CHAPTER

4a

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Reinforced Concrete Design — Fifth Edition

SHEAR IN BEAMS

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Part I – Concrete Design and Analysis

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ENCE 355 - Introduction to Structural Design

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Introduction

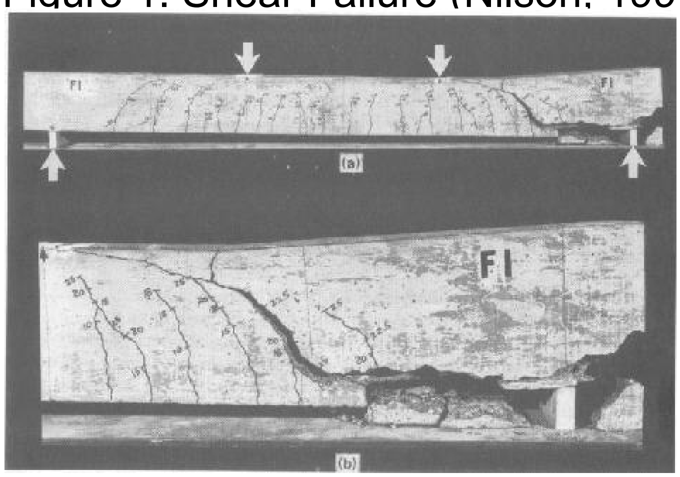
- The previous chapters dealt with the flexural strength of beams.
- Beams must also have an adequate safety margin against other types of failure such as shear, which may be more dangerous than flexural failure.
- The shear forces create additional tensile stresses that must be considered.

Introduction

- Shear Failure
  - Shear failure of reinforced concrete beam, more properly called "***diagonal tension failure***", is difficult to predict accurately.
  - In spite of many years of experimental research and the use of highly sophisticated computational tools, it is not fully understood.
  - If a beam without properly designed for shear reinforcement is overloaded to failure, shear collapse is likely to occur suddenly.

Introduction

Figure 1. Shear Failure (Nilson, 1997)



(a) Overall view, (b) detail near right support.

Introduction

- Shear Failure (cont'd)
  - Figure 1 shows a shear-critical beam tested under point loading.
  - With no shear reinforcement provided, the member failed immediately upon formation of the critical crack in the high-shear region near the right support.

Introduction

- Shear Failure (cont'd)

When are the shearing effects so large that they cannot be ignored as a design consideration?

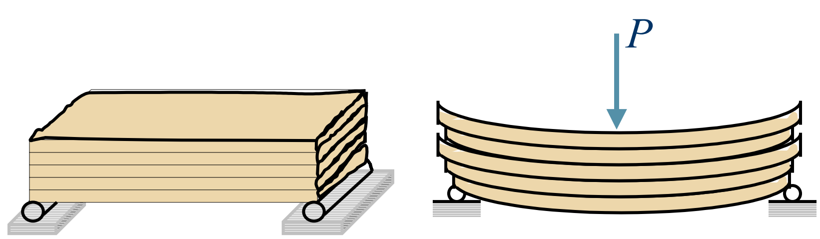
  - It is somehow difficult to answer this question.
  - Probably the best way to begin answering this question is to try to approximate the shear stresses on the cross section of the beam.

Introduction

- Shear Failure (cont'd)
  - Suppose that a beam is constructed by stacking several slabs or planks on top of another without fastening them together.
  - Also suppose this beam is loaded in a direction normal to the surface of these slabs.
  - When a bending load is applied, the stack will deform as shown in Fig. 2a.
  - Since the slabs were free to slide on one one another, the ends do not remain even but staggered.

Introduction

- Shear Failure (cont'd)



(a) Unloaded Stack of Slabs (b) Unglued Slabs loaded

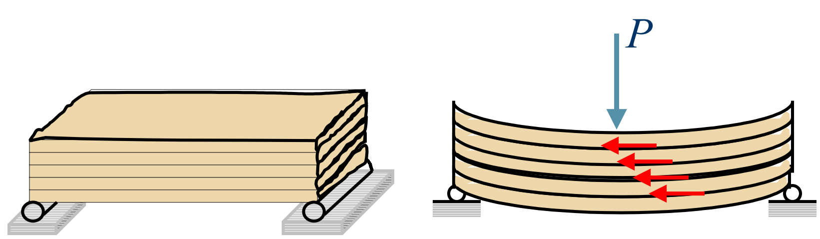
Figure 2a

Introduction

- Shear Failure (cont'd)
  - Each of the slabs behaves as independent beam, and the total resistance to bending of n slabs is approximately n times the resistance of one slab alone.
  - If the slabs of Fig. 2b is fastened or glued, then the staggering or relative longitudinal movement of slabs would disappear under the action of the force. However, shear forces will develop between the slabs.
  - In this case, the stack of slabs will act as a solid beam.

Introduction

- Shear Failure (cont'd)



(c) Glued Slabs Unloaded

(d) Glued Slabs loaded

Figure 2b

Introduction

- Shear Failure (cont'd)
  - The fact that this solid beam does not exhibit this relative movement of longitudinal elements after the slabs are glued indicates the presence of *shearing stresses* on longitudinal planes.
  - Evaluation of these shearing stresses will be discussed in the next couple of viewgraphs.

Introduction

- Theoretical Background
  - The concept of stresses acting in homogeneous beams are usually covered in various textbooks of mechanics of materials (strength of materials).
  - It can be shown that when the material is elastic, shear stresses can be computed from

$$v = \frac{VQ}{Ib} \qquad (1)$$

v = shear stress
V = external shear force
I = moment of inertia about neutral axis
Q = statical moment of area about N.A.
b = width of the cross section

Introduction

- Theoretical Background
  - Also, when the material is elastic, bending stresses can be computed from

$$f = \frac{Mc}{I} \qquad (2)$$

f = bending stress
M = external or applied moment
c = the distance from the neutral axis to out fiber of the cross section
I = moment of inertia of the cross section about N.A.

Introduction

- Theoretical Background
  - All points in the length of the beam, where the shear and bending moment are not zero, and at locations other than the extreme fiber or neutral axis, are subject to both shearing stresses and bending stresses.
  - The combination of these stresses produces maximum normal and shearing stresses in a specific plane inclined with respect to the axis of the beam.

Introduction

- Theoretical Background
  - The distributions of the bending and shear stresses acting individually are shown in Figs. 3, 4, 5, and 6.

$$f = \frac{Mc}{I}$$

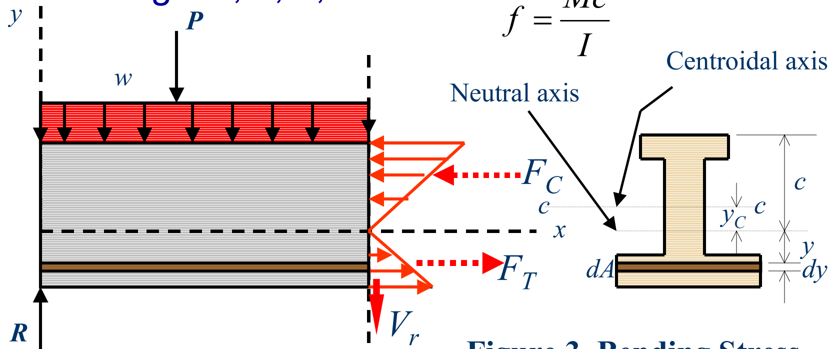
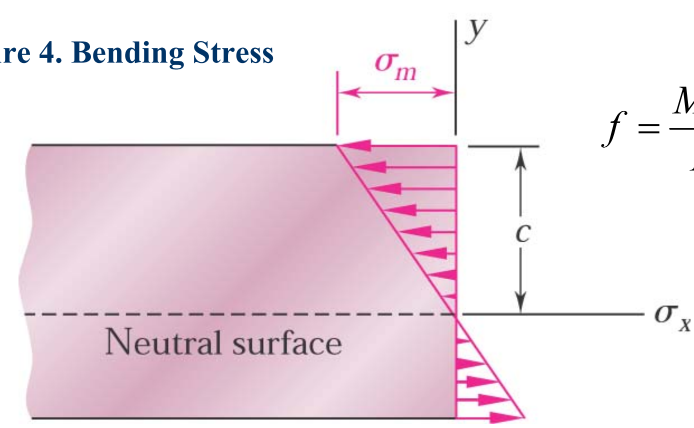


Figure 3. Bending Stress

Introduction

- Theoretical Background

Figure 4. Bending Stress



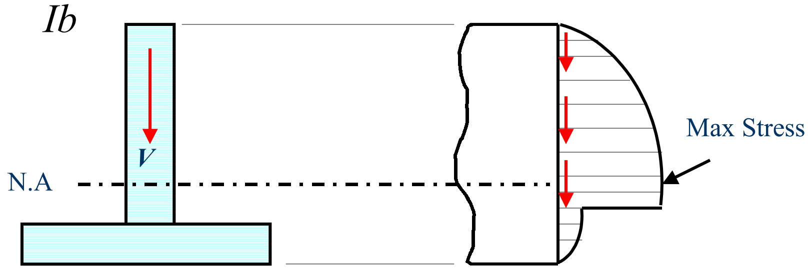
$$f = \frac{Mc}{I}$$

Introduction

- Theoretical Background

Figure 5. Vertical Shearing Stress

$$v = \frac{VQ}{Ib}$$

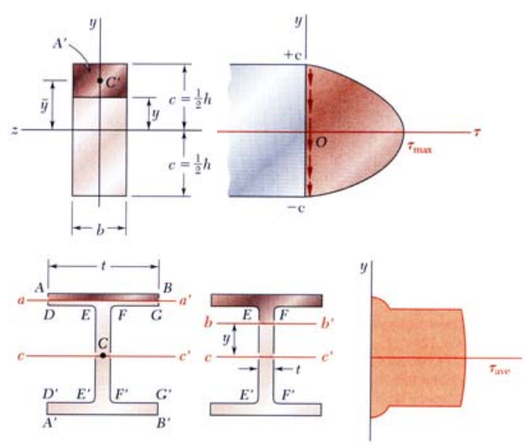


Introduction

- Theoretical Background

Figure 6. Vertical Shearing Stress

$$v = \frac{VQ}{Ib}$$

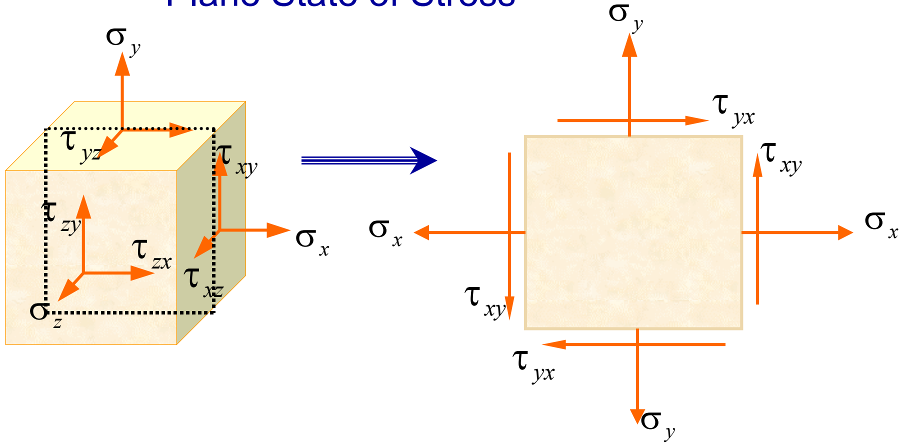


Introduction

- Principal Planes
  - The combination of bending moment and shearing stresses is of such a nature that maximum normal and shearing shearing stresses at a point in a beam exist on planes that are inclined with the axis of the beam.
  - These planes are commonly called ***principal planes***, and the stresses that act on them are referred to as ***principal stresses***.

Introduction

- Principal Planes
  - Plane State of Stress



Introduction

- Principal Planes
  - Plane State of Stress

Components:

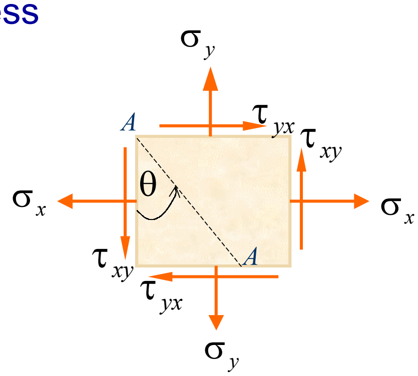
Normal Stress σ_x

Normal Stress σ_y

Shearing Stress τ_{xy}

Shearing Stress τ_{yx}

$$\tau_{xy} = \tau_{yx}$$



Introduction

- Principal Stresses
  - The principal stresses in a beam subjected to shear and bending may be computed using the following equation:

$$f_{pr} = \frac{f}{2} \pm \sqrt{\frac{f^2}{4} + v^2} \qquad (3)$$

f_{pr} = principal stress
f = bending stress computed from Eq. 2
v = shearing stress computed from Eq. 1

Introduction

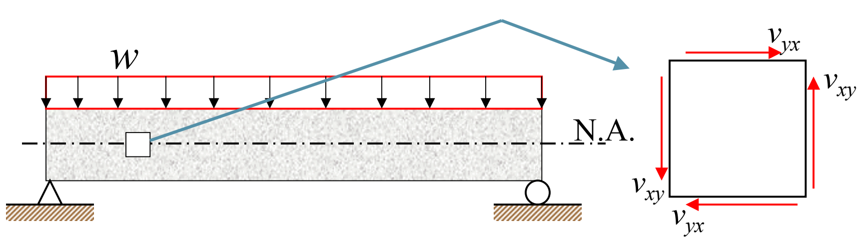
- Orientation Principal Planes
  - The orientation of the principal planes may be calculated using the following equation:

$$\alpha = \frac{1}{2}\tan^{-1}\left(\frac{2v}{f}\right) \qquad (4)$$

  - Note that at the neutral axis of the beam, the principal stresses will occur at a 45^0 angle.

Introduction

- State of Stress at the Neutral Axis of a Homogeneous Beam



(a) Beam under Uniform Loading

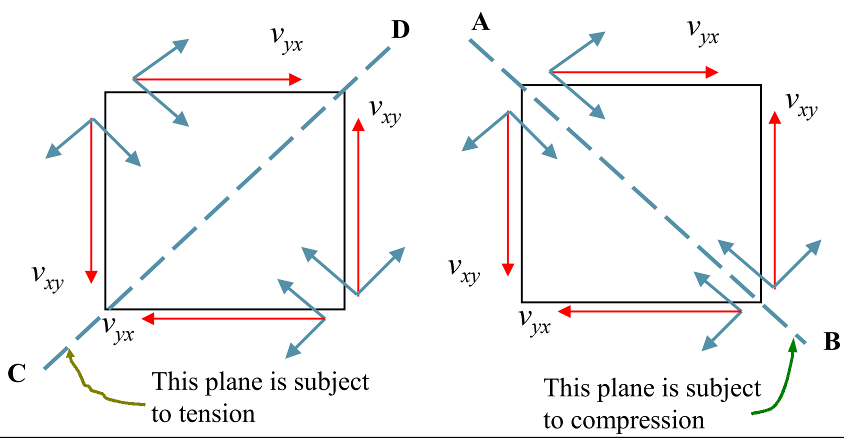
(b) Stresses on Unit Element

Figure 7. Shear Stress Relationship

Introduction

- State of Stress at the Neutral Axis of a Homogeneous Beam
  - Diagonal Tension

Figure 8



Introduction

- State of Stress at the Neutral Axis of a Homogeneous Beam
  - Diagonal Tension
    - Plane A-B is subjected to compression
    - While Plane C-D is subjected to tension.
    - The tension in Plane C-D is historically called "***diagonal tension***".
    - Note that concrete is strong in compression but weak in tension, and there is a tendency for concrete to crack on the plane subject to tension.
    - When the tensile stresses are so high, it is necessary to provide reinforcement.

Introduction

- Diagonal Tension Failure
  - In the beams with which we are concerned, where the length over which a shear failure could occur (the shear span) is in excess of approximately three times the effective depth, the diagonal tension failure would be the mode of failure in shear.
  - Such a failure is shown in Figs. 1 and 8.
  - For longer shear spans in plain concrete beams, cracks due to flexural tensile stresses would occur long before cracks due to diagonal tension.

Introduction

- Diagonal Tension Failure

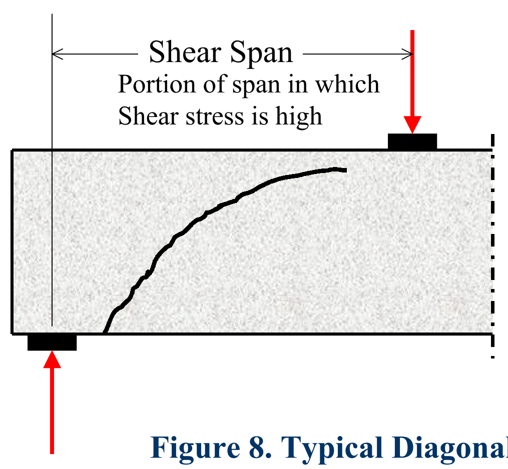
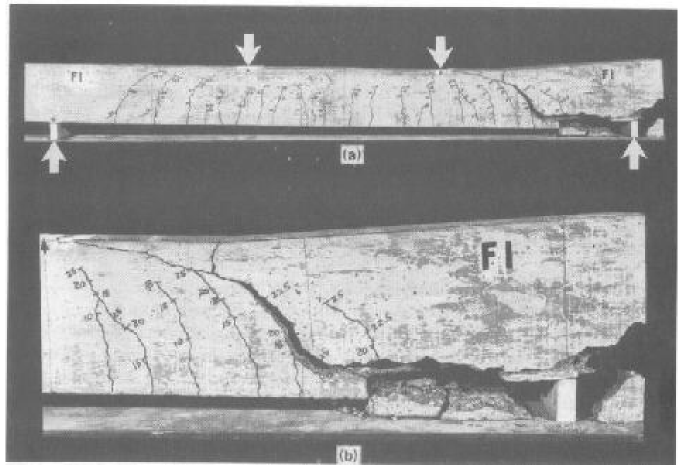


Figure 8. Typical Diagonal Tension Failure

Introduction

Figure 1. Shear Failure (Nilson, 1997)



(a) Overall view, (b) detail near right support.

Introduction

- Basis of ACI Design for Shear
  - The ACI provides design guidelines for shear reinforcement based on the vertical shear force V_u that develops at any given cross section of a member.
  - Although it is really the diagonal tension for which shear reinforcing must be provided, diagonal tensile forces (or stresses) are not calculated.
  - Traditionally, vertical shear force has been taken to be good indicator of diagonal tension present.

Shear Reinforcement Design Requirements

- Web Reinforcement
  - The basic rationale for the design of the shear reinforcement, or web reinforcement as it usually called in beams, is to provide steel to cross the diagonal tension cracks and subsequently keep them from opening.
  - In reference to Fig. 8, it is seen that the web reinforcement may take several forms such as:

Shear Reinforcement Design Requirements

- Web Reinforcement (cont'd)
  1. Vertical stirrups (see Fig. 9)
  2. Inclined or diagonal stirrups
  3. The main reinforcement bent at ends to act as inclined stirrups (see Fig. 10).
  - The most common form of web reinforcement used is the vertical stirrup.
  - This reinforcement appreciably increases the ultimate shear capacity of a bending member.

Shear Reinforcement Design Requirements

- Web Reinforcement (cont'd)
  – Vertical Stirrups

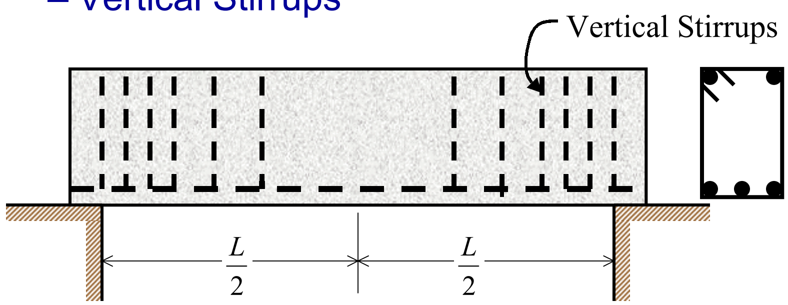


Figure 9. Types of Web Reinforcement

Shear Reinforcement Design Requirements

- Web Reinforcement (cont'd)
  – Bent-up Longitudinal Bars

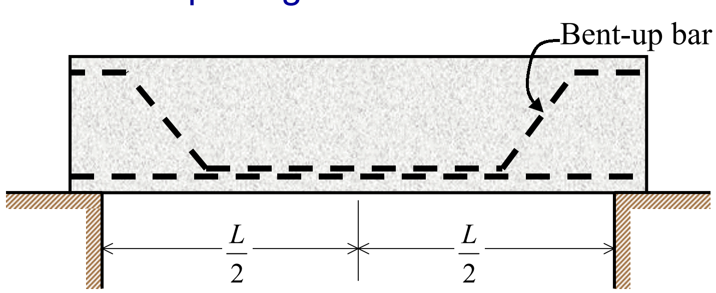


Figure 9. Type of Web Reinforcement

Shear Reinforcement Design Requirements

- ACI Code Provisions for Shear Reinforcement

  For member that are subject to shear and flexure only, the amount of shear force that the concrete (unreinforced for shear)can resist is

$$V_c = 2\sqrt{f_c'}\,b_w d \qquad (5)$$

Note, for rectangular beam $b_w = b$

Shear Reinforcement Design Requirements

- ACI Code Provisions for Shear Reinforcement
  - The design shear force V_u results from the application of factored loads.
  - Values of V_u are most conveniently determined using a typical shear force diagram.
  - Theoretically, no web reinforcement is required if

$$V_u \le \phi V_c \qquad (6)$$

Shear Reinforcement Design Requirements

Table 1. Strength Reduction Factors

Type of Loading	ϕ
Bending	0.90
Shear and Torsion	0.85
Compression members (spirally reinforced)	0.75
Compression Members (tied)	0.70
Bearing on Concrete	0.70

Shear Reinforcement Design Requirements

- ACI Code Provisions for Shear Reinforcement
  - However, the code requires that a minimum area of shear reinforcement be provided in all reinforced concrete flexural members when $V_u > \frac{1}{2}\phi V_c$, except as follows:
    - In slabs and footings
    - In concrete joist construction as defined in the code.
    - In beams with a total depth of less than 10 in., 2 ½ times the flange thickness, or one-half the width of the web, whichever is greater.

Shear Reinforcement Design Requirements

- ACI Code Provisions for Shear Reinforcement
  - In cases where shear reinforcement is required for strength or because $V_u > ½\ \phi V_c$, the minimum area of shear reinforcement shall be computed from

$$A_v = \frac{50 b_w s}{f_y} \qquad (7)$$

Shear Reinforcement Design Requirements

- ACI Code Provisions for Shear Reinforcement

Where

A_v = total cross-sectional area of web reinforcement within a distance s, for single loop stirrups, $A_v = 2A_s$
A_s = cross-sectional area of the stirrup bar (in^2)
b_w = web width = b for rectangular section (in.)
s = center-to-center spacing of shear reinforcement in a direction parallel to the longitudinal reinforcement (in.)
f_y = yield strength of web reinforcement steel (psi)

Shear Reinforcement Design Requirements

- ACI Code Provisions for Shear Reinforcement

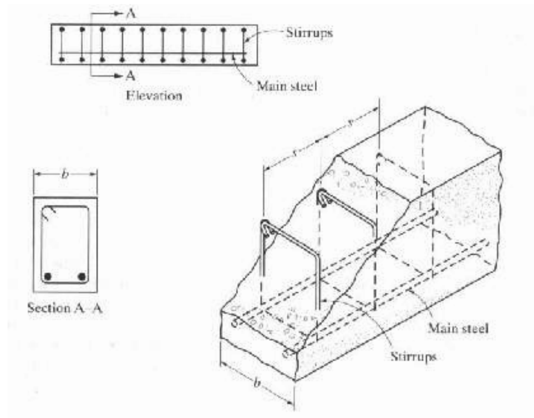


Figure 10. Isometric section showing stirrups partially exposed

Shear Reinforcement Design Requirements

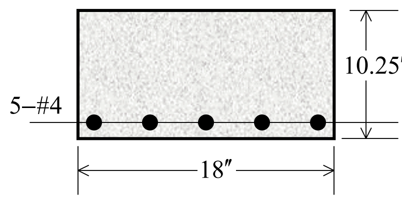
- Example

A reinforced concrete beam of rectangular cross section shown in the figure is reinforced for moment only (no shear reinforcement). Beam width b = 18 in., d = 10.25 in., and the reinforcing is five No. 4 bars. Calculate the maximum factored shear force V_u permitted on the member by the ACI Code. Use = 4,000 psi, and f_y = 60,000 psi.

Shear Reinforcement Design Requirements

- Example (cont'd)

Since no shear reinforcement
Is provided, the ACI Code
Requires that



$$\text{maximum } V_u = \frac{1}{2}\phi V_c$$

$$= \frac{1}{2}\phi\left(2\sqrt{f_c'}b_w d\right)$$

$$= \frac{1}{2}(0.85)(2)\left(\sqrt{4000}\right)(18)(10.25) = 9918 \text{ lb}$$