Design of Concrete Structures

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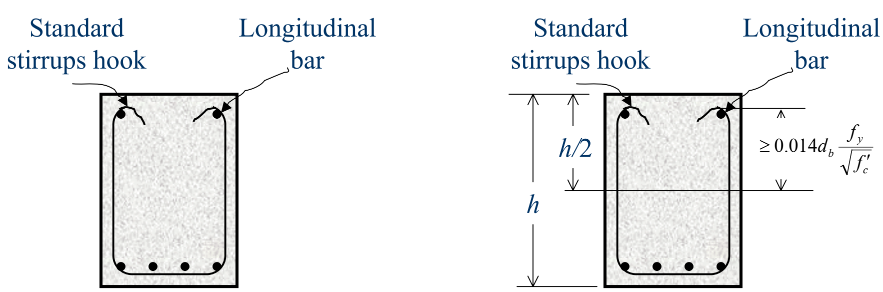
Development of Web Reinforcement

- Anchorage of web reinforcement must be provided in accordance with the ACI Code, Section 12.13.
- Stirrups must be carried as close to the compression and tension surfaces as possible.
- Close proximity to the compression face is necessary since flexural tension cracks penetrate deeply as ultimate load is approach.

Development of Web Reinforcement

- The ACI Code stipulates that ends of single-leg, simple U, or multiple U-stirrups shall be anchored by one of the following means (see Figure 8):
  1. For No. 5 bars and D31 wire and smaller, and for No. 6, 7, and 8 bars with f_y = 40,000 psi or less, a standard hook has to be used around the longitudinal reinforcement.
  2. For Nos. 6, 7, and 8 stirrups with f_y greater than 40,000 psi, anchorage is provided by a

Development of Web Reinforcement



For #5 stirrups and smaller and #6, #7, #8 stirrups with $f_y \leq$ 40,000 psi (ACI 12.13.2.1)

(a)

For #6, #7, #8 stirrups with f_y > 40,000 psi (ACI 12.13.2.2)

(b)

Figure 8. Web Reinforcement Anchorage

Development of Web Reinforcement

standard stirrup hook bent around a longitudinal bar *plus* an embedment between midheight of the member and the outside end of the hook equal to or greater than

$$0.014 d_b \frac{f_y}{\sqrt{f'_c}}$$

where d_b is the nominal diameter of bar or wire.

Development of Web Reinforcement

- A 135° or a 180° hook is preferred, but a 90° hook is acceptable provided that the free end of the hook is extended the full 12d_b as required in ACI Code, Section 7.1.3.
- In addition, the ACI Code, Section 12.13.5, establishes criteria with respect to lapping of double U-stirrups or ties (without hooks) to form a closed stirrup.
- Legs shall be considered properly spliced when lengths of lap are 1.3l_d as shown in Figure 9.

Development of Web Reinforcement

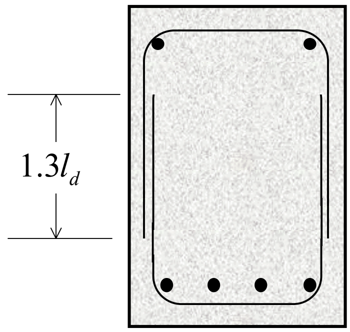


Figure 9. Overlapping U-stirrups to form a closed stirrups

Development of Web Reinforcement

- Each bend of each simple U-stirrup must enclose a longitudinal bar.
- If the lap of 1.3l_d cannot fit within the depth of shallow member (assuming h is at least 18 in.), double U-stirrups may be used if each U-stirrup extends the full available depth h of the member and the force in each leg does not exceed 9000 lb, that is $A_b f_y \leq 9000$ lb.

Splicing of Reinforcement

- The need to splice reinforcing steel is a reality due to the limited lengths of steel available.
- All bars are readily available in lengths up to 60 ft.
- No. 3 and No. 4 bars will tend to bend in handling when longer than 40 ft. Typical stock straight lengths are as follows:

No. 3 bar: 20, 40, 60 ft
No. 4 bar: 30, 40, 60 ft
No. 5 to 18 bars: 60 ft

Splicing of Reinforcement

- A general rule of thumb for maximum bar length is about 40 ft for shipping purposes.
- The most effective means of continuity in reinforcement is to weld the cut pieces without reducing the mechanical or strength properties of the welded bar at the weld.
- However, cost considerations require alternative methods.

Splicing of Reinforcement

- There are basically three types of splicing:
  1. **Lap splicing:** depends on full bond development of the two bars at the lap for bars of size not larger than No. 11.
  2. **Welding by fusion of the two bars at connection:** can become economically justifiable for bar sizes larger than No. 11 bars.

Splicing of Reinforcement

- Types of splices (cont'd)
  3. **Mechanical connecting:** can be achieved by mechanical sleeves threaded on the ends of the bars to be interconnected. Such connectors should have a yield strength at least 1.25 times the yield strength of the bars they interconnect. They are also more commonly used for larger-diameter bars.

Splicing of Reinforcement

- Lap Splicing
  - Splicing may be accomplished by welding, by mechanical means, or most commonly for No. 11 bars and smaller, by lapping bars, as shown in Figure 10.
  - The figure shows the force and stress distribution along the splice length l_s.
  - At failure, one bar slips relative to the other.
  - The idealized tensile stress distribution in the bars along the splice length l_s has a maximum value f_y at the splice end $0.5f_y$ at $l_s/2$.

Splicing of Reinforcement

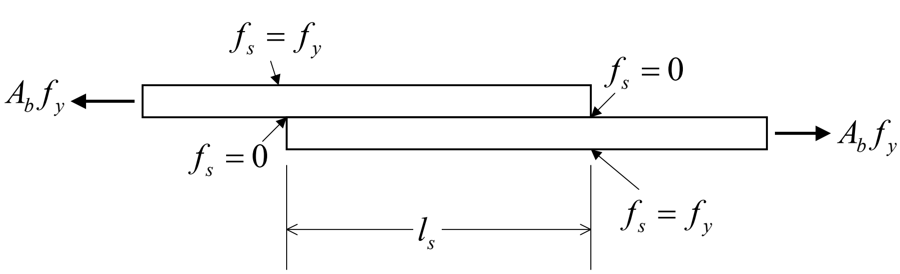


Figure 10. Stress transfer in tension lap splice

Splicing of Reinforcement

- Lap Splicing (cont'd)
  - At failure, the expected magnitude of slip is approximately $(0.5f_y/E_s) \times$ half splice length l_s in Figure 10.
- Tension Splices
  - Two classes of lap splices are specified by the ACI Code. The minimum length l_s, but not less than 12 in., is
    - Class A: $1.0l_d$
    - Class B: $1.3l_d$

Splicing of Reinforcement

- Compression Splices
  - The lap length l_s should be equal to at least the development length in compression as given previously.
  - l_s should also satisfy the following, but not be less than 12 in.:

$$f_y \leq 60{,}000 \text{ psi} \qquad l_s \geq 0.0005 f_y d_b \tag{10a}$$

$$f_y > 60{,}000 \text{ psi} \qquad l_s \geq (0.0009 f_y - 24) d_b \tag{10b}$$

Splicing of Reinforcement

- Compression Splices (cont'd)
  – If the compression strength f_c' of the concrete is less than 3000 psi, such as might occur in foundation, the splice length l_s has to be increased by one-third.
  – Modifying multipliers with values less than 1.0 are used in heavily reinforced tied compression members (**0.85**) and in spirally reinforced columns (**0.75**), but the lap length should not be less than **12 in**.

Simple-Span Bar Cutoffs and Bends

- Determination of Bar Cutoffs
  – Recall that the maximum required *As* for a beam is needed only where the moment is maximum.
  – This maximum steel may be reduced at points along a bending member where the bending moment is smaller.
  – This can be done by either stopping or bending the bars in a manner consistent with the theoretical requirements for the strength of the member and the ACI Code.

Simple-Span Bar Cutoffs and Bends

- Determination of Bar Cutoffs
  - In theory bars can be stopped or bent in bending members whenever they are no longer needed to resist the bending moment.
  - However, the ACI Code requires that each bar be extended beyond the point at which it is no longer needed for flexure a distance equal to the effective depth *d* of the cross section or 12*db*, whichever is greater.

Simple-Span Bar Cutoffs and Bends

- Determination of Bar Cutoffs
  - The ACI code gives the following exceptions to the previous rules:
    1. At supports of simple spans, and
    2. At free ends of cantilever beams.
  - This in effect prohibits the cutting off of a bar at the theoretical cutoff point, but can be bent at the theoretical cutoff point.
  - If bars are to be bent, it is common to start the bend at a distance equal to one-half the effective depth beyond the point.

Simple-Span Bar Cutoffs and Bends

- General Procedure for Determining the Theoretical Cutoff Point
  1. Establish a bar cutoff scheme (i.e., select the bars that will be cut off first).
  2. Plot the complete M_u diagram.
  3. Superimpose on the M_u diagram the values of ϕM_n corresponding to the bars of Step 1 that will not be stopped.
  4. The theoretical points are established where the ϕM_n lines intersect the M_u curve.

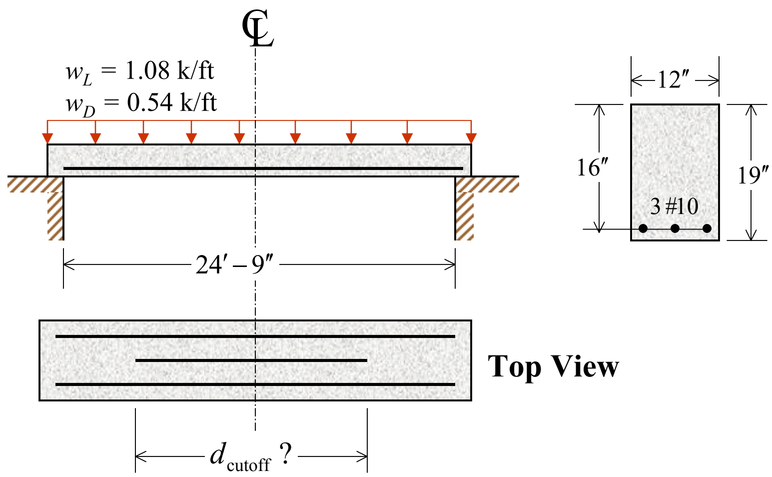
Simple-Span Bar Cutoffs and Bends

- Example 3: Bar Cutoff Point

  For the simply supported beam shown in the figure, determine the theoretical and actual cutoff point for the center No. 10 bar. The beam is to carry a distributed dead load of 0.72 kips/ft including its own weight, and live service load of 1.08 kips/ft. Material strengths specified are f_c' = 4,000 psi and f_y = 60,000 psi.

Simple-Span Bar Cutoffs and Bends

- Example 3 (cont'd):



Simple-Span Bar Cutoffs and Bends

- Example 3 (cont'd):

Determine factored distributed load:

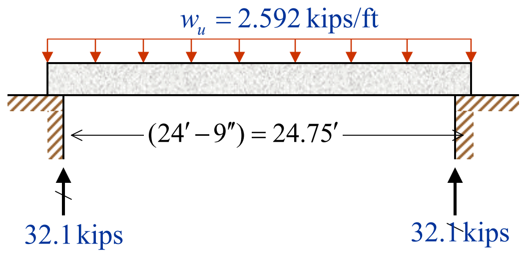
$$w_u = 1.2(0.72) + 1.6(1.08) = 2.592 \text{ kips/ft}$$

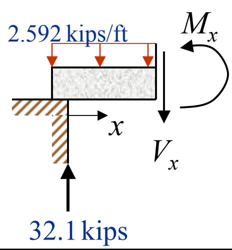
1. Bar cutoff scheme has been established for the center No. 10 bar.
2. **Plot of the complete M_u diagram:**

   In order to do that, we have to find an expression for M_u based on the loading

Simple-Span Bar Cutoffs and Bends

- Example 3 (cont'd):



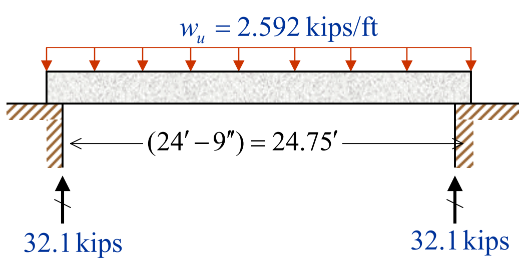


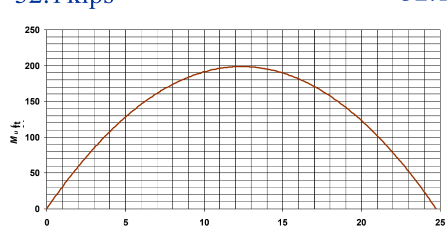
$$\boxed{M_x = 32.1x - 2.592\frac{x^2}{2} = 32.1x - 1.296x^2} \quad (11)$$

Note: $M_x = M_u(x)$

Simple-Span Bar Cutoffs and Bends

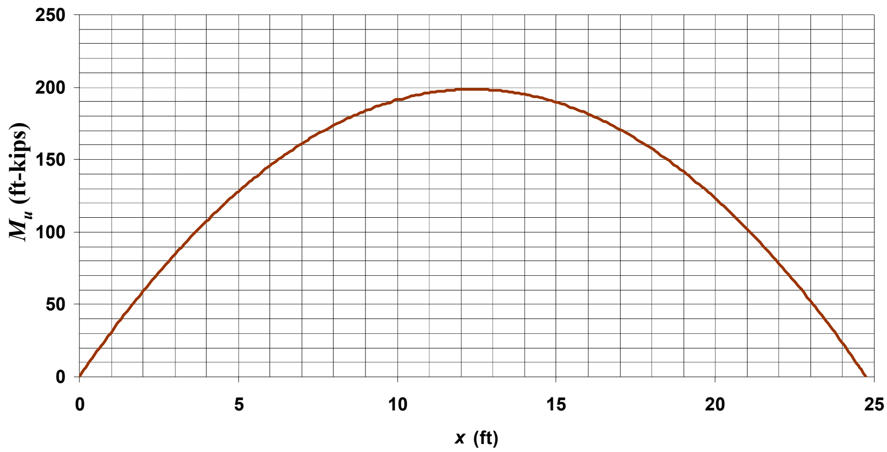
- Example 3 (cont'd):
  - Thus, the plot of M_u will appear as follows:





Simple-Span Bar Cutoffs and Bends

- Example 3 (cont'd):



Simple-Span Bar Cutoffs and Bends

- Example 3 (cont'd):
  3. **Superimpose on the M_u diagram the values of ϕM_u corresponding to 2 No. 10 bars:**

ϕM_n for 2 #10 bars:

$$\rho = \frac{A_s}{bd} = \frac{2.54}{12(16)} = 0.0132$$

From Table 5, R corresponding to 0.0132 is

$$R = 699.76 \text{ psi} = 0.6998 \text{ ksi}$$

$$A_{s,\min} = 0.0033(b)(d) = 0.0033(12)(16) = 0.63 \text{ in}^2 < 2.54 \text{ in}^2 \text{ OK}$$

Simple-Span Bar Cutoffs and Bends

- Example 3 (cont'd):

Table 2. Areas of Multiple of Reinforcing Bars (in^2)

Number of bars	Bar number								
	#3	#4	$5	#6	#7	#8	#9	#10	#11
1	0.11	0.20	0.31	0.44	0.60	0.79	1.00	1.27	1.56
2	0.22	0.40	0.62	0.88	1.20	1.58	2.00	2.54	3.12
3	0.33	0.60	0.93	1.32	1.80	2.37	3.00	3.81	4.68
4	0.44	0.80	1.24	1.76	2.40	3.16	4.00	5.08	6.24
5	0.55	1.00	1.55	2.20	3.00	3.95	5.00	6.35	7.80
6	0.66	1.20	1.86	2.64	3.60	4.74	6.00	7.62	9.36
7	0.77	1.40	2.17	3.08	4.20	5.53	7.00	8.89	10.92
8	0.88	1.60	2.48	3.52	4.80	6.32	8.00	10.16	12.48
9	0.99	1.80	2.79	3.96	5.40	7.11	9.00	11.43	14.04
10	1.10	2.00	3.10	4.40	6.00	7.90	10.00	12.70	15.60

Simple-Span Bar Cutoffs and Bends

- Example 3 (cont'd):

ρ	R (psi)
0.0128	681.26
0.0129	685.90
0.0130	690.53
0.0131	695.15
0.0132	699.76
0.0133	704.35
0.0134	708.94
0.0135	713.51

Table 5 (Table A6, Handout)

Simple-Span Bar Cutoffs and Bends

- Example 3 (cont'd):

$$\phi M_n = \phi b d^2 R = \frac{0.9(12)(16)^2(0.6998)}{12} = 161.2 \text{ ft-kips}$$

- The line ϕM_n = 161.2 intersects the curve of M_u at 7 in. and 17.7 in. Therefore, the theoretical cutoff point is located 7 ft. from the face of either support.
- The actual cutoff point:

$$d = 16'' = 1.33' \text{ and } 12d_b = 12(1.27) = 15.24'' = 1.27'$$

controls — Dia. No. 10 bar

Hence,

actual cutoff point = 7 - 1.33 = 5.7 ft from F.O.S

Simple-Span Bar Cutoffs and Bends

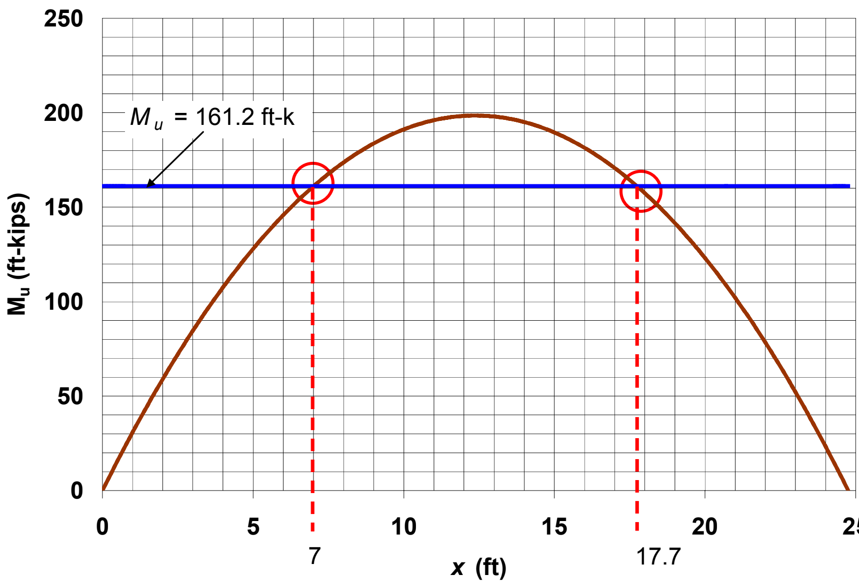
- Example 3 (cont'd):

Table 3. ASTM Standard - English Reinforcing Bars

Bar Designation	Diameter in	Area in²	Weight lb/ft
#3 [#10]	0.375	0.11	0.376
#4 [#13]	0.500	0.20	0.668
#5 [#16]	0.625	0.31	1.043
#6 [#19]	0.750	0.44	1.502
#7 [#22]	0.875	0.60	2.044
#8 [#25]	1.000	0.79	2.670
#9 [#29]	1.128	1.00	3.400
#10 [#32]	1.270	1.27	4.303
#11 [#36]	1.410	1.56	5.313
#14 [#43]	1.693	2.25	7.650
#18 [#57]	2.257	4.00	13.60

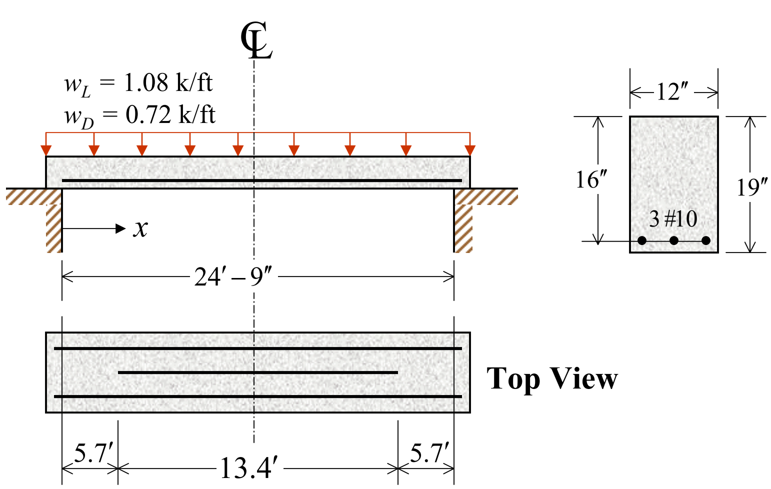
Note: Metric designations are in brackets

Simple-Span Bar Cutoffs and Bends



Simple-Span Bar Cutoffs and Bends

- Example 3 (cont'd):



Simple-Span Bar Cutoffs and Bends

- Example 3 (cont'd):
  - Alternative Method to Find the Theoretical Cutoff Point:
    - In method, the ϕM_n value for the continuous reinforcement can be substituted into Eq. 11, and consequently the distances from the face of the right support can be located analytically as follows:

$$\phi M_n = 161.2 = 32.1x - 1.296x^2$$

$$\text{or} \quad 1.296x^2 - 32.1x + 161.2 = 0$$

from which (quadratic formula):

$$x = \frac{32.1 \pm \sqrt{(32.1)^2 - 4(1.296)161.2}}{2(1.296)} = 7.0 \text{ in}, 17.7 \text{ in}$$

Code Requirements for Development of Positive Moment Steel

- The ACI Code, Section 12.11.3, contains requirements concerning the development of straight, positive moment bars at simple supports and at points of inflection.
- The intend of this code section is the same as the check on the development of the two bars extending into the support.
- The code approach does not require the use of a moment diagram.

Code Requirements for Development of Positive Moment Steel

- The ACI Code requirement places a restriction on the size of the bar that may be used such that

$$l_d \le \left(\frac{M_n}{V_n}\right) + l_a \qquad (12)$$

M_n	= nominal moment strength $\left[A_s f_y\left(d-\frac{a}{2}\right)\right]$, assuming all reinforcement at the section to be stressed to f_y
V_u	= total applied design shear force at the section
l_a (inflection point)	= the effective depth of the member or $12d_b$, whichever is greater
l_a (at support)	= the embedment length beyond the center of the support

Code Requirements for Development of Positive Moment Steel

- The effect of this code restriction is to require that bars be small enough so that they can become fully developed before the applied moment has increased to the magnitude where they must be capable of carrying f_y.
- The term M_n/V_u approximates the distance from the section in question to the location where applied M_u exists that is equal to ϕM_n.

Code Requirements for Development of Positive Moment Steel

- The distance from the end of the bar to the point where the bar must be fully developed is

$$\left(\frac{M_n}{V_n}\right) + l_a$$

- The bar must be chosen so that their l_d is less than this distance.

Code Requirements for Development of Positive Moment Steel

- The code allows M_n/V_u to be increased by **30%** when the ends of the reinforcement are confined by a compressive reaction such as is found in a simply supported beam, i.e., a beam supported by a wall.

Code Requirements for Development of Positive Moment Steel

- Example 4

At the support of a simply supported beam, a cross section as shown in the figure. Check the bar size (diameter) in accordance with the ACI Code. Assume a support width of 12 in., 1.5-in. cover, f_c' = 4000 psi, and f_y = 60,000 psi. Assume that V_u at the support is 80 kips. Normal-weight concrete is used. The stirrups begin at 3 in. from the face of the support. The bars are uncoated.

To be discussed and solved in class.

Code Requirements for Development of Positive Moment Steel

- Example 4 (cont'd)