CHAPTER

5b

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# DEVELOPMENT, SPLICES, AND SIMPLE SPAN BAR CUTOFFS

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Part I – Concrete Design and Analysis


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**ENCE 355 - Introduction to Structural Design**

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## Development Length: Compression Bars

- Deformed Bars in Compression
  - The method for determining the development length in compression $l_d$ involves finding the the basic development length $l_{db}$ and multiplying it by applicable modification factors.
  - The modification factors reflect special conditions.
  - Note: $l_d$ shall not be less than 8 in.

# Development Length: Compression Bars

- Basic Development Length (compression)

  The basic development length in compression is given by

$$l_{db} = 0.02 d_b \frac{f_y}{\sqrt{f'_c}} \qquad (1)$$

  But it shouldn't be less than $0.0003 f_y d_b$ according to the ACI Code, Section 12.3.



# Development Length: Compression Bars

- Modification Factors (Compression)
  - The following modification factors may be applied to the basic development length for compression bars:
    1. Reinforcement in excess of that required:

$$\frac{A_s \text{ required}}{A_s \text{ provided}}$$

    2. Bars enclosed within a spiral that is not less than ¼ in. in diameter and not more than 4 in. in pitch or within No. 4 ties and spaced at not more than 4 in. on center: USE 0.75

# Development Length: Compression Bars

- Tables 1a through 1c gives values of the basic development length $l_{db}$ for compression bars in inches for the following combinations of $f'_c$ and $f_y$:

$f'_c$: 3000, 4000, 5000, and 6000 psi

$f_y$: 40,000, 50,000, and 60,000 psi



# Development Length: Compression Bars

Table 1a. Basic Development Length $l_{db}$ for Compression Bars (in.) for $f_y$ = 40,000 psi

| Bar Size | $f'_c$ (normal-weight concrete), psi | | | |
|---|---|---|---|---|
| | 3000 | 4000 | 5000 | 6000 |
| 3 | 5.5 | 4.7 | 4.5 | 4.5 |
| 4 | 7.3 | 6.3 | 6.0 | 6.0 |
| 5 | 9.1 | 7.9 | 7.5 | 7.5 |
| 6 | 11.0 | 9.5 | 9.0 | 9.0 |
| 7 | 12.8 | 11.1 | 10.5 | 10.5 |
| 8 | 14.6 | 12.6 | 12.0 | 12.0 |
| 9 | 16.5 | 14.3 | 13.5 | 13.5 |
| 10 | 18.5 | 16.1 | 15.2 | 15.2 |
| 11 | 20.6 | 17.8 | 16.9 | 16.9 |
| 14 | 24.7 | 21.4 | 20.3 | 20.3 |
| 18 | 33.0 | 28.5 | 27.1 | 27.1 |

# Development Length: Compression Bars

Table 1b. Basic Development Length $l_{db}$ for Compression Bars (in.) for $f_y$ = 50,000 psi

| Bar Size | $f'_c$ (normal-weight concrete), psi | | | |
|---|---|---|---|---|
| | 3000 | 4000 | 5000 | 6000 |
| 3 | 6.8 | 5.9 | 5.6 | 5.6 |
| 4 | 9.1 | 7.9 | 7.5 | 7.5 |
| 5 | 11.4 | 9.9 | 9.4 | 9.4 |
| 6 | 13.7 | 11.9 | 11.3 | 11.3 |
| 7 | 16.0 | 13.8 | 13.1 | 13.1 |
| 8 | 18.3 | 15.8 | 15.0 | 15.0 |
| 9 | 20.6 | 17.8 | 16.9 | 16.9 |
| 10 | 23.2 | 20.1 | 19.1 | 19.1 |
| 11 | 25.7 | 22.3 | 21.2 | 21.2 |
| 14 | 30.9 | 26.8 | 25.4 | 25.4 |
| 18 | 41.2 | 35.7 | 33.9 | 33.9 |



# Development Length: Compression Bars

Table 1c. Basic Development Length $l_{db}$ for Compression Bars (in.) for $f_y$ = 60,000 psi

| Bar Size | $f'_c$ (normal-weight concrete), psi | | | |
|---|---|---|---|---|
| | 3000 | 4000 | 5000 | 6000 |
| 3 | 8.2 | 7.1 | 6.8 | 6.8 |
| 4 | 11.0 | 9.5 | 9.0 | 9.0 |
| 5 | 13.7 | 11.9 | 11.3 | 11.3 |
| 6 | 16.4 | 14.2 | 13.5 | 13.5 |
| 7 | 19.2 | 16.6 | 15.8 | 15.8 |
| 8 | 21.9 | 19.0 | 18.0 | 18.0 |
| 9 | 24.7 | 21.4 | 20.3 | 20.3 |
| 10 | 27.8 | 24.1 | 22.9 | 22.9 |
| 11 | 30.9 | 26.8 | 25.4 | 25.4 |
| 14 | 37.1 | 32.1 | 30.5 | 30.5 |
| 18 | 49.4 | 42.8 | 40.6 | 40.6 |

# Development Length: Standard Hooks in Tension

- Need for Hooks
  - In the event that the desired development length in tension cannot be furnished, it will be necessary to provide mechanical anchorage at the end of the bars



# Development Length: Standard Hooks in Tension

- Need for Hooks

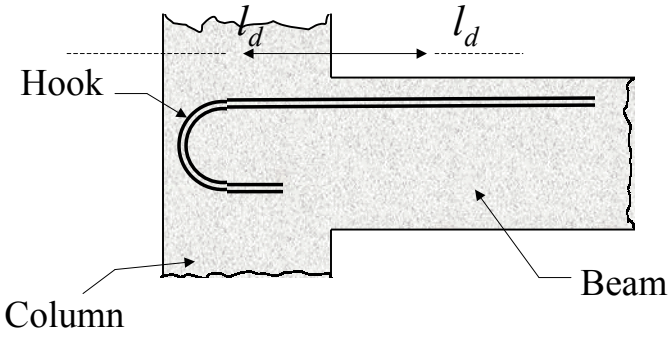



Figure 1. 180°-Hook

# Development Length: Standard Hooks in Tension

- Types of Hooks
  - Anchorage for main or primary reinforcement is usually accomplished by means of 90˚ or 180˚ hook.
  - The dimensions and bend radii for these hooks have been standardized by the ACI Code.
  - Standard reinforcement hooks are shown in Fig. 2.



# Development Length: Standard Hooks in Tension

- Types of Hooks

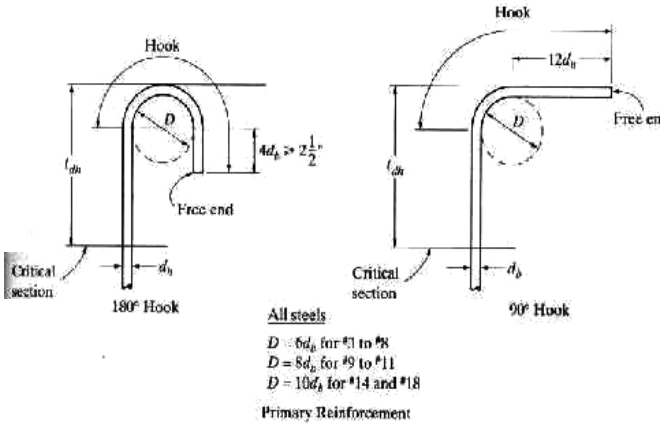



**Figure 2a. Standard Hooks**

# Development Length: Standard Hooks in Tension

- Types of Hooks

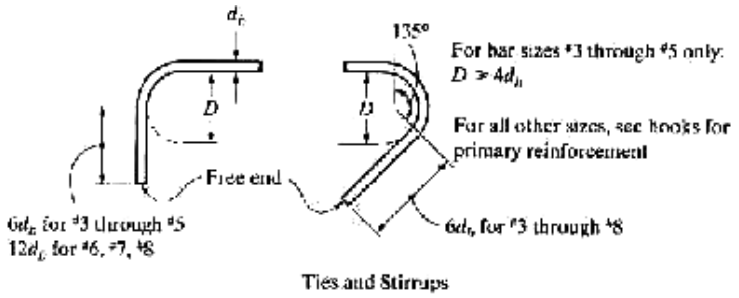



**Figure 2b. Standard Hooks**



# Development Length: Standard Hooks in Tension

- Types of Hooks

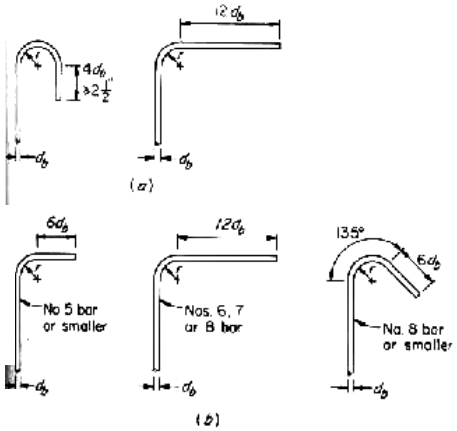



**Figure 3. Standard Hooks**

# Development Length: Standard Hooks in Tension

- ACI Code Specifications
  - The ACI Code specifies that the development length $l_{dh}$ (see Fig. 2) for deformed bars in tension, which terminate in a standard hook, be computed as the product of a basic development length $l_{hb}$ and any applicable modification factors.
  - Mathematically, this may expressed as

$$l_{dh} = l_{hb} \times \text{MF} \qquad (2)$$



# Development Length: Standard Hooks in Tension

- ACI Basic Development Length, $l_{hb}$
  - For a hooked bar with $f_y$ = 60,000 psi,

$$l_{hb} = \frac{1200 d_b}{\sqrt{f'_c}} \qquad (3)$$

  - Table 1 (Table A-13, Textbook) provides values for $l_{hb}$.

# Development Length: Standard Hooks in Tension

- ACI Modification Factors (MF)
  - Modification factors are to be used if applicable:
    1. Bars with $f_y$ other than 60,000 psi, USE

$$\text{MF} = \frac{f_y}{60{,}000} \qquad (4)$$

    2. Concrete cover for No. 3 through No. 11: Side cover (normal to the plane of the hook ) ≥ 2 ½ in. and, for 90° hooks, cover on bar extension beyond the bend ≥ 2 in.: USE 0.7 for MF



# Development Length: Standard Hooks in Tension

- ACI Modification Factors, MF (cont'd)
    3. Ties or stirrups: For No. 3 through No. 11 with hook enclosed vertically of horizontally within ties or stirrup ties spaced along the full development length $l_{dh}$ not greater than $3d_b$: USE MF = 0.8.
    4. Reinforcement in excess of that required, where anchorage or development for $f_y$ is not specifically required:

$$\text{MF} = \frac{A_s \text{ required}}{A_s \text{ provided}} \qquad (5)$$

# Development Length: Standard Hooks in Tension

- ACI Modification Factors, MF (cont'd)
  5. Lightweight aggregate concrete: USE

$$\text{MF} = 1.3 \qquad (6)$$

  6. Epoxy-coated reinforcement: USE

$$\text{MF} = 1.2 \qquad (7)$$



# Development Length: Standard Hooks in Tension

- ACI Modification Factors, MF (cont'd)
  - The basic development length $l_{hb}$ must be multiplied by the application factors outlined in the previous viewgraphs.
  - In no case may $l_{db}$ be less than $8d_b$ or 6 in., whichever is greater.

# Development Length: Standard Hooks in Tension

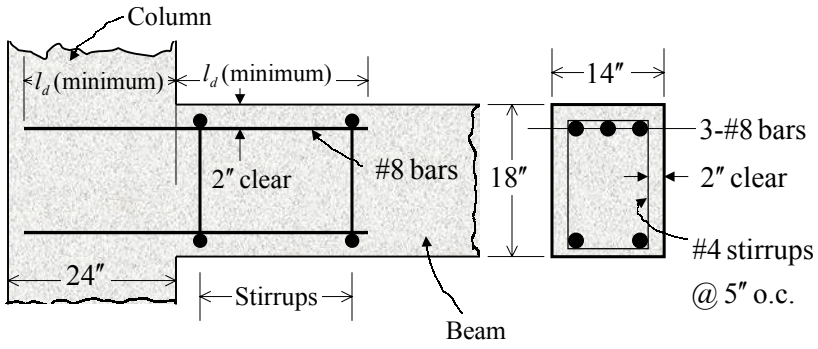
- Example 1

  Determine the anchorage or development length required for the conditions shown in the figure. Use $f'_c$ = 3,000 psi (normal-weight concrete) and $f_y$ = 60,000 psi. The No. 8 bars may be categorized as top bars. Assume a side cover on the main bars of 2 ½ in. minimum. Bars are uncoated.



# Development Length: Standard Hooks in Tension

- Example 1 (cont'd)



# Development Length: Standard Hooks in Tension

- Example 1 (cont'd)

**Anchorage into the exterior column:**

1. Establish values for the multiplying factors $\alpha$, $\beta$, $\gamma$, and $\lambda$:
   a. $\alpha$ = 1.3 (the bars are top bars).
   b. $\beta$ = 1.0 (the bars are uncoated).
   c. $\gamma$ = 1.0 (the bars are No. 8)
   d. $\lambda$ = 1.0 (normal-weight concrete used)
2. The product $\alpha \times \beta$ = 1.3 < 1.7 (OK)



# Development Length: Standard Hooks in Tension

- Example 1 (cont'd)

3. Determine $c$. Based on cover (center of bar to nearest concrete surface), consider the clear cover, the No. 4 stirrups diameter, and one-half the diameter of the No. 8 bar:

$$c = 2.0 + 0.5 + \frac{1.0}{2} = 3.0 \text{ in.}$$

Based on bar spacing:

$$c = \frac{14 - 2(2.0) - 2(0.5) - 2(0.5)}{2(2)} = 2.0 \text{ in.}$$ ← Controls

Therefore, use $c$ = 2.0 in (smallest)

# Development Length: Standard Hooks in Tension

Table 2. ASTM Standard - English Reinforcing Bars

| Bar Designation | Diameter in | Area $in^2$ | Weight lb/ft |
|---|---|---|---|
| #3 [#10] | 0.375 | 0.11 | 0.376 |
| #4 [#13] | 0.500 | 0.20 | 0.668 |
| #5 [#16] | 0.625 | 0.31 | 1.043 |
| #6 [#19] | 0.750 | 0.44 | 1.502 |
| #7 [#22] | 0.875 | 0.60 | 2.044 |
| #8 [#25] | 1.000 | 0.79 | 2.670 |
| #9 [#29] | 1.128 | 1.00 | 3.400 |
| #10 [#32] | 1.270 | 1.27 | 4.303 |
| #11 [#36] | 1.410 | 1.56 | 5.313 |
| #14 [#43] | 1.693 | 2.25 | 7.650 |
| #18 [#57] | 2.257 | 4.00 | 13.60 |

Note: Metric designations are in brackets



# Development Length: Standard Hooks in Tension

- Example 1 (cont'd)

4. The figure shows stirrups in the beam. However, there are no stirrups in the column, and $K_{tr}$ can be taken as zero for the column anchorage.
5. Check $(c + K_{tr})/d_b \le 2.5$:

$$\frac{c + K_{tr}}{d_b} = \frac{2.0 + 0}{1.0} = 2.0 < 2.5 \Rightarrow \text{USE } 2.0$$

6. The access reinforcement can be ignored and the factor applied can be omitted.

# Development Length: Standard Hooks in Tension

- Example 1 (cont'd)

7. Calculate $l_d$:

$$l_d = \frac{3}{40}\left(\frac{f_y}{\sqrt{f'_c}}\right)\left[\frac{\alpha\beta\gamma\lambda}{\left(\frac{c+k_{tr}}{d_b}\right)}\right]d_b$$

$$l_d = \frac{3}{40}\left(\frac{60{,}000}{\sqrt{3{,}000}}\right)\left[\frac{1.3(1)(1)(1)}{2.0}\right](1.0) = 53.4 \text{ in.} > 12 \text{ in.}$$ OK

Since 53.4 in > 24 in. (column width), use a standard hook, either a 90° hook or a 180° hook.



# Development Length: Standard Hooks in Tension

- Example 1 (cont'd)

**Anchorage using a standard 180° hook:**

1. The basic development length $l_{hb}$ for the standard hook shown in the figure can be computed from

$$l_{hb} = \frac{1200 d_b}{\sqrt{f'_c}} = \frac{1200(1)}{\sqrt{3000}} = 21.9 \text{ in. (also check Table 1c)}$$

2. The only applicable MF is based on side cover of 2 ½ in.

   Therefore, USE MF = 0.7

# Development Length: Standard Hooks in Tension

Table 1c. Basic Development Length $l_{db}$ for Compression Bars (in.) for $f_y = 60{,}000$ psi

| Bar Size | $f_c'$ (normal-weight concrete), psi | | | |
|---|---|---|---|---|
| | 3000 | 4000 | 5000 | 6000 |
| 3 | 8.2 | 7.1 | 6.8 | 6.8 |
| 4 | 11.0 | 9.5 | 9.0 | 9.0 |
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# Development Length: Standard Hooks in Tension

- Example 1 (cont'd)

3. The required development length is then calculated from

$$l_{dh} = l_{hb} \times \text{MF} = 21.9(0.7) = 15.3 \text{ in.}$$

Check minimum :

$$\text{minimum } l_{dh} = 8d_b \geq 6 \text{ in.}$$

$$8d_b = 8 \text{ in.} < 15.3 \text{ in.} \quad \text{OK}$$

The minimum width of column required is

$$15.3 + 2.5 = 17.8 \text{ in.} < 24 \text{ in. (column width)} \quad \text{OK}$$

Therefore, the hook will fit into the column.

# Development Length: Standard Hooks in Tension

- Example 1 (cont'd)

**Anchorage into beam:**

The development length required if bars are straight can be taken as 53.4 in. as determined previously. However, this number is conservative ($K_{tr} = 0$).

To determine a more accurate value, we have to take into consideration the transverse reinforcement index $K_{tr}$ because there are stirrups in the beam.



# Development Length: Standard Hooks in Tension

- Example 1 (cont'd)

Area of 2 #4 stirrups

$$K_{tr} = \frac{A_{tr} f_{yt}}{1500sn} = \frac{0.4(60{,}000)}{1500(5)(3)} = 1.067$$

$$\frac{c + K_{tr}}{d_b} = \frac{2.0 + 1.067}{1.0} = 3.07 < 2.5 \Rightarrow \text{USE } 2.5$$

$$l_d = \frac{3}{40}\left(\frac{f_y}{\sqrt{f'_c}}\right)\left[\frac{\alpha\beta\gamma\lambda}{\left(\frac{c + k_{tr}}{d_b}\right)}\right] d_b$$

$$l_d = \frac{3}{40}\left(\frac{60{,}000}{\sqrt{3{,}000}}\right)\left[\frac{1.3(1)(1)(1)}{2.5}\right](1.0) = 42.7 \text{ in.} > 12 \text{ in. OK}$$

# Development Length: Standard Hooks in Tension

Table 2. ASTM Standard - English Reinforcing Bars

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Note: Metric designations are in brackets



# Development Length: Standard Hooks in Tension

- Example 1 (cont'd)

**Anchorage into beam (cont'd):**

- The development length required if bars are straight is **42.7 in.**
- Therefore, the bars must extend at least this distance into the span.
- Figure 4 shows the detailed sketch for the development length.

# Development Length: Standard Hooks in Tension

- Example 1 (cont'd)

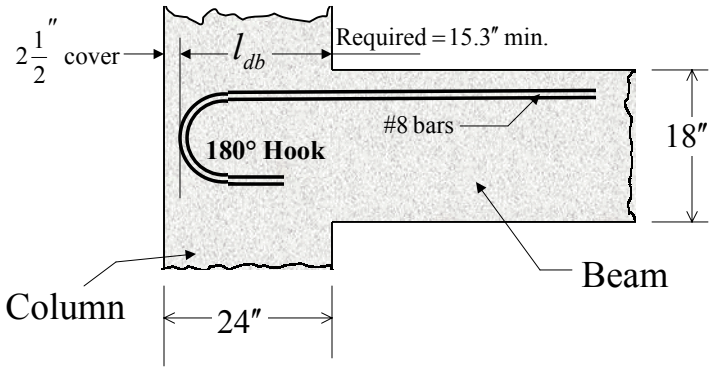


**Figure 4. Detailed Sketch for Example 1**