Solution to Homework Set #4
ENCE 454 – Design of Concrete Structures - SPRING 2004

Assigned T, 3/2    Due T, 3/9

Problem 1:
A reinforced concrete beam of rectangular cross section is reinforced for moment only and subjected to a shear \( V_u \) of 9000 lb. Beam width \( b = 12 \) in., \( d = 7.25 \) in., \( f'_c = 3000 \) psi, and \( f_y = 60,000 \) psi. Is the beam satisfactory for shear? Why? Justify your answer.

*** SOLUTION ***

Approximate \( h \):
\[
h = d + 1.5 + 0.5 = 7.25 + 1.5 + 0.5 = 9.25 \text{ in.} < 10.0 \text{ in.} \Rightarrow \text{beam is considered shallow.}
\]
max \( V_u = \phi V_c = \phi \lambda 2 \sqrt{f'_c} b w d = 0.75(1)(2)\sqrt{3000}(12)(7.25) = 7,148 \text{ lb} \)

7,148 lb < 9000 lb \( \Rightarrow \) Beam is N.G. in shear

Problem 2:
Assume that the beam of Problem 1 has an effective depth of 18 in. and is reinforced with No. 3 single-loop stirrups spaced at 10 in. on center. Determine the maximum shear \( V_u \) permissible.

*** SOLUTION ***

\[
V_u \leq \phi (V_c + V_s)
\]

\[
V_c = \lambda 2 \sqrt{f'_c} b w d = (1)(2)\sqrt{3000}(12)(18) = 23,661.61 \text{ lb} = 23.66 \text{ kips}
\]

\[
V_s = \frac{A_s f_y d}{s} = \frac{0.22(60)(18)}{10} = 23.76 \text{ kips}
\]

Therefore,
max \( V_u = \phi (V_c + V_s) = 0.75 (23.66 + 23.76) = 35.57 \text{ kips} \)
Problem 3:  
Textbook: 6.1

*** SOLUTION ***

\[ \omega_u = 3300 \text{ lb ft} \]

\[ h = 12 \text{ in. (305 mm)} \]

\[ d = 17 \text{ in. (432 mm)} \]

\[ k = 20 \text{ in. (508 mm)} \]

\[ A_s = 6.0 \text{ in.}^2 (3780 \text{ mm}^2) \]

\[ f' = 4000 \text{ psi (27.6 MPa), normal-weight concrete} \]

\[ f_s = 60.000 \text{ psi (415.7 MPa)} \]

Assume that no torsion exists.

\[ \omega_u = 1.2 (250 + 900) + 1.6 (1200) \]

\[ = 3300 \text{ lb ft} \]

\[ V_u \text{ (at support)} = \frac{3300 \times 22}{2} \]

\[ = 36,300 \text{ lb.} \]

\[ V_u \text{ (at } d) = 36,300 - 3300 (\frac{11}{12}) \]

\[ = 31,425 \text{ lb.} \]

\[ H_u \text{ (at } d) = 66,300 (\frac{11}{12}) - 3300 (\frac{11}{12})^2 \]

\[ = 48,113 \text{ ft-lb} \]

\[ = 3713 \text{ in-lb.} \]
**Simplified Method:**

\[ V_c = 2 \sqrt{fc} bd = 2(1000 \sqrt{4000}(12)(17)) = 25,804 \text{ lb} \]

\[ V_n = \frac{V_u}{q} = \frac{31,625}{0.75} = 42,167 \text{ lb} \]

\[ V_n > \frac{1}{2} V_c \Rightarrow \text{shear reinforcement is necessary} \]

\[ V_n = V_n - V_c = 42,167 - 25,804 = 16,363 \text{ lb} \]

\[ A_v = \frac{V_n}{\frac{1}{2} b d (60,000)(17)} = 0.016 \text{ in}^2/\text{in} \]

\[ \min A_v = \frac{50 b e}{m} = 50(12) = 0.01 < 0.016 \text{ in}^2/\text{in} \]

Try No. 3 stirrup, \( A_v = 2(0.11) = 0.22 \text{ in}^2 \)

\[ 3 = \frac{A_v}{(A_v/3)} = 13.8 \text{ in} \]

\[ V_n = 16,363 < 4 \sqrt{f_c} bd = 4 \sqrt{4000}(12)(17) = 51,408 \text{ lb} \]

\[ \Rightarrow \sigma_{max} = \min \left( \frac{f_{c}}{2} \right) = \frac{17}{2} = 8.5, 24 \]

\[ \Rightarrow \text{use No. 3 stirrups @ 8.5 in c-c.} \]
Refined Method:

\[ V_c = 19.65d \sqrt{f'c} + 2500 \rho_w \frac{V_{ud}}{H_u} b_w d \leq 3.5 b_w d \sqrt{f'c} \]

\[ \rho_w = \frac{A_s}{b_w d} = \frac{6}{12 \times 17} = 0.0294 \]

\[ V_{ud} = \frac{31.625 (17)}{577.363} = 0.93 \leq 1.0 \quad \text{OK} \]

\[ V_c = 19.65 (12 \times 17) \sqrt{4000} + 2500 (0.0294) (0.93) (12 \times 17) \leq 3.5 (12 \times 17) \sqrt{4000} \]

\[ = 38,458 \text{ lb} \leq 45,157 \text{ lb} \quad \therefore V_c = 38,458 \text{ lb}. \]

\[ V_n > \frac{1}{2} V_c; \quad \therefore \text{Shear reinforcement is required} \]

\[ V_3 = V_n - V_c = 42,167 - 38,458 = 3,709 \text{ lb} \]

\[ \frac{A_v}{A_d} = \frac{V_3}{8 \times 3.5 \times 17} = \frac{3709}{60,000 \times 17} = 0.0036 \text{ in}^2/\text{in} \]

\[ \min \frac{A_v}{A_d} = 0.01 \text{ in}^2/\text{in} \leftarrow \text{controls} \]

\[ s = \text{No. 3 stirrups, } A_v = 2(0.11) = 0.22 \text{ in}^2 \]

\[ s = \frac{A_v}{A_d} = \frac{0.22}{0.01} = 22 \text{ in} \]

\[ V_3 = 3,709 \text{ lb} < 4 \sqrt{f'c} b_d = 51,608 \text{ lb} \]

\[ \therefore V_{max} = 8.5 \text{ in} \]

\[ \Rightarrow \text{Use No. 3 stirrups @ 8.5 in o-c.} \]
Problem 4:
Textbook: 6.3

*** SOLUTION ***

\[ DL = 250 \text{ lb/ft} \]
\[ LL = 40,000 \text{ lb.} \]
\[ Self-weight = 10 \times 20 \times 150 = 208 \text{ lb/ft} \]
\[ DL = 1.2 (208) = 250 \text{ lb/ft} \]
\[ LL = 1.1 (25,000) = 40,000 \text{ lb.} \]

\[ V_u \text{ at support} = 250 \times 3.5 + 40,000 = 40,875 \text{ lb} \]
\[ V_u \text{ at } d = (40,875 - 40,000) \left( \frac{3.5 \times 12 - 17}{3.5 \times 12} \right) + 40,000 = 40,521 \text{ lb} \]

\[ V_n = \frac{V_u}{q} = \frac{40,521}{0.15} = 26,347 \text{ lb} \]

\[ V_c = 2.1 \sqrt{\frac{1}{2}} b w d = 2(10) \sqrt{8000} (10)(17) = 18,622 \text{ lb} \]

\[ V_n > V_c/2 \therefore 	ext{ stirrups are required} \]

\[ A_v = \frac{V_u}{b w d} = \frac{54,028 - 18,622}{10,000 \times 17} = 0.035 \text{ in}^2/\text{in} \]

\[ \min \frac{A_v}{s} = \frac{50 \text{ lb/ft}}{60,000} = 50 \text{ in} < 0.035 \therefore A_v = 0.035 \text{ in}^2/\text{in} \]
Try No. 4 stirrup, $A_v = 2(0.2) = 0.40 \text{ in}^2$

$$b = \frac{A_v}{A} = \frac{0.40}{0.85} = 1.1.4 \text{ in.}$$

$$v_b = 54,028 - 18,622 = 35,406 \text{ lb.} < 415000 \left( \frac{10 \times 11}{17} \right) = 87,243 \text{ lb.}$$

$$\therefore \quad b_{\text{max}} = \min \left\{ \frac{d}{2} : 8.5, 24 \text{ in} \right\} = 8.5 \text{ in.}$$

$$\Rightarrow \quad \text{Use No. 4 stirrups @ 8.5 in O-C.}$$