CHAPTER

Structural Steel Design
LRFD Method

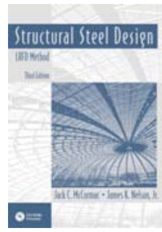
Third Edition

INTRODUCTION TO AXIALLY LOADED COMPRESSION MEMBERS

A. J. Clark School of Engineering •Department of Civil and Environmental Engineering

Part II – Structural Steel Design and Analysis

FALL 2002



By
Dr. Ibrahim. Assakkaf

5d

Prentice Hall

ENCE 355 - Introduction to Structural Design

Department of Civil and Environmental Engineering

University of Maryland, College Park

Long, Short, and Intermediate Columns

- The strength of a column and the manner in which it fails are greatly dependent on its effective length.
- A very short stocky steel column may be loaded until it reaches it yield point, and perhaps the strain hardening range.
- In essence, it can support about the same load in compression that it can in tension.

Long, Short, and Intermediate Columns

- As the effective length of a column increases, its buckling stress will decrease.
- The steel column is said to fail elastically if the buckling stress is less than the proportional limit of steel when the effective length exceeds a certain value.

Long, Short, and Intermediate Columns

- Long Columns
  - Long columns usually fails elastically.
  - The Euler formula predicts very well the strength of long columns where the axial compressive buckling stress remains below the proportional limit.

Long, Short, and Intermediate Columns

- Short Columns
  - The failure stress equals to the yield stress for short columns.
  - For a column to fall into this class, it would have to be so short as to have no practical application.

Long, Short, and Intermediate Columns

- Intermediate Columns
  - For intermediate columns some of the fibers will reach the yield stress and some will not.
  - The member will fail by both yielding and buckling, and their behavior is said to be inelastic.
  - Most columns fall into this range.

Column Formulas

- The Euler formula is used by the AISC LRFD Specification for long columns with elastic buckling.
- Other empirical (based on testing) equations are used by the LRFD for short and intermediate columns.
- With these equations, a critical or buckling stress F_{cr} is determined for a compression element.

Column Formulas

- LRFD General Design Equation for Columns

  The design strength of a compression member is determined as follows:

$$P_n = A_g F_{cr}$$

$$\phi_c P_n \leq \phi_c A_g F_{cr} \quad \text{with } \phi = 0.85 \qquad (1)$$

Column Formulas

- LRFD Critical Buckling Stress

  Two equations are provided by the LRFD for the critical buckling stress F_{cr}:

$$F_{cr} = \begin{cases} \left(0.658\right)^{\lambda_c^2} F_y & \text{for } \lambda_c \leq 1.5 \\ \left(\dfrac{0.877}{\lambda_c^2}\right) F_y & \text{for } \lambda_c > 1.5 \end{cases} \qquad (2)$$

Column Formulas

- LRFD Critical Buckling Stress

  The limiting λ_c value is given by

$$\lambda_c = \sqrt{\frac{F_y}{F_e}} \qquad (3)$$

Where F_e = Euler buckling stress = $\dfrac{\pi^2 E}{(KL/r)^2}$

Hence,

$$\lambda_c = \sqrt{\frac{F_y}{F_e}} = \sqrt{\frac{F_y}{\dfrac{\pi^2 E}{(KL/r)^2}}} = \sqrt{\frac{F_y (KL/r)^2}{\pi^2 E}} = \frac{KL}{r\pi}\sqrt{\frac{F_y}{E}} \qquad (4)$$

Column Formulas

- LRFD Critical Buckling Stress

So the limiting λ_c value to be used in Eq. 2 is given by

$$\lambda_c = \sqrt{\frac{F_y}{F_e}} = \frac{KL}{r\pi}\sqrt{\frac{F_y}{E}} \qquad (5)$$

where

F_y = yield strength of material (steel)

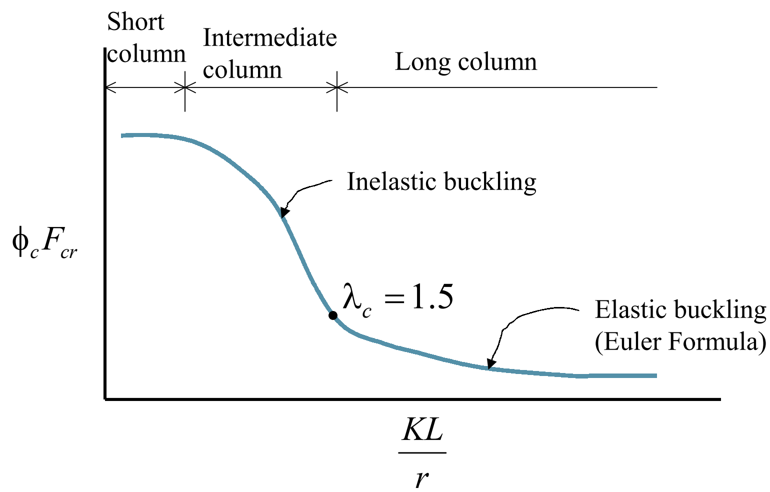
F_e = Euler critical buckling stress

Column Formulas

- LRFD Critical Buckling Stress
  - For inelastic flexural buckling, Eq. 2 can be used to compute the critical buckling stress F_{cr} when $\lambda_c \leq 1.5$.
  - For elastic flexural buckling, Eq. 2 can be used to compute the critical buckling stress F_{cr} when $\lambda_c > 1.5$.
  - Eq. 2 include the estimated effects of residual stresses and initial out-of-straightness of the members.
  - Eq. 2 is presented graphically in Fig. 1.

Column Formulas

Figure 1. LRFD Critical Buckling Stress



Column Formulas

- LRFD Critical Buckling Stress
  - To facilitate the design process, the LRFD Manual provides computed values $\phi_c F_{cr}$ values for steels with **F_y = 36** ksi and **50** ksi for ***KL/r* from 1 to 200** and has shown the results in **Tables 3.36 and 3.50** of the LRFD Specification located in Part 16 of the Manual.
  - Also, there is **Table 4 of the LRFD Specification** from which the user may obtain values for steel with any F_y values.

Column Formulas

- LRFD Manual Design Tables (P. 16.I-143)

TABLE 3-36

Design Stress for Compression Members of 36 ksi Specified Yield Stress Steel, $\phi_c = 0.85$[a]

$\frac{Kl}{r}$	$\phi_c F_{cr}$ ksi	$\frac{Kl}{r}$	$\phi_c F_{cr}$ ksi	$\frac{Kl}{r}$	$\phi_c F_{cr}$ ksi	$\frac{Kl}{r}$	$\phi_c F_{cr}$ ksi	$\frac{Kl}{r}$	$\phi_c F_{cr}$ ksi
1	30.6	41	28.0	81	21.7	121	14.2	161	8.23
2	30.6	42	27.9	82	21.5	122	14.0	162	8.13
3	30.6	43	27.8	83	21.3	123	13.8	163	8.03
4	30.6	44	27.6	84	21.1	124	13.6	164	7.93
5	30.6	45	27.5	85	20.9	125	13.4	165	7.84
6	30.5	46	27.4	86	20.7	126	13.3	166	7.74
7	30.5	47	27.2	87	20.5	127	13.1	167	7.65
8	30.5	48	27.1	88	20.4	128	12.9	168	7.56
9	30.5	49	27.0	89	20.2	129	12.7	169	7.47
10	30.4	50	26.8	90	20.0	130	12.6	170	7.38

Column Formulas

- LRFD Manual Design Tables (P. 16.I-145)

TABLE 3-50

Design Stress for Compression Members of 50 ksi Specified Yield Stress Steel, $\phi_c = 0.85$[a]

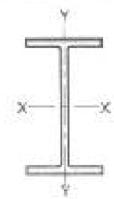
$\phi_c F_{cr}$ ksi	$\frac{Kl}{r}$	$\phi_c F_{cr}$ ksi	$\frac{Kl}{r}$	$\phi_c F_{cr}$ ksi	$\frac{Kl}{r}$	$\phi_c F_{cr}$ ksi	$\frac{Kl}{r}$	$\phi_c F_{cr}$ ksi
42.5	41	37.6	81	26.3	121	14.6	161	8.23
42.5	42	37.4	82	26.0	122	14.3	162	8.13
42.5	43	37.1	83	25.7	123	14.1	163	8.03
42.5	44	36.9	84	25.4	124	13.9	164	7.93
42.4	45	36.7	85	25.1	125	13.7	165	7.84
42.4	46	36.4	86	24.8	126	13.4	166	7.74
42.4	47	36.2	87	24.4	127	13.2	167	7.65
42.3	48	35.9	88	24.1	128	13.0	168	7.56
42.3	49	35.7	89	23.8	129	12.8	169	7.47
42.2	50	35.4	90	23.5	130	12.6	170	7.38

Column Formulas

- LRFD Manual Design Tables (P. 4-25)

= 50 ksi
$= 0.85 F_{cr} A_g$

Table 4-2 (cont.).
W-Shapes
Design Strength in Axial
Compression, $\phi_c P_n$, kips



Shape	W12×										
	106	96	87	79	72	65††	58	53	50	45	40
0	1330	1200	1090	986	897	812	723	663	621	557	497
6	1280	1150	1050	947	861	779	690	623	562	504	450
7	1260	1140	1030	933	848	767	686	610	543	486	434
8	1240	1120	1010	917	834	754	645	594	521	456	416
9	1210	1100	994	900	818	739	631	577	497	445	396
10	1190	1070	973	880	800	723	611	559	472	422	376

Maximum Slenderness Ratios

- Compression members preferably should be designed with

$$\frac{KL}{r} \leq 200 \qquad (6)$$

as specified in Section B7 of the LRFD Manual.

Note that LRFD Tables 3.36 and 3.50 give a value of 5.33 ksi for the design stress $\phi_c F_{cr}$ when $KL/r = 200$. If $KL/r > 200$, it is then necessary to substitute into the column formulas to get the the stress.

Maximum Slenderness Ratios

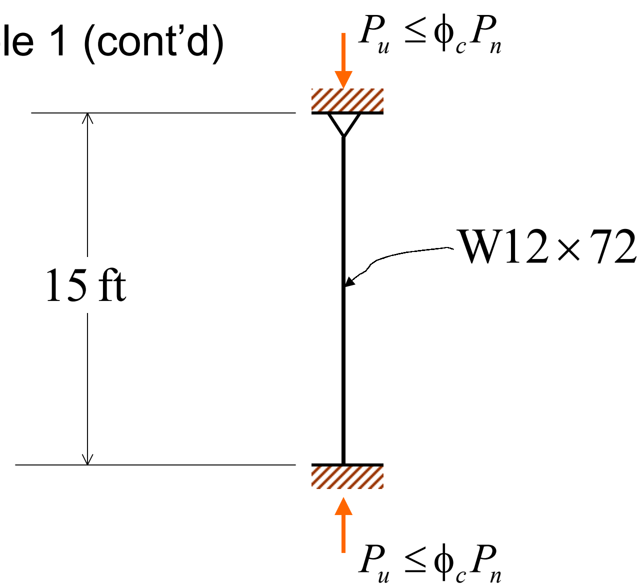
- More Simplification by the LRFD Manual for Design
  - It is to be noted that the LRFD Manual in its Part 4 has further simplified the calculations required by computing the column design strength $\phi_c F_{cr} A_g$ for each of the shapes normally used as columns for commonly used effective lengths or KL values.
  - These were determined with respect to the least radius of gyration for each section.

Example Problems

- Example 1
  a. Using the column design stress values shown in Table 3.50, part 16 of the LRFD manual, determine the design strength, $\phi_c P_n$ of the F_y = 50 ksi axially loaded column shown in the figure.
  b. Repeat the problem using the column tables of part 4 of the Manual.
  c. Check local buckling for the section selected using the appropriate values from Table 5.2.

Example Problems

- Example 1 (cont'd)



Example Problems

- Example 1 (cont'd)
  a. The properties of the W12 × 72 are obtained from the LRFD Manual as

$A = 21.1\text{ in}^2 \quad r_x = 5.31\text{ in} \quad r_y = 3.04\text{ in}$

$d = 12.3\text{ in} \quad b_f = 12.00\text{ in} \quad t_f = 0.670\text{ in}$

$k = 1.27\text{ in} \quad t_w = 0.430\text{ in}$

$K = 0.80$ from Table 1 (Table 5.1, Text)

Since $(r_y = 3.04\text{ in}) < (r_x = 5.31\text{ in})$, r_y controls

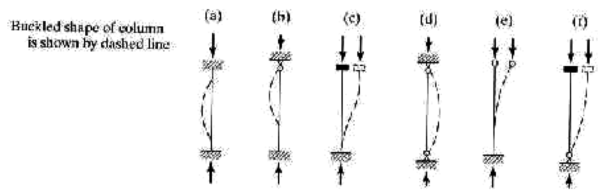
and

$$\frac{KL}{r_y} = \frac{0.80(12\times 15)}{3.04} = 47.37$$

Example Problems

■ Example 1 (cont'd)

Table 1



Buckled shape of column is shown by dashed line	(a)	(b)	(c)	(d)	(e)	(f)
Theoretical K value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.10	2.0

End condition code:
- Rotation fixed and translation fixed
- Rotation free and translation fixed
- Rotation fixed and translation free
- Rotation free and translation free

Source: Load and Resistance Factor Design Specification for Structural Steel Buildings, December 27, 1999 (Chicago: AISC)

Example Problems

■ Example 1 (cont'd)

For KL/r = 47 and 48, Table 3-50 of the LRFD Manual, Page 16.I-145, gives respectively the following values for $\phi_c F_{cr}$: 36.2 ksi and 35.9 ksi.

Using interpolation,

47	36.2
47.37	$\phi_c F_{cr}$
48	35.9

$$\Rightarrow \frac{\phi_c F_{cr} - 36.2}{35.9 - 36.2} = \frac{47.37 - 47}{48 - 47} \Rightarrow \phi_c F_{cr} = 36.09 \text{ ksi}$$

Example Problems

- Example 1 (cont'd)

TABLE 3-50 P. 16.I-145

Design Stress for Compression Members of 50 ksi Specified Yield Stress Steel, $\phi_c = 0.85$[a]

$\phi_c F_{cr}$ ksi	$\frac{Kl}{r}$	$\phi_c F_{cr}$ ksi	$\frac{Kl}{r}$	$\phi_c F_{cr}$ ksi	$\frac{Kl}{r}$	$\phi_c F_{cr}$ ksi	$\frac{Kl}{r}$	$\phi_c F_{cr}$ ksi
42.5	41	37.6	81	26.3	121	14.6	161	8.23
42.5	42	37.4	82	26.0	122	14.3	162	8.13
42.5	43	37.1	83	25.7	123	14.1	163	8.03
42.5	44	36.9	84	25.4	124	13.9	164	7.93
42.4	45	36.7	85	25.1	125	13.7	165	7.84
42.4	46	36.4	86	24.8	126	13.4	166	7.74
42.4	47	36.2	87	24.4	127	13.2	167	7.65
42.3	48	35.9	88	24.1	128	13.0	168	7.56
42.3	49	35.7	89	23.8	129	12.8	169	7.47
42.2	50	35.4	90	23.5	130	12.6	170	7.38

Example Problems

- Example 1 (cont'd)

  Therefore,

$$\phi_c P_n = \phi_c F_{cr} A_g = 36.09(21.1) = 761.5\ \text{k}$$

b. Entering column tables Part 4 of the LRFD Manual with $K_y L_y$ in feet:

$$K_y L_y = 0.80(15) = 12\ \text{ft}$$

$$P_u = \phi_c P_n = 761\ \text{k}$$

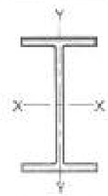
Example Problems

- Example 1 (cont'd)

= 50 ksi
$= 0.85 F_{cr} A_g$

Table 4-2 (cont.).
W-Shapes
Design Strength in Axial
Compression, $\phi_c P_n$, kips

P. 4-25



Shape	W12×										
	106	96	87	79	72	65††	58	53	50	45	40
0	1330	1200	1090	986	897	812	723	663	621	557	497
6	1280	1150	1050	947	861	779	690	623	562	504	450
7	1260	1140	1030	933	848	767	666	610	543	486	434
8	1240	1120	1010	917	834	754	645	594	521	456	416
9	1210	1100	994	900	818	739	631	577	497	445	396
10	1190	1070	973	880	800	723	611	559	472	422	376
11	1160	1050	950	860	781	706	590	539	445	398	354
12	1130	1020	926	838	781	687	568	518	419	374	332
13	1100	995	901	814	740	668	545	496	390	349	310

Example Problems

- Example 1 (cont'd)

c. Checking W12× 72 for compactness:

For flange (See Table 2)

$$\frac{h}{2t_f} = \frac{12.0}{2(0.670)} = 8.96 < 0.56\sqrt{\frac{E}{F_y}} = 0.56\sqrt{\frac{29\times10^3}{50}} = 13.49 \quad \text{OK}$$

For web, noting $h = d - 2k = 12.3 - 2\ (1.27) = 9.76$ in

$$\frac{h}{t_w} = \frac{9.76}{0.430} = 22.7 < 1.49\sqrt{\frac{E}{F_y}} = 1.49\sqrt{\frac{29\times10^3}{50}} = 35.88 \quad \text{OK}$$

See Table 3

Example Problems

- Table 2. Limiting Width-Thickness Ratios for Compression Elements

	Description of Element	Width Thickness Ratio	Limiting Width-Thickness Ratios λ_p (compact)	λ_r (noncompact)
Unstiffened Elements	Flanges of I-shaped rolled beams and channels in flexure	b/t	$0.38\sqrt{E/F_y}$ [c]	$0.83\sqrt{E/F_L}$ [e]
	Flanges of I-shaped hybrid or welded beams in flexure	b/t	$0.38\sqrt{E/F_{yf}}$	$0.95\sqrt{E/(F_L/k_c)}$ [e], [f]
	Flanges projecting from built-up compression members	b/t	NA	$0.64\sqrt{E/(F_y/k_c)}$ [f]
	Flanges of I-shaped sections in pure compression, plates projecting from compression elements; outstanding legs of pairs of angles in continuous contact; flanges of channels in pure compression	b/t	NA	$0.56\sqrt{E/F_y}$
	Legs of single angle struts; legs of double angle struts with separators; unstiffened elements, i.e., supported along one edge	b/t	NA	$0.45\sqrt{E/F_y}$
	Stems of tees	d/t	NA	$0.75\sqrt{E/F_y}$

Example Problems

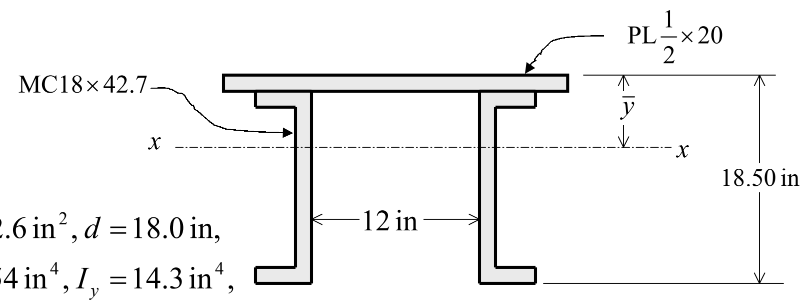
- Table 3. (cont'd) Limiting Width-Thickness Ratios for Compression Elements

Stiffened Elements	Webs in combined flexural and axial compression	h/t_w	for $P_u/\phi_b P_y \le 0.125$ [c], [g] $3.76\sqrt{\frac{E}{F_y}}\left(1-\frac{2.75P_u}{\phi_b P_y}\right)$ for $P_u/\phi_b P_y > 0.125$ [c],[g] $1.12\sqrt{\frac{E}{F_y}}\left(2.33-\frac{P_u}{\phi_b P_y}\right) \ge 1.49\sqrt{\frac{E}{F_y}}$	[h] $5.70\sqrt{\frac{E}{F_y}}\left(1-0.74\frac{P_u}{\phi_b P_y}\right)$
	All other uniformly compressed stiffened elements, i.e., supported along two edges	b/t h/t_w	NA	$1.49\sqrt{E/F_y}$
	Circular hollow sections In axial compression In flexure	D/t	[d] NA $0.07E/F_y$	$0.11E/F_y$ $0.31E/F_y$

Example Problems

- Example 2

  Determine the design strength $\phi_c P_n$ of the axially loaded column shown in the figure if KL = 19 ft and 50 ksi steel is used.



$[A = 12.6\text{ in}^2, d = 18.0\text{ in},$
$I_x = 554\text{ in}^4, I_y = 14.3\text{ in}^4,$
$\bar{x} = 0.877\text{ in from back of C}]$

Example Problems

- Example 2 (cont'd)

$$A = 20\left(\frac{1}{2}\right) + 2(12.6) = 35.2\text{ in}^2$$

$$\bar{y}\text{ from top} = \frac{(0.5\times 20)(0.25)+(2\times 12.6)(9.5)}{35.2} = 6.87\text{ in}$$

$$I_x = 2(554) + 2\left[12.6(9.25-6.69)^2\right] + \frac{20(0.5)^3}{12} + 10(6.69-0.25)^2 = 1{,}688\text{ in}^4$$

$$I_y = 2(14.3) + 2\left[12.6(6+0.877)^2\right] + \frac{0.5(20)^3}{12} = 1554\text{ in}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{1688}{35.2}} = 6.92\text{ in}$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{1554}{35.2}} = 6.64\text{ in}$$ ← Controls

Example Problems

- Example 2 (cont'd)

$$\frac{KL}{r} = \frac{KL}{r_y} = \frac{(12 \times 19)}{6.64} = 34.34$$

For KL/r = 34 and 35, Table 3-50 of the LRFD Manual, Page 16.I-145, gives respectively the following values for $\phi_c F_{cr}$: 39.1 ksi and 38.9 ksi

$$\begin{matrix} 34 & 39.1 \\ 34.34 & \phi_c F_{cr} \\ 35 & 38.9 \end{matrix} \Rightarrow \frac{\phi_c F_{cr} - 39.1}{38.9 - 39.1} = \frac{34.34 - 34}{35 - 34} \Rightarrow \phi_c F_{cr} = 39.03 \text{ ksi}$$

Therefore, the design strength = $\phi_c P_n = \phi_c A_g F_{cr}$

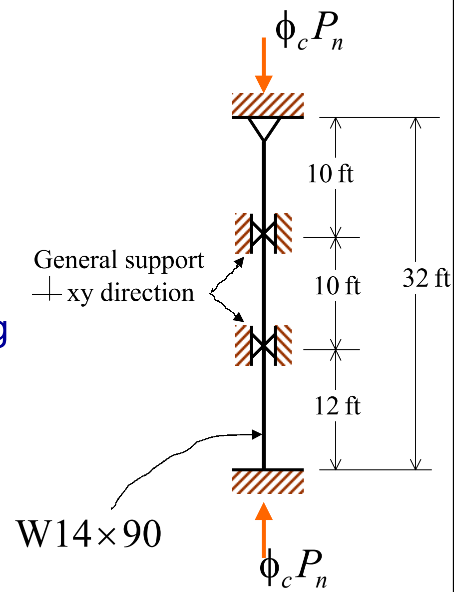
=39.03 (35.2) = 1374 k

Example Problems

- Example 3
  a. Using Table 3.50 of Part 16 of the LRFD Manual, determine the design strength $\phi_c P_n$ of the 50 ksi axially loaded W14 × 90 shown in the figure. Because of its considerable length, this column is braced perpendicular to its weak axis at the points shown in the figure. These connections are assumed to permit rotation of the member in a plane parallel to the plane of the flanges. At the same time, however, they are assumed to prevent translation or sideway and twisting

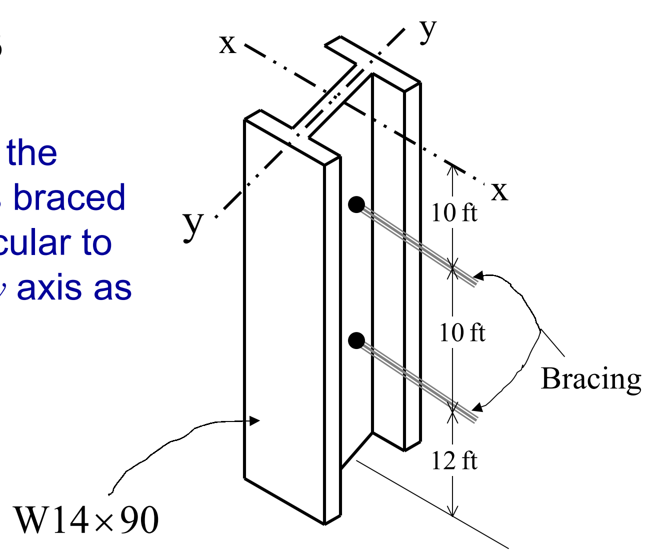
Example Problems

- Example 3 (cont'd)
  of the cross section about a longitudinal axis passing through the shear center of the cross section.
  - Repeat part (a) using the column tables of Part 4 of the LRFD Manual.



Example Problems

- Example 3 (cont'd)
  - Note that the column is braced perpendicular to its weak y axis as shown.



Example Problems

- Example 3 (cont'd)
  a. The following properties of the W14 × 90 can be obtained from the LRFD Manual as

$$A = 26.5 \text{ in}^2 \qquad r_x = 6.14 \text{ in} \qquad r_y = 3.70 \text{ in}$$

Determination of effective lengths:

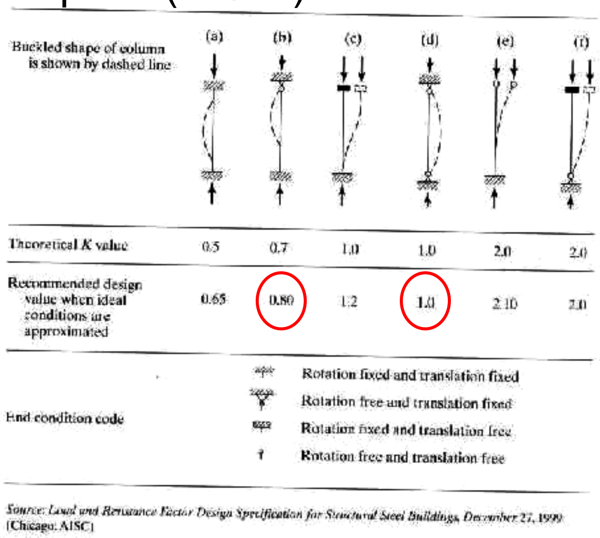
$$K_x L_x = (0.8)(32) = 25.6 \text{ ft}$$
$$K_y L_y = (1.0)(10) = 10 \text{ ft} \longleftarrow \text{Governs for } K_y L_y$$
$$K_x L_y = (0.8)(12) = 9.6 \text{ ft}$$

See Table for the K values

Example Problems

- Example 3 (cont'd)

Table 1



Buckled shape of column is shown by dashed line	(a)	(b)	(c)	(d)	(e)	(f)
Theoretical K value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.10	2.0

End condition code:
- Rotation fixed and translation fixed
- Rotation free and translation fixed
- Rotation fixed and translation free
- Rotation free and translation free

Source: Load and Resistance Factor Design Specification for Structural Steel Buildings, December 27, 1999 (Chicago: AISC)

Example Problems

- Example 3 (cont'd)

Computations of slenderness ratios:

$$\left(\frac{KL}{r}\right)_x = \frac{12\times 25.6}{6.14} = 50.03 \leftarrow \text{Governs}$$

$$\left(\frac{KL}{r}\right)_y = \frac{12\times 10}{3.70} = 32.43$$

Design Strength:

$$\frac{KL}{r} = 50.03 \approx 50, \text{Table 3-50 gives } \phi_c F_{cr} = 35.4 \text{ ksi}$$

$$\therefore \phi_c P_n = \phi_c F_{cr} A_g = 35.4(26.5) = 938 \text{ k}$$

Example Problems

- Example 3 (cont'd)

b. Using columns tables of Part 4 of LRFD Manual:

Note: from part (a) solution, there are two different KL values:

$$K_x L_x = 25.6 \text{ ft and } K_y L_y = 10 \text{ ft}$$

Which value would control? This can accomplished as follows:

$$\frac{K_x L_x}{r_x} = \text{Equivalent } \frac{K_y L_y}{r_y}$$

Example Problems

- Example 3 (cont'd)

$$\text{Equivalent } K_y L_y = r_y \frac{K_x L_x}{r_x} = \frac{K_x L_x}{r_x / r_y}$$

The controlling K_y L_y for use in the tables is larger of the real K_y L_y = 10 ft, or equivalent K_y L_y:

$$\frac{r_x}{r_y} \text{ for W14} \times 90 \text{ from bottom of column tables} = 1.66$$

$$\text{Equivalent } K_y L_y = \frac{25.6}{1.66} = 15.43 > K_y L_y = 10 \text{ ft}$$

For $K_y L_y = 15.42$ and by interpolation:

$$\phi_c P_n = 938 \text{ k}$$

Example Problems

- Example 3 (cont'd)

The Interpolation Process:

- For K_y L_y = 15 ft and 16 ft, column table (P. 4-23) of Par 4 of the LRFD Manual, gives respectively the following values for ϕ_c P_n: 947 k and 925 k. Therefore, by interpolation:

$$\begin{matrix} 15 & 947 \\ 15.42 & \phi_c P_n \\ 16 & 925 \end{matrix} \Rightarrow \frac{\phi_c P_n - 947}{925 - 947} = \frac{15.42 - 15}{16 - 15} \Rightarrow \phi_c P_n = 938 \text{ k}$$