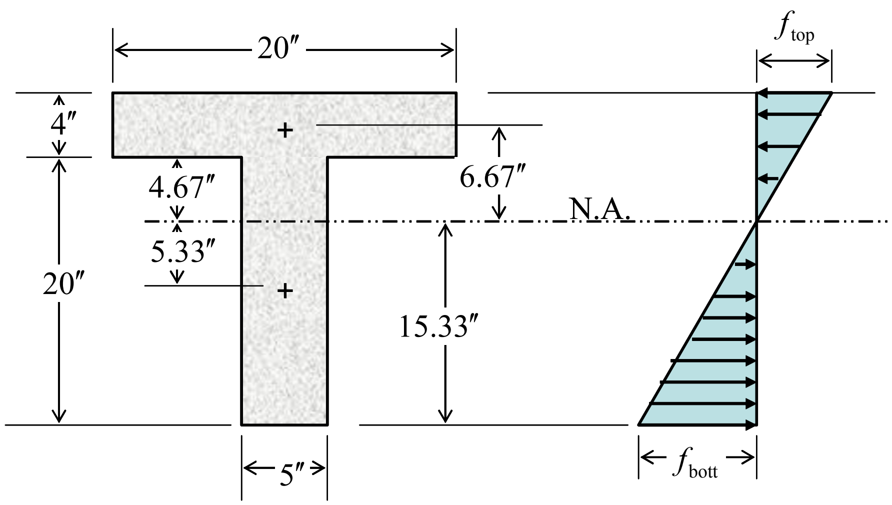
Solution to Homework Set #2

ENCE 454 – Design of Concrete Structures – SPRING 2004

Assigned T, 2/17   Due T, 2/24

Problem 1:

Calculate the cracking moment (resisting moment) for the T-shaped ***unreinforced*** concrete beam shown in the figure. Use f_c' = 4000 psi. Assume positive moment (compression in the top). Use the internal couple method and check using the flexure formula.



***** SOLUTION *****

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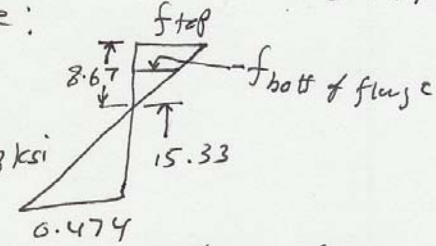
Problem 1:

$$f_{bott} = f_r = 7.5\sqrt{f'_c} = 7.5\sqrt{4000} = 474 \text{ psi}$$
$$= 0.474 \text{ ksi}$$

Using similar triangle:

$$\frac{f_{top}}{0.474} = \frac{8.67}{15.33}$$

or $f_{top} = \frac{8.67}{15.33}(0.474) = 0.268 \text{ ksi}$



Similarly, the stress at the bottom of the flange is

$$f_{bott\ of\ flange} = \frac{4.67}{15.33}(0.474) = 0.1444 \text{ ksi}$$

T = average stress × area

$$= \frac{1}{2}(0.474)(15.33)(5) = 18.17 \text{ kips}$$

It's location below N.A. is calculated from

$$\tfrac{2}{3}(15.33) = 10.22 \text{ in}$$

$$C_1 = 0.1444\,(20)(4) = 11.55 \text{ kips}$$
$$C_2 = \frac{1}{2}(0.1236)(20)(4) = 4.94 \text{ kips}$$
$$C_3 = \frac{1}{2}(0.1444)(5)(4.67) = 1.686 \text{ kips}$$
$$C = C_1 + C_2 + C_3 = 18.18 \simeq T$$
$$Z_1 = 10.22 + 4.67 + \tfrac{1}{2}(4) = 16.89 \text{ in}$$
$$Z_2 = 10.22 + 4.67 + \tfrac{2}{3}(4) = 17.56 \text{ in}$$
$$Z_3 = 10.22 + \tfrac{2}{3}(4.67) = 13.33 \text{ in.}$$

Problem 1 (con'd):

$$M_1 = 11.55(16.89) = 195.1 \text{ in-kips}$$
$$M_2 = 4.94(17.56) = 86.7 \text{ in-kips}$$
$$M_3 = 1.686(13.33) = 22.5 \text{ in-kips}$$
$$\therefore \quad M_{cr} = M_1 + M_2 + M_3 = 304 \text{ in-kips}$$

Check using the flexure formula:

$$I = \Sigma I_c + \Sigma A d^2 \quad \text{parallel axis theroen}$$
$$= \frac{1}{12}(20)(4)^3 + \frac{1}{12}(5)(20)^3 + 4(20)(6.67)^2 + 5(20)(5.33)^2$$
$$= 9840 \text{ in}^4$$

$$\therefore \quad M_{cr} = \frac{f_r I}{c} = \frac{0.474(9840)}{15.33}$$
$$= 304 \text{ in-kips} \qquad \text{[Checks O.K.]}$$

Problem 2:

Textbook: 5.1 (part b only)

b) $\beta_1 = 0.85 - 0.05\left(\dfrac{7{,}000 - 4{,}000}{1{,}000}\right) = 0.70$

$f'_c = 7{,}000$ psi

$A_s = 5$ in^2

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(5)(60{,}000)}{0.85(7{,}000)(12)} = 4.20 \text{ inches}$$

$$c = \frac{a}{\beta_1} = \frac{4.2}{0.70} = 6.0 \text{ inches}$$

$$\frac{c}{d_t} = \frac{6}{24} = 0.25 < 0.375$$

$\therefore$ Tension - controlled and steel yields before concrete crushes

$$\rho = \frac{A_s}{bd} = \frac{5}{(12)(24)} = 0.017 \text{ in/in.}$$

$$\rho_{min} = \max\left\{\frac{3\sqrt{7{,}000}}{60{,}000} = 0.0042,\ \frac{200}{60{,}000} = 0.0033\right\} = 0.0042 \text{ in/in.}$$

$0.017 > 0.0042$ $\therefore$ O.K. satisfies ACI CODE

Problem 3:

Textbook: 5.2 (part a only)

Solution:

a) $f'_c = 5{,}000$ psi

$\beta_1 = 0.80$

$A_s = 3 \times 0.9 = 3 \text{ in}^2$

$b = 10$ in.

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(3)(60{,}000)}{0.85(5{,}000)(10)} = 4.24 \text{ inches}$$

$$C = \frac{a}{\beta_1} = \frac{4.24}{0.80} = 5.29 \text{ in.}$$

$$\frac{c}{d_t} = \frac{5.29}{16} = 0.33 < 0.375 \quad \therefore \text{ Tension-Controlled}$$

$\phi = 0.90$

$$\rho = \frac{A_s}{bd} = \frac{3}{(10)(16)} = 0.019$$

$$\rho_{min} = \max\left\{\frac{3\sqrt{f'_c}}{f_y}, \frac{200}{f_y}\right\} = 0.0035 < 0.019 \quad \underline{\underline{\text{ok.}}}$$

$$M_n = A_s f_y (d - a/2) = (3)(60{,}000)\left(16 - \frac{4.24}{2}\right) = 2{,}498{,}824 \text{ in-lb}$$

$$M_u = \phi M_n = 0.90\,(2{,}498{,}824) = 2{,}248{,}941 \text{ in-lb.}$$

Problem 4:

Textbook: 5.4

Solution:

Design as a 1 ft. wide singly reinforced section.

Try, $\min h = \frac{L}{20} = \frac{(12)(12)}{20} = 7.2 \text{ in.}$

Try $h = 8$ in, $d_t = 7$ in. ; $b = 12$ in.

$$\text{Self-weight} = (150)\frac{(8)(12)}{144} = 100 \text{ lb/ft.}$$

$$DL = \left(50 \frac{\text{lb}}{\text{ft}^2}\right)(1 \text{ ft}) = 50 \text{ lb/ft}$$

$$LL = \left(100 \frac{\text{lb}}{\text{ft}^2}\right)(1 \text{ ft}) = 100 \text{ lb/ft.}$$

$$\therefore\ w_u = 1.2(100+50) + 1.6(100) = 340 \text{ lb/ft.}$$

$$M_u = \frac{w_u \ell^2}{8} = \frac{(340 \text{ lb/ft})(12 \text{ ft})^2}{8} = 6{,}120 \text{ ft-lb} = 73{,}440 \text{ in.-lb.}$$

Required nominal moment strength:

$$M_n = \frac{73{,}440}{0.90} = 81{,}600 \text{ in-lb.}$$

Assume $(d - a/2) \simeq 0.9d = 0.9(7) = 6.3$ in.

$$M_n = A_s f_y (d - a/2)$$

$$81{,}600 = A_s (60{,}000)(6.3)$$

$$A_s = 0.22 \text{ in}^2/\text{12-in strip}$$

$$\rho_{min} = \max\left\{\frac{3\sqrt{f_c'}}{f_y}, \frac{200}{f_y}\right\} = 0.0033 \qquad \therefore \min A_s = (0.0033)(12)(7) = 0.27 \text{ in}^2/\text{12-in strip}$$

$\therefore$ try $A_s = 0.28 \text{ in}^2/\text{12-in strip}$ (#4 bars at 8.5 in c-c)

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{(0.28)(60{,}000)}{0.85(4{,}000)(12)} = 0.41 \text{ in.}$$

$$c = \frac{a}{\beta_1} = \frac{0.41}{0.85} = 0.48 \text{ in.}$$

$$\frac{c}{d_t} = \frac{0.48}{7} = 0.069 < 0.375 \quad \therefore \text{Tension-controlled} \quad \phi = 0.90$$

Actual nominal moment strength.

$$M_n = A_s f_y (d - a/2) = (0.28)(60{,}000)\left(7 - \frac{0.41}{2}\right) = 114{,}156 \text{ in-lb}$$

$114{,}156$ in-lb > Req'd $81{,}600$ in-lb <u>O.K.</u>

<u>Shrinkage and Temperature Reinforcement</u> :

Req'd steel area $= 0.0018(12)(8) = 0.17 \text{ in}^2/\text{12-in strip}$

maximum spacing $= \min\{5(8) = 40 \text{ in}, 18 \text{ in}\} = 18$ in.

$\therefore$ Use #4 bar @ 14 in. c-c ($A_s = 0.17 \text{ in}^2/\text{12-in strip}$)

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Problem 5:

Textbook: 5.5 (the beam of part b only)

b) Try $h = 15$ in. $d = 13$ in. $b = 8$ in.

Self-weight $= \dfrac{(150)(8)(15)}{144} = 125$ lb/ft.

M_u due to dead load $= \dfrac{(1.2)(125)(20)^2}{8} = 1500$ ft-lb $= 90{,}000$ in-lb

M_u due to live load $= \dfrac{(1.6)(10{,}000)(10)}{2} = 80{,}000$ ft-lb $= 960{,}000$ in-lb

Required nominal moment strength :

$$M_n = \frac{90{,}000 + 960{,}000}{0.90} = 1{,}166{,}667 \text{ in-lb.}$$

Assume $c/d_t = 0.30$

$C = (0.30)(13) = 3.90$ in.

$a = \beta_1 c = (0.80)(3.90) = 3.12$ in.

$C = T$ $\quad 0.85 f_c' ba = A_s f_y \quad A_s = \dfrac{0.85(5{,}000)(8)(3.12)}{60{,}000} = 1.77 \text{ in}^2$

Try 2 no. 9 bars $A_s = 2.0 \text{ in}^2$

$a = \dfrac{(2)(60{,}000)}{0.85(5000)(8)} = 3.53$ in.

$c = \dfrac{a}{\beta_1} = \dfrac{3.53}{0.80} = 4.41$ in.

$$\frac{c}{d_t} = \frac{4.41}{13} = 0.34 < 0.375 \quad \therefore \text{ Tension-Controlled}$$

$$\phi = 0.90$$

$$\rho = \frac{A_s}{bd} = \frac{2}{(8)(13)} = 0.019$$

$$\rho_{min} = \max\left\{\frac{3\sqrt{f_c'}}{f_y}, \frac{200}{f_y}\right\} = 0.0035 \quad 0.019 \quad \therefore \text{ satisfies ACI CODE Requirements}$$

Actual moment strength:

$$M_n = A_s f_y (d - a/2) = (2)(60{,}000)\left(13 - \frac{3.53}{2}\right) = 1{,}348{,}235 \text{ in-lb}$$

$$1{,}348{,}235 > 1{,}666{,}667 \quad \therefore \quad \underline{\text{O.K.}}$$