CHAPTER

9b

Prentice Hall

Structural Steel Design
LRFD Method

Third Edition

DESIGN OF BEAMS FOR MOMENTS

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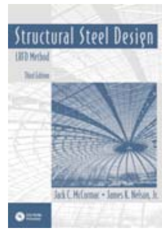
Part II – Structural Steel Design and Analysis

FALL 2002



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ENCE 355 - Introduction to Structural Design

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Lateral Support of Beams

- Most beams are designed in such a way that their flanges are restrained against lateral buckling.
- The upper flanges of beams used to support concrete building and bridge floors are often incorporated in these concrete floors.
- Therefore, these type of beams fall into **Zone 1**.

Lateral Support of Beams

- If the compression flange of a beam is without lateral support for some distance, it will have a stress distribution similar to that of columns.
- When the compression flange of a beam is long enough and slender enough, it may buckle unless lateral support is provided

Lateral Support of Beams

- Twisting or Torsion of Beams
  - When the compression flange begin to buckle, twisting or torsion will occur, and the smaller the torsional strength of the beam the more rapid will be the failure.
  - Standard shapes such as W, S, and channels used for beam sections do not have a great deal of resistance to lateral buckling and the resulting torsion.
  - Some other shapes, notably the built-up box shapes are tremendously stronger.

Lateral Support of Beams

- Lateral Support not Provided by Slab
  – Should lateral support of the compression flange not be provided by a floor slab, it is possible that such support my be provided with connecting beams or with special members inserted for that purpose.
  – Depending on the spacing of the support, the beam will fall into Zones 1, 2, or 3.

Introduction to Inelastic Buckling, Zone 2

- Inelastic buckling can occur when the bracing is insufficient to permit the member to develop and reach a full plastic strain distribution before buckling occurs.
- Because of the presence of residual stresses, yielding will begin in a section at applied stresses equal to

$$F_y - F_r \qquad (1)$$

Introduction to Inelastic Buckling, Zone 2

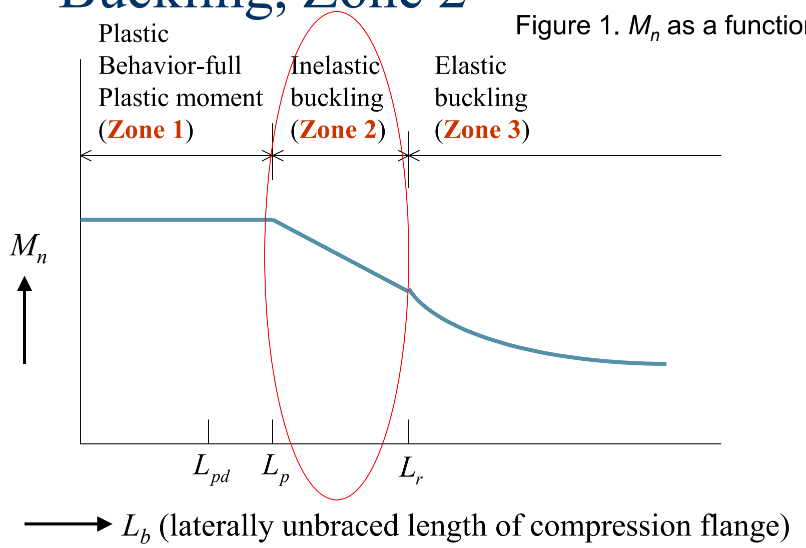
- In Eq. 1, F_y = yield stress of the web, and F_r = compressive residual stress, and assumed equal to 10 ksi for rolled shapes and 16.5 ksi for welded shapes.
- When a constant moment occurs along the unbraced length L_b of a compact I- or C-shaped section and L_b is larger than L_p, the beam will fail inelastically unless L_b is greater than a distance L_r.

Introduction to Inelastic Buckling, Zone 2

- Lateral Buckling of Beams
  - Fig. 1 shows that beams have three distinct ranges or **zones** of behavior depending on their lateral bracing situation:
    - Zone 1: closely spaced lateral bracing, beams fail plastically.
    - Zone 2: moderate unbraced lengths, beams fail inelastically.
    - Zone 3: Larger unbraced lengths, beams fail elastically

Introduction to Inelastic Buckling, Zone 2

Figure 1. M_n as a function of L_b



Introduction to Inelastic Buckling, Zone 2

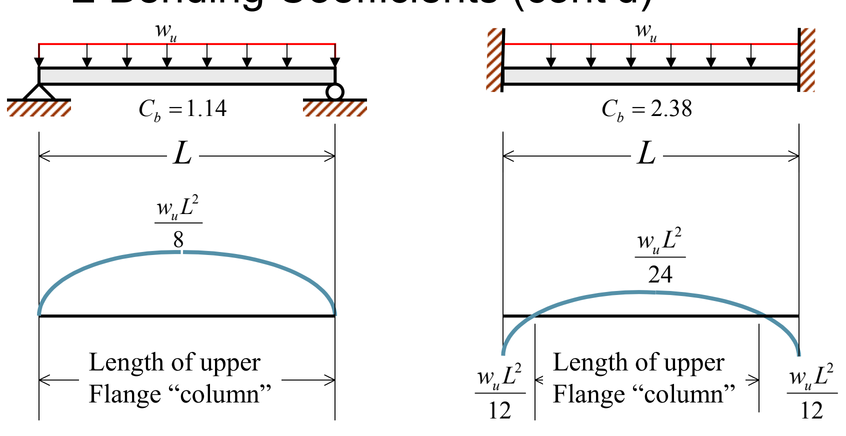
- Bending Coefficients
  - A moment coefficient, designated by C_b, is included in design formulas to account for the effect of different moment gradients on lateral-torsional buckling.
  - The use of this coefficient is to take into account the effect of the end restraint and loading condition of the member on lateral buckling.

Introduction to Inelastic Buckling, Zone 2

- Bending Coefficients (cont'd)
  - In Fig 2a, the moment in the unbraced beam causes a worse compression flange situation than does the moment in the unbraced beam of Fig. 2b.
  - For one reason, the upper flange in Fig. 2a is in compression for its entire length, while in Fig. 2b the length of the "*column*", that is the length of the upper flange that is in compression is much less (shorter column).

Introduction to Inelastic Buckling, Zone 2

- Bending Coefficients (cont'd)



(a) Single curvature — Figure 2 — (b) Double curvature

Introduction to Inelastic Buckling, Zone 2

- Bending Coefficients (cont'd)
  - Values of C_b:
    - For the simply supported beam of Fig .2a:

      $C_b = 1.14$

    - For the fixed-end beam of Fig. 2b:

      $C_b = 2.38$

  - The basic moment capacity equations for Zones 2 and 3 were developed for laterally unbraced beams subjected to single curvature with

    $C_b = 1.0$

Introduction to Inelastic Buckling, Zone 2

- LRFD Specification
  - LRFD Specification provides moment or C_b coefficients larger than 1.0 which are to be multiplied by the computed M_n values.
  - The results are higher moment capacities.
  - The value of $C_b = 1.0$ is a conservative value.
  - In should be noted that that value obtained by multiplying M_n by C_b may not be larger than the plastic moment M_p of Zone 1, which is equal to F_yZ.

Introduction to Inelastic Buckling, Zone 2

■ LRFD Specification

The Manual provides an equation for calculating the coefficient C_b as follows:

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C} \qquad (2)$$

M_{max} = largest moment in unbraced segment of a beam
M_A = moment at the ¼ point
M_B = moment at the ½ point
M_C = moment at the ¾ point

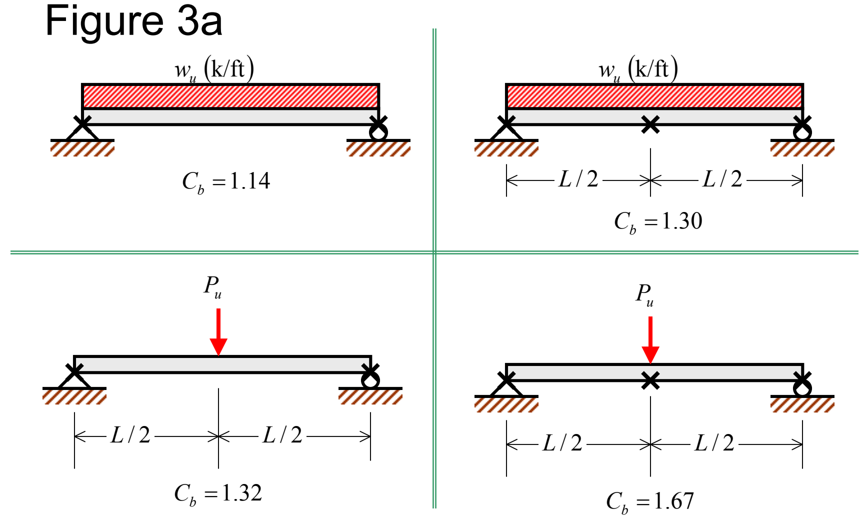
Introduction to Inelastic Buckling, Zone 2

■ LRFD Specification

– C_b = 1.0 for cantilevers or overhangs where the free end is unbraced.
– Some special values of C_b calculated with Eq. 2 are shown in Fig. 3 for various beam moment situations.
– Most of these values are also given in **Table 5.1 of the LRFD Manual**

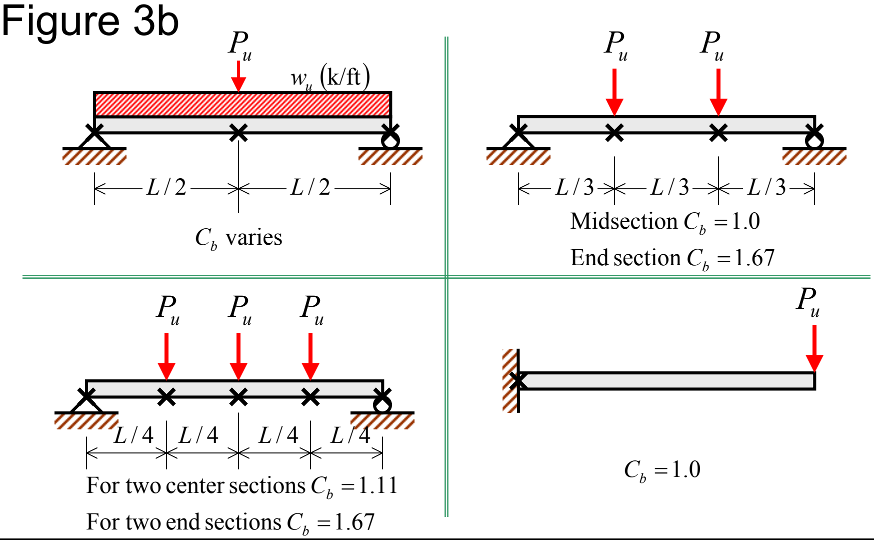
Introduction to Inelastic Buckling, Zone 2

Figure 3a



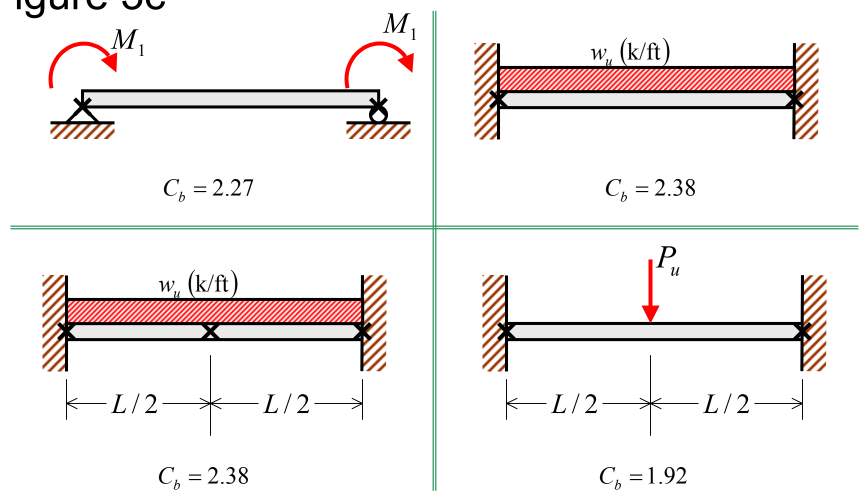
Introduction to Inelastic Buckling, Zone 2

Figure 3b



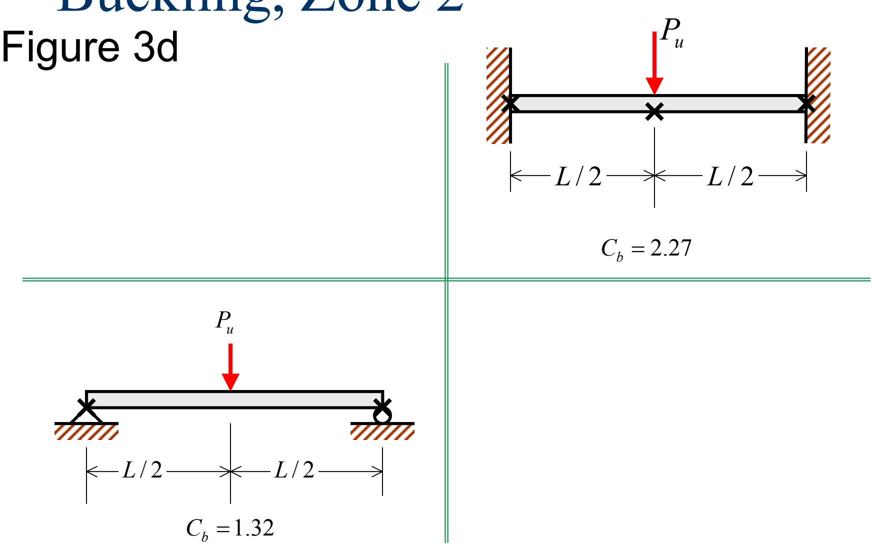
Introduction to Inelastic Buckling, Zone 2

Figure 3c



Introduction to Inelastic Buckling, Zone 2

Figure 3d



Moment Capacity, Zone 2

- If the distance between points of torsional bracing is increased beyond L_p (see Fig. 1), the moment capacity of the section will become smaller and smaller.
- Finally, at an unbraced length L_r, the section will buckle elastically as soon as the yield stress is reached.

Moment Capacity, Zone 2

- Effect of Residual Stresses
  - Due to the rolling operation on steel shapes, there is residual stress in the section equal to F_r.
  - Thus, the elastically computed stress caused by bending can only reach $F_y - F_r$ as given by Eq. 1. Assuming C_b = 1, the design moment for a compact I- or C –shaped section may be determined as follows if $L_b = L_r$:

$$\phi_b M_r = \phi_b S_x \left(F_y - F_r\right) \qquad (3)$$

Moment Capacity, Zone 2

- Provisions for Zone 2 Design by the LRFD
  - If decrease the unbraced length L_b from L_r to L_p, buckling does not not occur when the yield stress is first reached.
  - This range between L_r and L_p is called Zone 2 and is illustrated in Fig. 1.
  - For these cases, when the unbraced length falls between L_r and L_p, the design moment strength will fall approximately on a straight line between

$$\phi M_n = \phi_b F_y Z \text{ at } L_p \quad \text{and } \phi_b S_x \left(F_y - F_r\right) \text{ at } L_r$$

Moment Capacity, Zone 2

- Provisions for Zone 2 Design by the LRFD
  - For intermediate values of the unbraced length, the moment capacity may be determined by proportions or by substituting into expressions.
  - If C_b is larger than 1.0, the section will resist additional moment but not more than

$$\phi_b F_y Z = \phi_b M_p \qquad (4)$$

Moment Capacity, Zone 2

- Provisions for Zone 2 Design by the LRFD

  The moment capacity can determined by the following two expressions:

$$\phi_b M_{nx} = C_b\left[\phi_b M_{px} - BF\left(L_b - L_p\right)\right] \leq \phi_b M_{px} \quad (5)$$

  or

$$M_n = C_b\left[M_p - \left(M_p - M_r\right)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \leq M_p \quad (6)$$

Moment Capacity, Zone 2

- Provisions for Zone 2 Design by the LRFD
  - In Eq. 5, BF is a factor given in **LRFD Table 5-3** for each section, which enables us to do the proportioning with simple formula.
  - Note that in Eq. 6, after the moment M_n has been computed, it should be multiplied by ϕ_b to obtain $\phi_b M_n$.

Moment Capacity, Zone 2

- Example 1

Determine the moment capacity of a W24 × 62 with F_y = 50 ksi if L_b = 8.0 ft and C_b = 1.0.

For F_y = 50 ksi, the LRFD Table 5-3 (P. 5-46) gives the following for W24 × 62:

$$L_p = 4.84 \text{ ft}, \quad L_r = 13.3 \text{ ft},$$
$$\phi_b M_r = 396 \text{ kip - ft}$$
$$\phi_b M_p = 578 \text{ kip - ft, and}$$
$$BF = 21.6 \text{ kips}$$

Moment Capacity, Zone 2

- Example 1 (cont'd)

Since $(L_p = 4.84') < (L_b = 8.0') < (L_r = 13.3')$

The moment capacity falls in Zone 2, and

$$\phi_b M_{nx} = C_b\left[\phi_b M_{px} - BF\left(L_b - L_p\right)\right] \leq \phi_b M_{px} \qquad \text{Eq. 5}$$
$$\phi_b M_n = C_b\left[\phi_b M_p - BF\left(L_b - L_p\right)\right] \leq \phi_b M_p$$
$$\phi_b M_n = 1.0[578 - 21.6(8.0 - 4.84)] = 509.7 \leq \phi_b M_p = 578$$

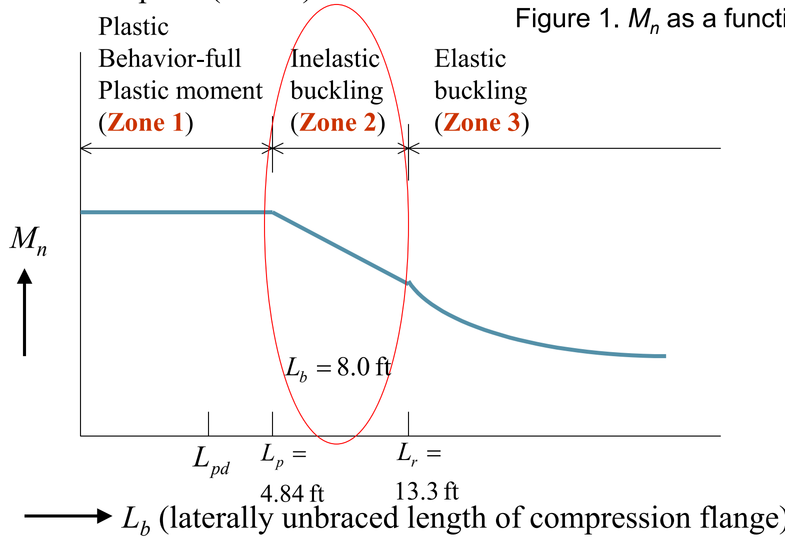
Therefore,

The moment capacity = 509.7 ft - kip

Moment Capacity, Zone 2

Example 1 (cont'd)

Figure 1. M_n as a function of L_b



Moment Capacity, Zone 2

- Example 2

Select the lightest available section for a factored moment of 290 ft-kips if L_b = 10.0 ft. Use 50 ksi steel and assume C_b = 1.0.

Enter LRFD Table 5-3 (P. 5-47) and notice that $\phi_b M_p$ for W18 × 40 is 294 ft-kip.

For this section:

$$L_p = 4.49 \text{ ft}, \quad L_r = 12.0 \text{ ft},$$
$$\phi_b M_r = 205 \text{ kip-ft}$$
$$\phi_b M_p = 294 \text{ kip-ft, and}$$
$$BF = 11.7 \text{ kips}$$

Moment Capacity, Zone 2

- Example 2 (cont'd)

Since $(L_p = 4.49') < (L_b = 10.0') < (L_r = 12.0')$

The moment capacity falls in Zone 2, and

$$\phi_b M_{nx} = C_b\left[\phi_b M_{px} - BF(L_b - L_p)\right] \leq \phi_b M_{px}$$
$$\phi_b M_n = C_b\left[\phi_b M_p - BF(L_b - L_p)\right] \leq \phi_b M_p$$
$$\phi_b M_n = 1.0\left[294 - 11.7(10.0 - 4.49)\right] = 229.5 \leq \phi_b M_p = 294$$

Therefore,

The moment capacity $= 229$ ft - kip $< M_u = 290$ ft - kip **NG**

Moment Capacity, Zone 2

- Example 2 (cont'd)

Moving up in the table and after several trials, try a W21 × 48 that has the following properties:

$$L_p = 6.09 \text{ ft}, \quad L_r = 15.4 \text{ ft},$$
$$\phi_b M_r = 279 \text{ kip - ft}$$
$$\phi_b M_p = 401 \text{ kip - ft}, \text{ and}$$
$$BF = 13.2 \text{ kips}$$

Since $(L_p = 6.09') < (L_b = 10.0') < (L_r = 15.4')$

The moment capacity falls in Zone 2, and

Moment Capacity, Zone 2

- Example 2 (cont'd)

$$\phi_b M_{nx} = C_b\left[\phi_b M_{px} - BF\left(L_b - L_p\right)\right] \leq \phi_b M_{px}$$
$$\phi_b M_n = C_b\left[\phi_b M_p - BF\left(L_b - L_p\right)\right] \leq \phi_b M_p$$
$$\phi_b M_n = 1.0\left[401 - 13.2(10.0 - 6.09)\right] = 249 \leq \phi_b M_p = 401$$

Therefore,

The moment capacity $= 349$ ft-kip $> M_u = 290$ ft-kip OK

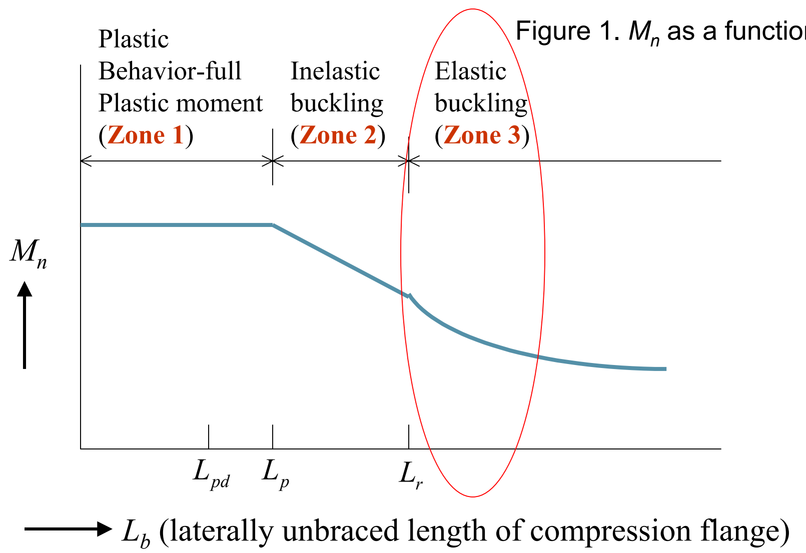
Hence, USE W21 × 48

Elastic Buckling, Zone 3

- When a beam is not fully braced, it may fail due to buckling of the compression portion of the cross section laterally about the weak axis.
- This will be accompanied also with twisting of the entire cross section about the beam's longitudinal axis between points of lateral bracing.
- For the moment capacity to fall into Zone 3, $L_b \geq L_r$

Elastic Buckling, Zone 3

Figure 1. M_n as a function of L_b



Elastic Buckling, Zone 3

- LRFD Specifications for Zone 3

  The classic equation for determining the flexural-torsional buckling moment is given by

$$M_{cr} = C_b \frac{\pi}{L_b} \sqrt{EI_y GJ + \left(\frac{\pi E}{L_b}\right)^2 I_y C_w} \qquad (7)$$

G = shear modulus of steel = 11,200 ksi
J = torsional constant (in^4)
C_w = warping constant (in^6)

NOTE: These properties are provided in Tables 1.25 to 1.35 Of the LRFD Manual for rolled sections

Elastic Buckling, Zone 3

- Example 3

Compute ϕM_{cr} for a W18 × 97 consisting of 50 ksi steel if the the unbraced length L_b is 38 ft. Assume $C_b = 1.0$.

For W18 × 97, Table 5-3 (P. 5-46) of the Manual gives

$$L_p = 9.36 \text{ ft}, \quad L_r = 27.5 \text{ ft},$$
$$\phi_b M_r = 564 \text{ kip-ft}$$
$$\phi_b M_p = 791 \text{ kip-ft, and}$$
$$BF = 12.8 \text{ kips}$$

Elastic Buckling, Zone 3

- Example 3 (cont'd)

$$\text{Since } (L_p = 9.36') < (L_r = 27.5') < (L_b = 38')$$

The moment capacity falls in Zone 3, and

From P.1-17

From Part 1 of the Manual, tables for torsion properties (P. 1-91):

$$I_y = 201 \text{ in}^4, \quad J = 5.86 \text{ in}^4, \text{ and } C_w = 15{,}800 \text{ in}^6$$

Therefore, using Eq. 7

$$M_{cr} = (1)\left(\frac{\pi}{12\times 38}\right)\sqrt{29\times10^3(201)(11{,}200)(5.86) + \left(\frac{\pi\, 29\times10^3}{38\times12}\right)^2 (201)(15{,}800)}$$
$$= 4{,}916.9 \text{ in-kip} = 410 \text{ ft-kip}$$

Therefore, $\phi_b M_{cr}$ = 0.9 (410) = 369 ft-kip