CHAPTER

3e

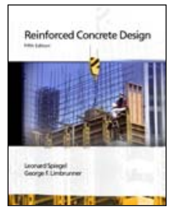
Prentice Hall

REINFORCED CONCRETE BEAMS: T-BEAMS AND DOUBLY REINFORCED BEAMS

A. J. Clark School of Engineering •Department of Civil and Environmental Engineering

Part I – Concrete Design and Analysis

FALL 2002



By
Dr. Ibrahim. Assakkaf

ENCE 355 - Introduction to Structural Design

Department of Civil and Environmental Engineering
University of Maryland, College Park

Procedure for Design of Doubly Reinforced Beams for Moment

1. Assume that $d = h - 4$ in.
2. Establish the total design moment M_u.
3. Check if a doubly reinforced beam is necessary. From tables for coefficient of resistance (Table 1, Tables A-7 to A-11, Textbook), obtain the maximum $\bar{k}$ and compute maximum ϕM_n for a singly reinforced beam:

$$\text{maximum}\, \phi M_n = \phi b d^2 \bar{k}$$

Procedure for Design of Doubly Reinforced Beams for Moment

Table A-10 Textbook

Sample Values

Table 1.
Coefficient of Resistance

ρ	$\bar{k}$
0.0010	0.0595
0.0011	0.0654
0.0012	0.0712
0.0013	0.0771
0.0014	0.0830
0.0015	0.0888
0.0016	0.0946
0.0017	0.1005
0.0018	0.1063
0.0019	0.1121
0.0020	0.1179
0.0021	0.1237

Procedure for Design of Doubly Reinforced Beams for Moment

4. If $\phi M_n < M_u$, design the beam as a doubly reinforced beam. If $\phi M_n \geq M_u$, the beam can be designed as a beam reinforced with tension steel only.

For a Doubly Reinforced Beam

5. Provide a concrete-steel couple having the steel ratio

$$\rho = 0.9\rho_{\max} = 0.9(0.75\rho_b)$$

Procedure for Design of Doubly Reinforced Beams for Moment

with this value of ρ, enter the appropriate table and determine $\bar{k}$.

6. Determine the moment capacity of the concrete-steel couple:

$$\phi M_{n1} = \phi b d^2 \bar{k}$$

Find the steel required for the concrete-steel couple:

$$\text{required } A_{s1} = \rho b d$$

Procedure for Design of Doubly Reinforced Beams for Moment

7. Find the remaining moment that must be resisted by the steel-steel couple:

$$\text{required}\, \phi M_{n2} = M_u - \phi M_{n1}$$

8. Considering the steel-steel couple, find the required compressive force in the steel (assume that d'= 3 in.):

$$N_{C2} = \frac{\phi M_{n2}}{\phi\left(d - d'\right)}$$

Procedure for Design of Doubly Reinforced Beams for Moment

9. Since $N_{C2} = A_s' f_s'$, compute f_s' so that may eventually be determined. This can be accomplished by using the neutral-axis location of the concrete-steel couple and checking the strain in the compression steel with ε_y. Thus

$$a = \frac{A_{s1} f_y}{0.85 f_c' b}$$

Procedure for Design of Doubly Reinforced Beams for Moment

$$c = \frac{a}{\beta_1} \qquad \varepsilon_y = \frac{f_y}{E_s} = \frac{f_y}{29 \times 10^6}$$

$$\varepsilon_s' = \frac{0.003(c - d')}{c}$$

If $\varepsilon_s' \geq \varepsilon_y$, the compressive steel has yielded at the ultimate moment and $f_s' = f_y$. If $\varepsilon_s' \geq \varepsilon_y$, then calculate

$$f_s' = \varepsilon_s' E_s$$

Procedure for Design of Doubly Reinforced Beams for Moment

10. Since $N_{C2} = A_s' f_s'$,

$$\text{required } A_s' = \frac{N_{C2}}{f_s'}$$

11. Determine the required A_{s2}:

$$A_{s2} = \frac{f_s' A_s'}{f_y}$$

12. Find the total tension steel required:

$$A_s = A_{s1} + A_{s2}$$

Procedure for Design of Doubly Reinforced Beams for Moment

13. Select the compressive steel, A_s'.
14. Select the tensile steel, A_s. Check the required beam width. Preferably, place the bars in one layer.
15. Check the actual d and compare it with the actual d. If the actual d is slightly in excess of the assumed d, the design will be slightly conservative (on the safe side). If the actual d is less

Procedure for Design of Doubly Reinforced Beams for Moment

than the assumed d, the design may be on the unconservative side and an analysis and possibly revision should be considered.

16. Sketch the detailed design.

Procedure for Design of Doubly Reinforced Beams for Moment

- Example 1

  Design a rectangular reinforced concrete beam to resist a total design moment M_u of 780 ft-kips (this includes the moment due to the weight of the beam). The beam size is limited to 15 in. maximum width and 30 in. maximum overall depth. Use f_c' = 3000 psi and f_y = 60,000 psi. If compressive steel is required, make d'= 2.5 in.

Procedure for Design of Doubly Reinforced Beams for Moment

■ Example 1 (cont'd)

Assume that $d = 30 - 4 = 26$ in.

Given:

$$f_c' = 3\text{ ksi},\quad f_y = 60\text{ ksi},\quad \text{and}\quad M_u = 780\text{ ft-kips}$$

– For singly reinforced beam:

- Max ρ=0.0161 (Table 2, Table A-5, Text)
- Therefore, $\bar{k}$ = 0.7831 ksi (Table 3, Table A-8, Text)

$$\phi M_{n1} = \phi b d^2 \bar{k} = \frac{0.9(15)(26)^2(0.7831)}{12} = 595.6\text{ ft-kips}$$

Procedure for Design of Doubly Reinforced Beams for Moment

■ Example 1 (cont'd)

Table A-5 Textbook

Table 2
Design Constants

f_c' (psi)	$\left[\frac{3\sqrt{f_c'}}{f_y} \geq \frac{200}{f_y}\right]$	$\rho_{max} = 0.75\,\rho_b$	Recommended Design Values ρ_b	Recommended Design Values $\bar{k}$ (ksi)
F_y = 40,000 psi				
3,000	0.0050	0.0278	0.0135	0.4828
4,000	0.0050	0.0372	0.0180	0.6438
5,000	0.0053	0.0436	0.0225	0.8047
6,000	0.0058	0.0490	0.0270	0.9657
F_y = 50,000 psi				
3,000	0.0040	0.0206	0.0108	0.4828
4,000	0.0040	0.0275	0.0144	0.6438
5,000	0.0042	0.0324	0.0180	0.8047
6,000	0.0046	0.0364	0.0216	0.9657
F_y = 60,000 psi				
3,000	0.0033	0.0161	0.0090	0.4828
4,000	0.0033	0.0214	0.0120	0.6438
5,000	0.0035	0.0252	0.0150	0.8047
6,000	0.0039	0.0283	0.0180	0.9657
F_y = 75,000 psi				
3,000	0.0027	0.0116	0.0072	0.4828
4,000	0.0027	0.0155	0.0096	0.6438
5,000	0.0028	0.0182	0.0120	0.8047
6,000	0.0031	0.0206	0.0144	0.9657

Value used in example

Procedure for Design of Doubly Reinforced Beams for Moment

- Example 1 (cont'd)

Table 3. Steel Ratio Versus Coefficient of Resistance for $f_c' = 3{,}000$ psi and $f_y = 60{,}000$ psi

ρ	$\bar{k}$	ρ	$\bar{k}$	ρ	$\bar{k}$
0.0082	0.4446	0.0118	0.6098	0.0154	0.7567
0.0083	0.4494	0.0119	0.6141	0.0155	0.7605
0.0084	0.4542	0.0120	0.6184	0.0156	0.7643
0.0085	0.4590	0.0121	0.6227	0.0157	0.7681
0.0086	0.4638	0.0122	0.6270	0.0158	0.7719
0.0087	0.4686	0.0123	0.6312	0.0159	0.7756
0.0088	0.4734	0.0124	0.6355	0.0160	0.7794
0.0089	0.4781	0.0125	0.6398	0.0161	0.7831
0.0090	0.4828	0.0126	0.6440	0.0162	0.7868
0.0091	0.4876	0.0127	0.6482	0.0163	0.7905

Values used in example

Procedure for Design of Doubly Reinforced Beams for Moment

- Example 1 (cont'd)

Since ($\phi M_n = 595.6$ ft-k) < ($M_u = 780$ ft-k)

Design the beam as doubly reinforced

For concrete-steel couple:

Use $\rho = 0.9\,(\rho_{max}) = 0.9\,(0.0161) = 0.0145$

Therefore,

$\bar{k} = 0.7216$ ksi (from Table 4, Table A-8, Text)

and

$$\phi M_{n1} = \phi b d^2 \bar{k} = \frac{0.9(15)(26)^2(0.7216)}{12} = 549 \text{ ft - kips}$$

CHAPTER 3e. R/C BEAMS: T-BEAMS AND DOUBLY REINFORCED BEAMS — Slide No. 16

ENCE 355 ©Assakkaf

Procedure for Design of Doubly Reinforced Beams for Moment

- Example 1 (cont'd)

Table 4. Steel Ratio Versus Coefficient of Resistance for = 3,000 psi and f_y = 60,000 psi

ρ	$\bar{k}$	ρ	$\bar{k}$	ρ	$\bar{k}$
0.0104	0.5477	0.0140	0.7017	0.0176	0.8374
0.0105	0.5522	0.0141	0.7057	0.0177	0.8409
0.0106	0.5567	0.0142	0.7097	0.0178	0.8444
0.0107	0.5612	0.0143	0.7137	0.0179	0.8479
0.0108	0.5657	0.0144	0.7177	0.0180	0.8514
0.0109	0.5702	0.0145	0.7216	0.0181	0.8548
0.0110	0.5746	0.0146	0.7256	0.0182	0.8583
0.0111	0.5791	0.0147	0.7295	0.0183	0.8617
0.0112	0.5835	0.0148	0.7334	0.0184	0.8651
0.0113	0.5879	0.0149	0.7373	0.0185	0.8685

Values used in example

CHAPTER 3e. R/C BEAMS: T-BEAMS AND DOUBLY REINFORCED BEAMS — Slide No. 17

ENCE 355 ©Assakkaf

Procedure for Design of Doubly Reinforced Beams for Moment

- Example 1 (cont'd)

$$\text{required } A_{s1} = \rho bd = 0.0145(15)(26) = 5.66 \text{ in}^2$$

$$\text{required } \phi M_{n2} = M_u - \phi M_{n1} = 780 - 549 = 231 \text{ ft-kips}$$

$$d' = 2.5 \text{ in. (given)}$$

– Therefore, the required force for the steel-steel couple is

$$N_{C2} = \frac{\phi M_{n2}}{\phi(d - d')} = \frac{12 \times 231}{0.9(26 - 2.5)} = 131 \text{ kips}$$

Procedure for Design of Doubly Reinforced Beams for Moment

- Example 1 (cont'd)
  - Check compression steel stress:

$$a = \frac{A_{s1} f_y}{0.85 f_c' b} = \frac{5.66(60)}{0.85(3)(15)} = 8.88 \text{ in.}$$

  - Thus, c can be calculated as follows:

$$a = \beta_1 c \Rightarrow c = \frac{a}{\beta_1} = \frac{8.88}{0.85} = 10.45 \text{ in.}$$

See Eq. 1 (next slide)

  - and

$$\varepsilon_s' = \frac{0.003(c - d')}{c} = \frac{0.003(10.45 - 2.5)}{10.45} = 0.00228$$

Procedure for Design of Doubly Reinforced Beams for Moment

- Example 1 (cont'd)
  - The value of β_1 may be determined from the following equation:

$$\beta_1 = \begin{cases} 0.85 & \text{for } f_c' \leq 4{,}000 \text{ psi} \\ 1.05 - 5\times10^{-5} f_c' & \text{for } 4{,}000 \text{ psi} < f_c' \leq 8{,}000 \text{ psi} \\ 0.65 & \text{for } f_c' > 8{,}000 \text{ psi} \end{cases} \quad (1)$$

Procedure for Design of Doubly Reinforced Beams for Moment

- Example 1 (cont'd)
  - The yield strain of steel can be computed as

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60{,}000}{29 \times 10^6} = 0.00207$$

  - Since ($\varepsilon_s' = 0.00228$) > ($\varepsilon_y = 0.00207$), the compressive steel has yielded at the ultimate moment and $f_s' = f_y$.

Procedure for Design of Doubly Reinforced Beams for Moment

- Example 1 (cont'd)
  - Since $N_{C2} = A_s' f_s' = A_s' f_y$,

$$\text{required } A_s' = \frac{N_{C2}}{f_y} = \frac{131}{60} = 2.18 \text{ in}^2$$

  - Select steel bars:
    - Use 2 #10 (A_s' = 2.54 in^2) for compression rebars.

$$A_s = A_{s1} + A_{s2} = 5.66 + 2.18 = 7.84 \text{ in}^2$$

    - Use 8 #9 (A_s = 8.00 in^2) for tension rebars in two layers.

Procedure for Design of Doubly Reinforced Beams for Moment

- Example 1 (cont'd)

Table 5. Areas of Multiple of Reinforcing Bars (in^2)

Number	Bar number								
of bars	#3	#4	$5	#6	#7	#8	#9	#10	#11
1	0.11	0.20	0.31	0.44	0.60	0.79	1.00	1.27	1.56
2	0.22	0.40	0.62	0.88	1.20	1.58	2.00	2.54	3.12
3	0.33	0.60	0.93	1.32	1.80	2.37	3.00	3.81	4.68
4	0.44	0.80	1.24	1.76	2.40	3.16	4.00	5.08	6.24
5	0.55	1.00	1.55	2.20	3.00	3.95	5.00	6.35	7.80
6	0.66	1.20	1.86	2.64	3.60	4.74	6.00	7.62	9.36
7	0.77	1.40	2.17	3.08	4.20	5.53	7.00	8.89	10.92
8	0.88	1.60	2.48	3.52	4.80	6.32	8.00	10.16	12.48
9	0.99	1.80	2.79	3.96	5.40	7.11	9.00	11.43	14.04
10	1.10	2.00	3.10	4.40	6.00	7.90	10.00	12.70	15.60

Table A-2 Textbook

Procedure for Design of Doubly Reinforced Beams for Moment

- Example 1 (cont'd)
  - Check the ACI Code requirements for minimum width of 4 #9 bars in one layer: From Table 6, min b = 12.0 in < 15 in. OK

$$\text{Actual } d = 30 - 1.5 - \underbrace{0.375}_{\text{Dia. \#3 stirrup}} - \underbrace{1.128}_{\text{Dia. \#9 bar}} - \underbrace{0.5}_{\text{Half spacing between layers}} = 26.5 \text{ in.}$$

$$(\text{actual } d = 26.5'') > (\text{assumed } d = 26.0'') \quad \text{OK}$$

Procedure for Design of Doubly Reinforced Beams for Moment

■ Example 1 (cont'd)

Table 6. Minimum Required Beam Width, *b* (in.)

Number	Bar number							
of bars	**# 3 and #4**	**$5**	**#6**	**#7**	**#8**	**#9**	**#10**	**#11**
2	6.0	6.0	6.5	6.5	7.0	7.5	8.0	8.0
3	7.5	8.0	8.0	8.5	9.0	9.5	10.5	11.0
4	9.0	9.5	10.0	10.5	11.0	12.0	13.0	14.0
5	10.5	11.0	11.5	12.5	13.0	14.0	15.5	16.5
6	12.0	12.5	13.5	14.0	15.0	16.5	18.0	19.5
7	13.5	14.5	15.0	16.0	17.0	18.5	20.5	22.5
8	15.0	16.0	17.0	18.0	19.0	21.0	23.0	25.0
9	16.5	17.5	18.5	20.0	21.0	23.0	25.5	28.0
10	18.0	19.0	20.5	21.5	23.0	25.5	28.0	31.0

Table A-3 Textbook

Procedure for Design of Doubly Reinforced Beams for Moment

Table 7. ASTM Standard - English Reinforcing Bars

Bar Designation	Diameter in	Area in^2	Weight lb/ft
#3 [#10]	0.375	0.11	0.376
#4 [#13]	0.500	0.20	0.668
#5 [#16]	0.625	0.31	1.043
#6 [#19]	0.750	0.44	1.502
#7 [#22]	0.875	0.60	2.044
#8 [#25]	1.000	0.79	2.670
#9 [#29]	1.128	1.00	3.400
#10 [#32]	1.270	1.27	4.303
#11 [#36]	1.410	1.56	5.313
#14 [#43]	1.693	2.25	7.650
#18 [#57]	2.257	4.00	13.60

Note: Metric designations are in brackets

Procedure for Design of Doubly Reinforced Beams for Moment

■ Example 1 (cont'd)

$$\text{Actual } d' = 1.5 + 0.375 - \frac{1.27}{2} = 2.51 \text{ in.} \quad \text{OK}$$

Dia. #3 stirrup (0.375); Half Dia. of #10 bar ($\frac{1.27}{2}$)

– Check steel ductility:

$$\rho \frac{A_{s1}}{bd} = \frac{8.00 - 2.54}{15(26.5)} = 0.0137$$

$$(\rho = 0.0137) < (\rho_{max} = 0.0161) \quad \text{OK}$$

Procedure for Design of Doubly Reinforced Beams for Moment

■ Example 1 (cont'd)

Final Detailed Sketch: