REINFORCED CONCRETE

A Fundamental Approach - Fifth Edition

CHAPTER

10a

BOND DEVELOPMENT OF REINFORCING BARS

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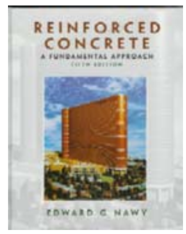

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ENCE 454 – Design of Concrete Structures

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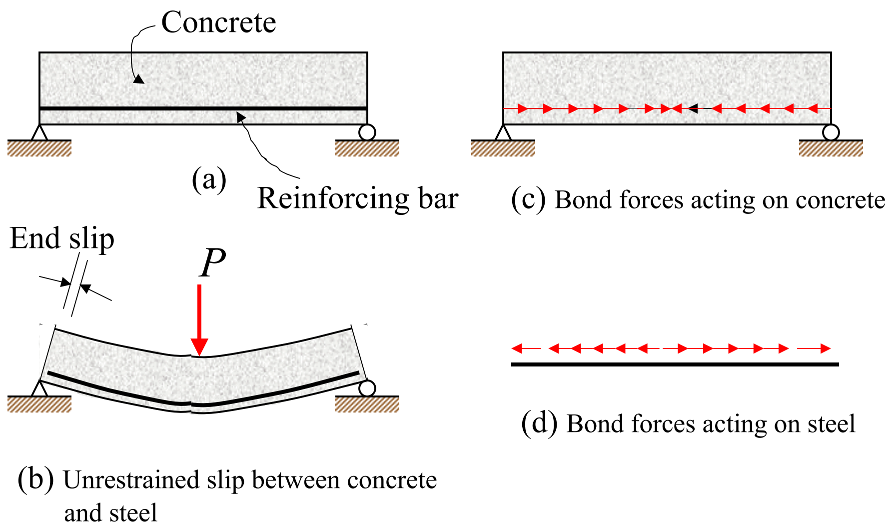
Introduction

- If the reinforced concrete beam shown in Figure 1 were constructed using plain round reinforcing bars, and in addition, if those bars were to be greased or otherwise lubricated before the concrete were poured, the beam would be as strong as it was made of plain concrete, without reinforcement.

Introduction

Figure. 1. Bond Stresses due to Flexure



Introduction

- If a load is applied as shown Figure 1b, the bars would tend to maintain its original length as the beam deflects.
- The bars would slip longitudinally with respect to adjacent concrete, which would experience tensile strain due to flexure.
- The assumption that the strain in an embedded reinforcing bar is the same as that in surrounding concrete, would not be valid.

Introduction

- In order for reinforced concrete to behave as intended, it is essential that "**bond forces**" be developed on the interface between concrete and steel, such as to prevent significant slip from occurring at the interface.
- It is through the action of these interface bond forces that the slip of Figure 5b is prevented.

Introduction

- The assumptions for the design of reinforced concrete include:
  1. Perfect bonding between the concrete and steel exist, and
  2. No slippage occur.
- Based on these assumptions, it follows that some form of bond stress exists at the contact surface between the concrete and steel bars.

Introduction

- In beams, this bond stress is caused by the change in bending moment along the length of the beam and the accompanying change in the tensile stress in the bars (flexural bond).
- The actual distribution of bond stresses along the reinforcing steel is highly complex, due mainly to the presence of concrete cracks.

Introduction

- Large local variations in bond stress are caused by flexural and diagonal cracks.
- High bond stresses have been measured adjacent to these cracks.
- The high bond stress may result in:
  - Small local slips adjacent to the crack.
  - Increased deflection.
- In general, this is harmless as long as failure does not propagate all along the bar with complete loss of bond.

Introduction

- Basic Development Length
  - End anchorage may be considered reliable if the bar is embedded into concrete a prescribed distance known as the "**development length**" of the bar.
  - In a beam, if the actual extended length of the bar is equal or greater than this required development length, then no bond failure will occur.

Introduction

- Basic Development Length

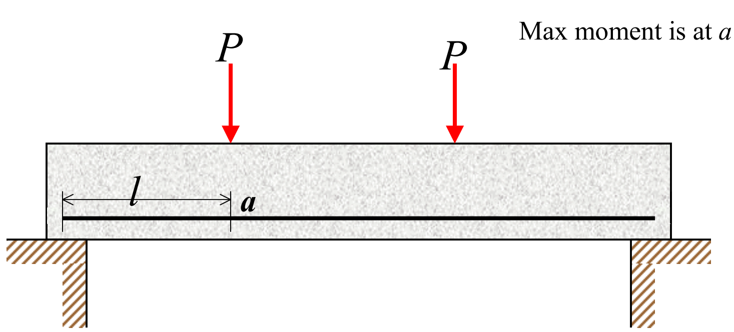
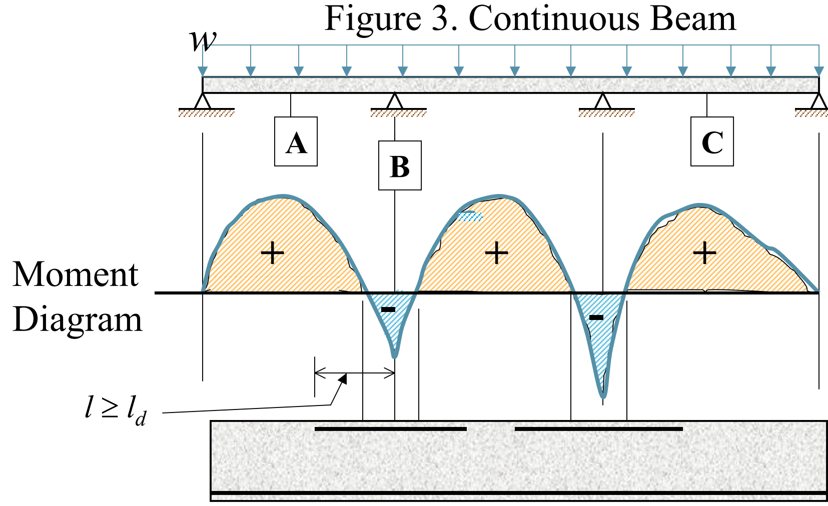


Figure 2. Development length
l should be at least equal to l_b

Introduction

- Need for Development Length

Figure 3. Continuous Beam



Introduction

- Anchorages Versus Development Length
  - If the actual available length is inadequate for full development, special anchorages ,such as hooks, must be provided to ensure adequate strength.

Introduction

- ACI-318 Code
  - The provisions of the ACI Code are directed toward providing adequate length of embedment, past the location at which the bar is fully stressed, which will ensure development of the full strength of the bar.
  - Therefore, the current method based on ACI disregard high localized bond stress even though it may result in localized slip between steel and concrete adjacent to the cracks.

Development of Bars in Tension

- Methods for Determining the Development Length, l_d
  - The ACI allows the determination of the development length by two methods:
    - Tabular criteria (ACI Section 12.2.2).
    - General equation (ACI Section 12.2.3).
  - In either case, l_d shall not be less than 12 in.
  - The general equation of the ACI Code offers a simple approach that allows the user to see the effect of all variables controlling the development length.

Development of Bars in Tension

- Methods for Determining the Development Length, l_d (cont'd)
  - This equation (ACI Eq. 12-1) is provided in Section 12.2.3 of the ACI Code, and it is as follows:

$$l_d = \frac{3}{40}\left(\frac{f_y}{\sqrt{f_c'}}\right)\left[\frac{\alpha\beta\gamma\lambda}{\left(\frac{c+k_{tr}}{d_b}\right)}\right]d_b \qquad (1)$$

Development of Bars in Tension

- Notations of Eq. 1:

$(c + k_{tr})/d_b$: shall not be taken greater than 2.5

l_d = development length (in.)

f_y = yield strength of nonprestressed reinforcement (psi)

= compressive strength of concrete (psi); the value of $\sqrt{f_c'}$ shall not exceed 100 psi (ACI Code, Section 12.1.2)

d_b = nominal diameter of bar or wire (in.)

Development of Bars in Tension

- Modifying Multipliers for Eq. 1:
  1. α is a reinforcement location factor that accounts for the position of the reinforcement in freshly place concrete.

     α = 1.3 (ACI Code, Section 12.2.4) where horizontal reinforcement is so placed that more than 12 in. of fresh concrete is cast in member below the development length or splice.

     α = 1.0 for other reinforcement.
  2. β is a coating factor reflecting the effects of epoxy coating.

     For epoxy-coated reinforcement having cover less than $3d_b$ or clear spacing between bars less than $6d_b$, use **β = 1.5**

Development of Bars in Tension

- Modifying Multipliers for Eq. 1 (cont'd):

     For all other conditions, use **β = 1.2**

     For uncoated reinforcement, use **β = 1.0**

     The product of α and β need not be taken greater than **1.7** (ACI Code, Section 12.2.4)
  3. γ is a reinforcement size factor.

     Where No. 6 and smaller bars are used, **γ = 0.8**

     Where No. 7 and larger bars used, **γ = 0.1**
  4. λ is a lightweight-aggregate concrete factor.

     For lightweight-aggregate concrete when the average splitting tensile strength f_{ct} is not specified, use **λ = 1.3**

Development of Bars in Tension

- Modifying Multipliers for Eq. 1 (cont'd):

When f_{ct} is specified, use

$$\lambda = 6.7\frac{\sqrt{f'_c}}{f_{ct}} \geq 1.0$$

When normal-weight concrete is used, λ = **1.0** (ACI Code, Section 12.2.4)

5. c represents a spacing or cover dimension (in.)

   The value of c will be the smaller of either the distance from the center of the bar to the nearest concrete cover (surface) or one-half the center-to-center spacing of the bars being developed (spacing).

Development of Bars in Tension

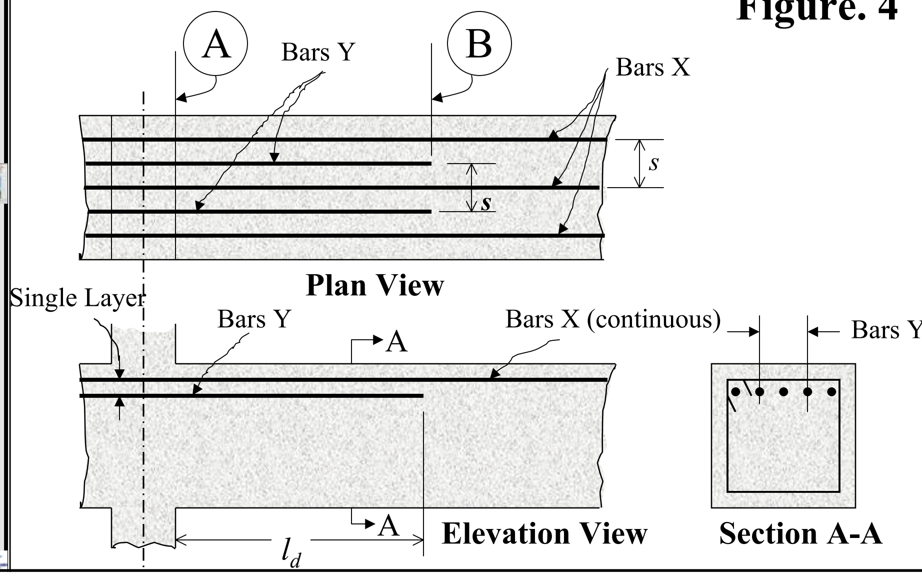
- Modifying Multipliers for Eq. 1 (cont'd):

The bar spacing will be the actual center-to-center spacing between the bars if adjacent bars are all being developed at the same location. If, however, an adjacent bar has been developed at another location, the spacing to be used will be greater than the actual spacing to the adjacent bar.

Note in Figure 4 that the spacing for bars Y may be taken the same as for bars X, since bars Y are developed in length AB, whereas bars X are developed at a location other than AB.

Development of Bars in Tension

Figure. 4



Development of Bars in Tension

- Modifying Multipliers for Eq. 1 (cont'd):

  6. The transverse reinforcement index K_{tr} is to be calculated from

$$K_{tr} = \frac{A_{tr} f_{yt}}{1500sn}$$

where

A_{tr} = total cross-sectional area of all transverse reinforcement that is within the spacing s and that crosses the potential plane of splitting through the reinforcement being developed (in^2)

f_{yt} = yield strength of transverse reinforcement (psi)

Development of Bars in Tension

- Modifying Multipliers for Eq. 1 (cont'd):

s = maximum center-to-center spacing of transverse reinforcement within the development length l_d (in.)

n = number of bars or wires being developed along the plane of splitting.

Development of Bars in Tension

- Reduction in Development Length
  - A reduction in the development length l_d is permitted where reinforcement is in excess of that required by analysis (except where anchorage or development for f_y is specifically required or where the design includes provisions for seismic considerations).
  - The reduction factor K_{ER} is given by

$$K_{ER} = \frac{A_s \text{ required}}{A_s \text{ provided}} \qquad (2)$$

Procedure for l_d Calculation

- Determine multiplying factors (use 1.0 unless otherwise determined).
  - Use α = 1.3 for top reinforcement, when applicable.
  - Coating factor β applies to epoxy-coated bars. Determine cover and clear spacing as multiples of *db*. Use β = 1.5 if cover < 3*db* or clear space < 6*db*. Use β = 1.2 otherwise.
  - Use γ = 0.8 for No. 6 bars and smaller.
  - Use λ = 1.3 for lightweight concrete with f_{ct} not specified. Use $\lambda = 6.7\dfrac{\sqrt{f'_c}}{f_{ct}} \geq 1.0$ if f_{ct} specified.

Procedure for l_d Calculation

2. Check $\alpha\beta \leq 1.7$.
3. Determine c, the smaller of cover or half-spacing (both referenced to the center of the bar).
4. Calculate

$$K_{tr} = \frac{A_{tr} f_y}{1500 sn}, \text{ or use } K_{tr} = 0 \text{ (conservative)}$$

5. Check

$$\frac{c + K_{tr}}{d_b} \leq 2.5$$

Procedure for l_d Calculation

6. Calculate K_{ER} if applicable:

$$K_{ER} = \frac{A_s \text{ required}}{A_s \text{ provided}}$$

7. Calculate l_d from Eq. 1 (ACI Code Eq. 12-1):

$$l_d = \frac{3}{40}\left(\frac{f_y}{\sqrt{f_c'}}\right)\left[\frac{\alpha\beta\gamma\lambda}{\left(\frac{c+k_{tr}}{d_b}\right)}\right]d_b$$

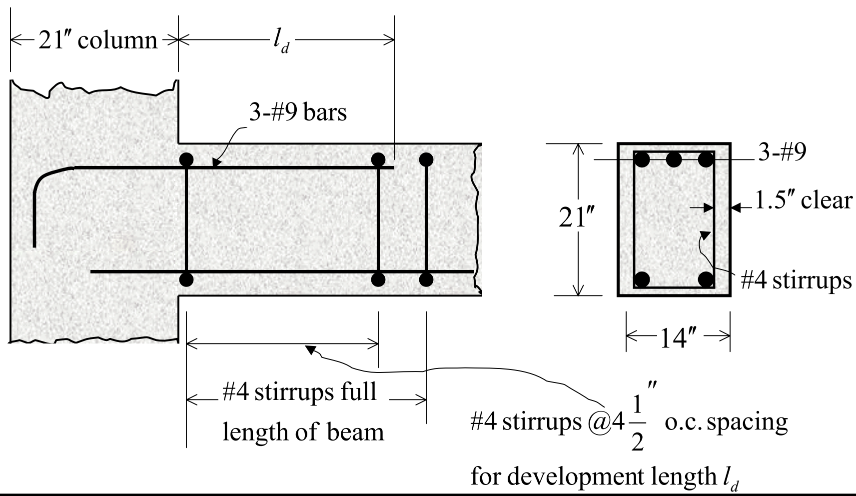
Procedure for l_d Calculation

- Example 1

  Calculate the required development length l_d into the beam for the negative moment steel shown so as to develop the tensile strength of the steel at the face of the column. Required A_s = 2.75 in^2, f_c' =4,000 psi, and f_y = 60,000 psi. Assume normal-weight concrete.

Procedure for l_d Calculation

- Example 1 (cont'd)



Procedure for l_d Calculation

- Example 1 (cont'd)

3#9 bars: $d_b = 1.128''$ ← From Table 1

(1) $\alpha = 1.3,\ \beta = 1.0,\ \gamma = 1.0,\ \text{and}\ \lambda = 1.0$

(2) $\alpha\beta = (1.3)(1) = 1.3 < 1.7$ OK

(3) cover: $c = 1.5 + 0.5 + \frac{1.128}{2} = 2.56''$

Dia. #4 stirrup (→ 0.5)

Half-spacing: $c = \frac{14 - 2(1.5) - 2(0.5) - 1.128}{2(2)} = 2.22''$ ← Controls

(4) $K_{tr} = \frac{A_{tr} f_{yt}}{1500sn} = \frac{0.4(60{,}000)}{1500(4.5)(3)} = 1.185$

Area of 2 #4 stirrups (→ 0.4)

Procedure for l_d Calculation

- Example 1 (cont'd)

Table 1. ASTM Standard - English Reinforcing Bars

Bar Designation	Diameter in	Area in^2	Weight lb/ft
#3 [#10]	0.375	0.11	0.376
#4 [#13]	0.500	0.20	0.668
#5 [#16]	0.625	0.31	1.043
#6 [#19]	0.750	0.44	1.502
#7 [#22]	0.875	0.60	2.044
#8 [#25]	1.000	0.79	2.670
#9 [#29]	1.128	1.00	3.400
#10 [#32]	1.270	1.27	4.303
#11 [#36]	1.410	1.56	5.313
#14 [#43]	1.693	2.25	7.650
#18 [#57]	2.257	4.00	13.60

Note: Metric designations are in brackets

Procedure for l_d Calculation

- Example 1 (cont'd)

Table 2. Areas of Multiple of Reinforcing Bars (in^2)

Number of bars	Bar number								
	#3	#4	$5	#6	#7	#8	#9	#10	#11
1	0.11	0.20	0.31	0.44	0.60	0.79	1.00	1.27	1.56
2	0.22	0.40	0.62	0.88	1.20	1.58	2.00	2.54	3.12
3	0.33	0.60	0.93	1.32	1.80	2.37	3.00	3.81	4.68
4	0.44	0.80	1.24	1.76	2.40	3.16	4.00	5.08	6.24
5	0.55	1.00	1.55	2.20	3.00	3.95	5.00	6.35	7.80
6	0.66	1.20	1.86	2.64	3.60	4.74	6.00	7.62	9.36
7	0.77	1.40	2.17	3.08	4.20	5.53	7.00	8.89	10.92
8	0.88	1.60	2.48	3.52	4.80	6.32	8.00	10.16	12.48
9	0.99	1.80	2.79	3.96	5.40	7.11	9.00	11.43	14.04
10	1.10	2.00	3.10	4.40	6.00	7.90	10.00	12.70	15.60

Procedure for l_d Calculation

- Example 1 (cont'd)

$$(5)\quad \frac{c+K_{tr}}{d_b}=\frac{2.22+1.185}{1.128}=3.02>2.5,\ \text{Therefore, use } 2.5$$

$$(6)\quad K_{ER}=\frac{A_s \text{ required}}{A_s \text{ provided}}=\frac{2.75}{3.00}=0.917$$

(7) Calculate the development length l_d using Eq. 1:

$$l_d=\frac{3}{40}\left(\frac{f_y}{\sqrt{f'_c}}\right)\left[\frac{\alpha\beta\gamma\lambda}{\left(\frac{c+k_{tr}}{d_b}\right)}\right]d_b$$

Procedure for l_d Calculation

- Example 1 (cont'd)

Reduction factor

$$l_d=K_{ER}\times\frac{3}{40}\left(\frac{f_y}{\sqrt{f'_c}}\right)\left[\frac{\alpha\beta\gamma\lambda}{\left(\frac{c+k_{tr}}{d_b}\right)}\right]d_b$$

$$l_d=0.917\times\frac{3}{40}\left(\frac{60{,}000}{\sqrt{4{,}000}}\right)\left[\frac{1.3(1)(1)(1)}{2.5}\right](1.128)=38.3''$$

38.3 in. > 12 in OK

Development of Bars in Compression

- Deformed Bars in Compression
  - The method for determining the development length in compression l_d involves finding the the basic development length l_{db} and multiplying it by applicable modification factors.
  - The modification factors reflect special conditions.
  - Note: l_d shall not be less than 8 in.

Development of Bars in Compression

- Basic Development Length (compression)

  The basic development length in compression is given by

$$l_{db} = 0.02 d_b \frac{f_y}{\sqrt{f'_c}} \qquad (3)$$

  But it shouldn't be less than $0.0003 f_y d_b$ according to the ACI Code, Section 12.3.

Development of Bars in Compression

- Modification Factors (Compression)
  - The following modification factors may be applied to the basic development length for compression bars:
    - Reinforcement in excess of that required:

$$\frac{A_s \text{ required}}{A_s \text{ provided}}$$

    - Bars enclosed within a spiral that is not less than ¼ in. in diameter and not more than 4 in. in pitch or within No. 4 ties and spaced at not more than 4 in. on center: USE 0.75

Development of Bars in Compression

- Tables 3a through 3c gives values of the basic development length l_{db} for compression bars in inches for the following combinations of f_c' and f_y:

f_c': 3000, 4000, 5000, and 6000 psi

f_y: 40,000, 50,000, and 60,000 psi

Development of Bars in Compression

Table 3a. Basic Development Length l_{db} for Compression Bars (in.) for f_y = 40,000 psi

Bar Size	f_c' (normal-weight concrete), psi			
	3000	4000	5000	6000
3	5.5	4.7	4.5	4.5
4	7.3	6.3	6.0	6.0
5	9.1	7.9	7.5	7.5
6	11.0	9.5	9.0	9.0
7	12.8	11.1	10.5	10.5
8	14.6	12.6	12.0	12.0
9	16.5	14.3	13.5	13.5
10	18.5	16.1	15.2	15.2
11	20.6	17.8	16.9	16.9
14	24.7	21.4	20.3	20.3
18	33.0	28.5	27.1	27.1

Development of Bars in Compression

Table 3b. Basic Development Length l_{db} for Compression Bars (in.) for f_y = 50,000 psi

Bar Size	f_c' (normal-weight concrete), psi			
	3000	4000	5000	6000
3	6.8	5.9	5.6	5.6
4	9.1	7.9	7.5	7.5
5	11.4	9.9	9.4	9.4
6	13.7	11.9	11.3	11.3
7	16.0	13.8	13.1	13.1
8	18.3	15.8	15.0	15.0
9	20.6	17.8	16.9	16.9
10	23.2	20.1	19.1	19.1
11	25.7	22.3	21.2	21.2
14	30.9	26.8	25.4	25.4
18	41.2	35.7	33.9	33.9

Development of Bars in Compression

Table 3c. Basic Development Length l_{db} for Compression Bars (in.) for f_y = 60,000 psi

Bar Size	f'_c (normal-weight concrete), psi			
	3000	**4000**	**5000**	**6000**
3	8.2	7.1	6.8	6.8
4	11.0	9.5	9.0	9.0
5	13.7	11.9	11.3	11.3
6	16.4	14.2	13.5	13.5
7	19.2	16.6	15.8	15.8
8	21.9	19.0	18.0	18.0
9	24.7	21.4	20.3	20.3
10	27.8	24.1	22.9	22.9
11	30.9	26.8	25.4	25.4
14	37.1	32.1	30.5	30.5
18	49.4	42.8	40.6	40.6

Mechanical Anchorage and Hooks

- Need for Hooks
  - In the event that the desired development length in tension cannot be furnished, it will be necessary to provide mechanical anchorage at the end of the bars.

Mechanical Anchorage and Hooks

- Need for Hooks

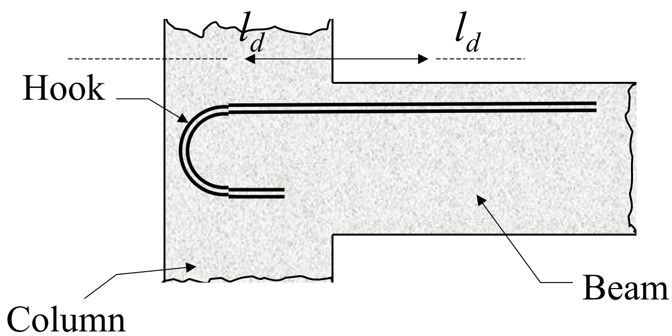


Figure 5. 180°-Hook

Mechanical Anchorage and Hooks

- Types of Hooks
  - Anchorage for main or primary reinforcement is usually accomplished by means of 90˚ or 180˚ hook.
  - The dimensions and bend radii for these hooks have been standardized by the ACI Code.
  - Standard reinforcement hooks are shown in Figure 6.

Mechanical Anchorage and Hooks

- Types of Hooks

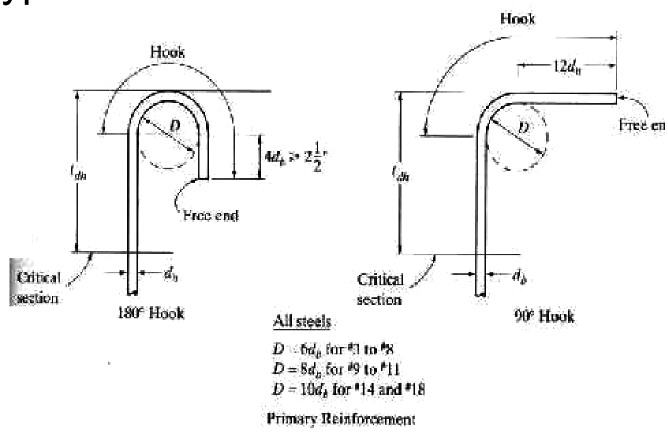


Figure 5a. Standard Hooks

Mechanical Anchorage and Hooks

- Types of Hooks

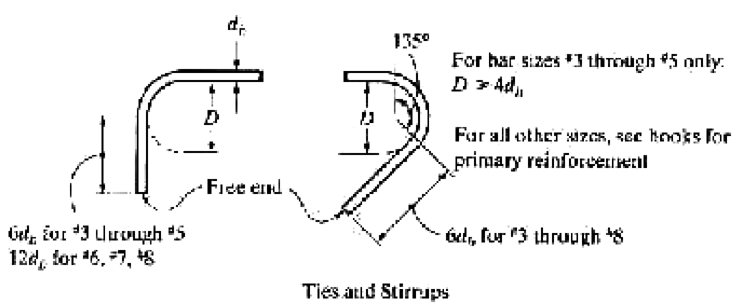


Figure 5b. Standard Hooks

Mechanical Anchorage and Hooks

- Types of Hooks

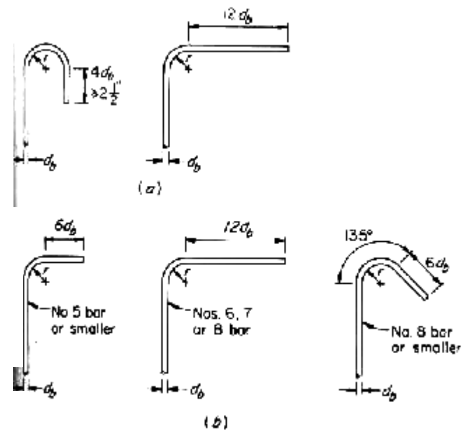


Figure 6. Standard Hooks

Mechanical Anchorage and Hooks

- ACI Code Specifications
  - The ACI Code specifies that the development length l_{dh} (see Fig. 5) for deformed bars in tension, which terminate in a standard hook, be computed as the product of a basic development length l_{hb} and any applicable modification factors.
  - Mathematically, this may expressed as

$$l_{dh} = l_{hb} \times \text{MF} \qquad (4)$$

Mechanical Anchorage and Hooks

- ACI Basic Development Length, l_{hb}
  - For a hooked bar with f_y = 60,000 psi,

$$l_{hb} = \frac{1200 d_b}{\sqrt{f'_c}} \qquad (5)$$

  - Table 4 provides values for l_{hb}.

Mechanical Anchorage and Hooks

Table 4a. Basic Development Length l_{hb} for Hooked Bars (in.) with f_y = 40,000 psi

Bar Size	f'_c (normal-weight concrete), psi			
	3000	**4000**	**5000**	**6000**
3	5.5	4.7	4.2	3.9
4	7.3	6.3	5.7	5.2
5	9.1	7.9	7.1	6.5
6	11.0	9.5	8.5	7.7
7	12.8	11.1	9.9	9.0
8	14.6	12.6	11.3	10.3
9	16.5	14.3	12.8	11.6
10	18.5	16.1	14.4	13.1
11	20.6	17.8	16.0	14.6
14	24.7	21.4	19.2	17.5
18	33.0	28.5	25.5	23.3

Mechanical Anchorage and Hooks

Table 4b. Basic Development Length l_{hb} for Hooked Bars (in.) with f_y = 50,000 psi

Bar Size	f_c' (normal-weight concrete), psi			
	3000	**4000**	**5000**	**6000**
3	6.8	5.9	5.3	4.8
4	9.1	7.9	7.1	6.5
5	11.4	9.9	8.8	8.1
6	13.7	11.9	10.6	9.7
7	16.0	13.8	12.4	11.3
8	18.3	15.8	14.1	12.9
9	20.6	17.8	16.0	14.6
10	23.2	20.1	18.0	16.4
11	25.7	22.3	19.9	18.2
14	30.9	26.8	23.9	21.9
18	41.2	35.7	31.9	29.1

Mechanical Anchorage and Hooks

Table 4c. Basic Development Length l_{hb} for Hooked Bars (in.) with f_y = 60,000 psi

Bar Size	f_c' (normal-weight concrete), psi			
	3000	**4000**	**5000**	**6000**
3	8.2	7.1	6.4	5.8
4	11.0	9.5	8.5	7.7
5	13.7	11.9	10.6	9.7
6	16.4	14.2	12.7	11.6
7	19.2	16.6	14.8	13.6
8	21.9	19.0	17.0	15.5
9	24.7	21.4	19.1	17.5
10	27.8	24.1	21.6	19.7
11	30.9	26.8	23.9	21.8
14	37.1	32.1	28.7	26.2
18	49.4	42.8	38.3	35.0

Mechanical Anchorage and Hooks

- ACI Modification Factors (MF)
  - Modification factors are to be used if applicable:
    1. Bars with f_y other than 60,000 psi, USE

$$\text{MF} = \frac{f_y}{60{,}000} \quad \text{or use Tables 4a and 4b} \qquad (6)$$

    2. Concrete cover for No. 3 through No. 11: Side cover (normal to the plane of the hook) ≥ 2 ½ in. and, for 90° hooks, cover on bar extension beyond the bend ≥ 2 in.: USE 0.7 for MF

Mechanical Anchorage and Hooks

- ACI Modification Factors, MF (cont'd)
    3. Ties or stirrups: For No. 3 through No. 11 with hook enclosed vertically of horizontally within ties or stirrup ties spaced along the full development length l_{dh} not greater than $3d_b$: USE MF = 0.8.
    4. Reinforcement in excess of that required, where anchorage or development for f_y is not specifically required:

$$\text{MF} = \frac{A_s \text{ required}}{A_s \text{ provided}} \qquad (7)$$

Mechanical Anchorage and Hooks

- ACI Modification Factors, MF (cont'd)
  5. Lightweight aggregate concrete: USE

$$MF = 1.3 \qquad (8)$$

  6. Epoxy-coated reinforcement: USE

$$MF = 1.2 \qquad (9)$$

Mechanical Anchorage and Hooks

- ACI Modification Factors, MF (cont'd)
  – The basic development length l_{hb} must be multiplied by the application factors outlined in the previous viewgraphs.
  – In no case may l_{db} be less than $8d_b$ or 6 in., whichever is greater.

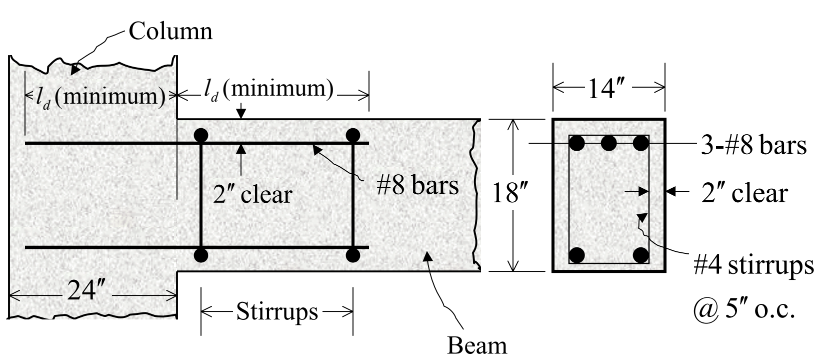
Mechanical Anchorage and Hooks

- Example 2

  Determine the anchorage or development length required for the conditions shown in the figure. Use $f_c' = 3{,}000$ psi (normal-weight concrete) and $f_y = 60{,}000$ psi. The No. 8 bars may be categorized as top bars. Assume a side cover on the main bars of 2 ½ in. minimum. Bars are uncoated.

Mechanical Anchorage and Hooks

- Example 2 (cont'd)



Mechanical Anchorage and Hooks

- Example 2 (cont'd)

Anchorage into the exterior column:

1. Establish values for the multiplying factors α, β, γ, and λ:
   a. α = 1.3 (the bars are top bars).
   b. β = 1.0 (the bars are uncoated).
   c. γ = 1.0 (the bars are No. 8)
   d. λ = 1.0 (normal-weight concrete used)
2. The product $\alpha \times \beta = 1.3 < 1.7$ (OK)

Mechanical Anchorage and Hooks

- Example 2 (cont'd)

3. Determine c. Based on cover (center of bar to nearest concrete surface), consider the clear cover, the No. 4 stirrups diameter, and one-half the diameter of the No. 8 bar:

$$c = 2.0 + 0.5 + \frac{1.0}{2} = 3.0 \text{ in.}$$

Based on bar spacing:

$$c = \frac{14 - 2(2.0) - 2(0.5) - 2(0.5)}{2(2)} = 2.0 \text{ in.} \quad \leftarrow \text{Controls}$$

Therefore, use c = 2.0 in. (smallest)

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Table 2. ASTM Standard - English Reinforcing Bars

Bar Designation	Diameter in	Area in^2	Weight lb/ft
#3 [#10]	0.375	0.11	0.376
#4 [#13]	0.500	0.20	0.668
#5 [#16]	0.625	0.31	1.043
#6 [#19]	0.750	0.44	1.502
#7 [#22]	0.875	0.60	2.044
#8 [#25]	1.000	0.79	2.670
#9 [#29]	1.128	1.00	3.400
#10 [#32]	1.270	1.27	4.303
#11 [#36]	1.410	1.56	5.313
#14 [#43]	1.693	2.25	7.650
#18 [#57]	2.257	4.00	13.60

Note: Metric designations are in brackets

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- Example 2 (cont'd)
  4. The figure shows stirrups in the beam. However, there are no stirrups in the column, and K_{tr} can be taken as zero for the column anchorage.
  5. Check $(c + K_{tr})/d_b \leq 2.5$:

$$\frac{c + K_{tr}}{d_b} = \frac{2.0 + 0}{1.0} = 2.0 < 2.5 \Rightarrow \text{USE 2.0}$$

  6. The access reinforcement can be ignored and the factor applied can be omitted.

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- Example 2 (cont'd)

7. Calculate l_d:

$$l_d = \frac{3}{40}\left(\frac{f_y}{\sqrt{f'_c}}\right)\left[\frac{\alpha\beta\gamma\lambda}{\left(\frac{c+k_{tr}}{d_b}\right)}\right]d_b$$

$$l_d = \frac{3}{40}\left(\frac{60{,}000}{\sqrt{3{,}000}}\right)\left[\frac{1.3(1)(1)(1)}{2.0}\right](1.0) = 53.4\text{ in.} > 12\text{ in.} \quad \text{OK}$$

Since 53.4 in > 24 in. (column width), use a standard hook, either a 90° hook or a 180° hook.

Mechanical Anchorage and Hooks

- Example 2 (cont'd)

Anchorage using a standard 180° hook:

1. The basic development length l_{hb} for the standard hook shown in the figure can be computed from

$$l_{hb} = \frac{1200 d_b}{\sqrt{f'_c}} = \frac{1200(1)}{\sqrt{3000}} = 21.9\text{ in. (also see Table 4c)}$$

2. The only applicable MF is based on side cover of 2 ½ in.

   Therefore, USE MF = 0.7

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Table 4c. Basic Development Length l_{hb} for Hooked Bars (in.) with f_y = 60,000 psi

Bar Size	f_c' (normal-weight concrete), psi			
	3000	4000	5000	6000
3	8.2	7.1	6.4	5.8
4	11.0	9.5	8.5	7.7
5	13.7	11.9	10.6	9.7
6	16.4	14.2	12.7	11.6
7	19.2	16.6	14.8	13.6
8	21.9	19.0	17.0	15.5
9	24.7	21.4	19.1	17.5
10	27.8	24.1	21.6	19.7
11	30.9	26.8	23.9	21.8
14	37.1	32.1	28.7	26.2
18	49.4	42.8	38.3	35.0

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- Example 2 (cont'd)

3. The required development length is then calculated from

$$l_{dh} = l_{hb} \times \text{MF} = 21.9(0.7) = 15.3 \text{ in.} \qquad \leftarrow \text{EQ. 4}$$

Check minimum:

$$\text{minimum } l_{dh} = 8d_b \geq 6 \text{ in.}$$

$$8d_b = 8 \text{ in.} < 15.3 \text{ in.} \qquad \text{OK}$$

The minimum width of column required is

$$15.3 + 2.5 = 17.8 \text{ in.} < 24 \text{ in. (column width)} \qquad \text{OK}$$

Therefore, the hook will fit into the column.

Mechanical Anchorage and Hooks

- Example 2 (cont'd)

Anchorage into beam:

The development length required if bars are straight can be taken as 53.4 in. as determined previously. However, this number is conservative ($K_{tr} = 0$).

To determine a more accurate value, we have to take into consideration the transverse reinforcement index K_{tr} because there are stirrups in the beam.

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- Example 2 (cont'd)

Area of 2 #4 stirrups

$$K_{tr} = \frac{A_{tr} f_{yt}}{1500sn} = \frac{0.4(60{,}000)}{1500(5)(3)} = 1.067$$

$$\frac{c + K_{tr}}{d_b} = \frac{2.0 + 1.067}{1.0} = 3.07 < 2.5 \Rightarrow \text{USE } 2.5$$

$$l_d = \frac{3}{40}\left(\frac{f_y}{\sqrt{f'_c}}\right)\left[\frac{\alpha\beta\gamma\lambda}{\left(\frac{c + k_{tr}}{d_b}\right)}\right] d_b$$

$$l_d = \frac{3}{40}\left(\frac{60{,}000}{\sqrt{3{,}000}}\right)\left[\frac{1.3(1)(1)(1)}{2.5}\right](1.0) = 42.7 \text{ in.} > 12 \text{ in. OK}$$

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Table 2. ASTM Standard - English Reinforcing Bars

Bar Designation	Diameter in	Area in^2	Weight lb/ft
#3 [#10]	0.375	0.11	0.376
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Note: Metric designations are in brackets

Mechanical Anchorage and Hooks

- Example 2 (cont'd)

Anchorage into beam (cont'd):

- The development length required if bars are straight is **42.7 in.**
- Therefore, the bars must extend at least this distance into the span.
- Figure 7 shows the detailed sketch for the development length.

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- Example 2 (cont'd)

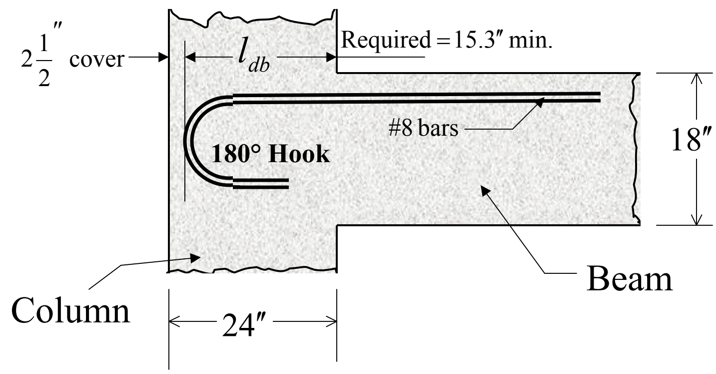

Figure 7. Detailed Sketch for Example 1