Reinforced Concrete Design Fifth Edition

CHAPTER

3b

REINFORCED CONCRETE BEAMS: T-BEAMS AND DOUBLY REINFORCED BEAMS

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Part I – Concrete Design and Analysis

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Development of T-Beam $A_{s,\max}$

- Basic Relationships

(a)

$$c_b = \frac{87{,}000}{f_y + 87{,}000} d \qquad (1)$$

From Fig. 1:

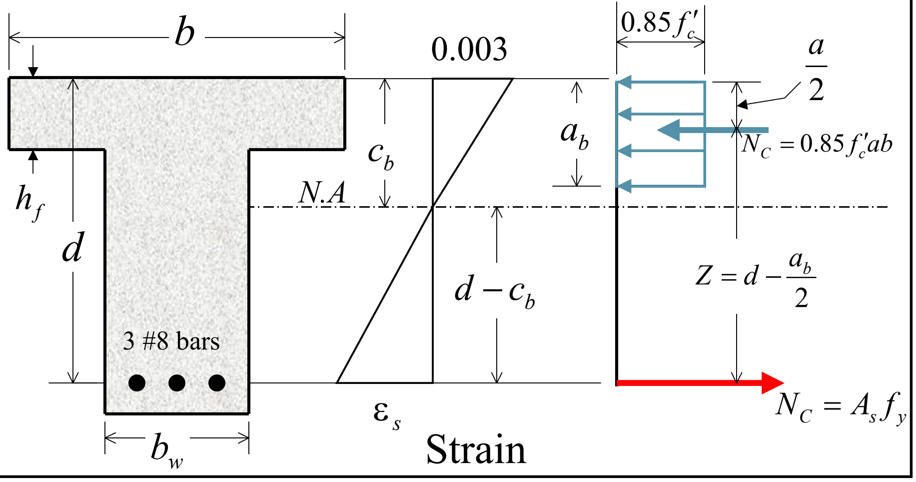
$$\frac{c_b}{0.003} = \frac{d}{0.003 + \varepsilon_s} = \frac{d}{0.003 + \frac{f_y}{E_s}} = \frac{d}{0.003 + \frac{f_y}{29 \times 10^6}}$$

$$c_b = \frac{0.003}{0.003 + \frac{f_y}{29 \times 10^6}} d = \frac{87{,}000}{87{,}000 + f_y} d$$

Development of T-Beam $A_{s,\max}$

- Basic Relationships

Figure 1



Development of T-Beam $A_{s,\max}$

- Basic Relationships
- (b) $a_b = 0.85c_b$ (where $\beta_1 = 0.85$) (2)
- (c) $N_{Cb} = 0.85f'_c\left[bh_f + b_w\left(a_b - h_f\right)\right]$ (3)
- (d) $N_{Cb} = N_{Tb} = A_{sb}f_y$ (4)
- (e) $A_{s,\max} = 0.75A_{sb}$ (5)

Development of T-Beam $A_{s,\max}$

- Basic Relationships

Combining Eqs. 1, 2, 3, 4, and 5, and solving for $A_{s,\max}$, the following expression is obtained:

$$A_{s,\max} = \frac{0.638}{f_y} f'_c h_f \left\{ b + b_w \left[\frac{\beta_b}{h_f} \left(\frac{87{,}000}{87{,}000 + f_y} d \right) - 1 \right] \right\} \qquad (6)$$

Substituting for various combinations of f'_c and f_y, $A_{s,\max}$ expressions result as listed in Table 1 (Table 3-1 Text)

Development of T-Beam $A_{s,\max}$

Table 1. Expressions for $A_{s,\max}$ (T-Beams)

f'_c (psi)	f_y (psi)	$A_{s,\max}$ (in^2)
3,000	40,000	$0.0478h_f\left\{b + b_w\left[\frac{0.582}{h_f}d - 1\right]\right\}$
	60,000	$0.0319h_f\left\{b + b_w\left[\frac{0.503}{h_f}d - 1\right]\right\}$
4,000	40,000	$0.0638h_f\left\{b + b_w\left[\frac{0.582}{h_f}d - 1\right]\right\}$
	60,000	$0.0425h_f\left\{b + b_w\left[\frac{0.503}{h_f}d - 1\right]\right\}$

Development of T-Beam $A_{s,\max}$

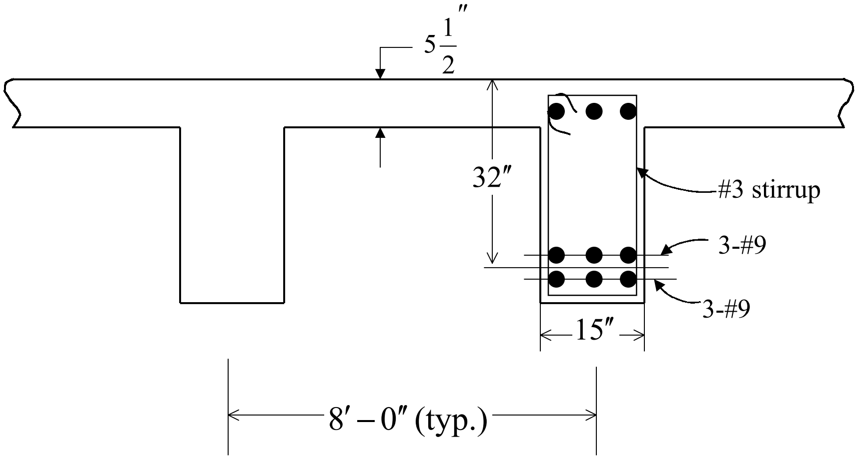
- Maximum Steel Reinforcement (ACI)
  - The maximum steel reinforcement as governed by the ACI Code can be obtained using Table 1.
  - If A_s exceeds $A_{s,\max}$, then the beam should be analyzed using $A_{s,\max}$ as an effective steel area.

Development of T-Beam $A_{s,\max}$

- Example 1
  - Find the practical moment strength ϕM_n for the T-beam in the floor system shown. The beam span is 31 ft-6 in. Use f_y = 60,000 psi and f'_c = 4,000 psi. Check the steel to ensure that it is within allowable limits.

Development of T-Beam $A_{s,\max}$

- Example 1 (cont'd)



Development of T-Beam $A_{s,\max}$

- Example 1 (cont'd)

Determine *b*: $A_s = 6.0\text{ in}^2$ (6 No. 9 bars, See Table 2)

$$\frac{\text{span}}{4} = \frac{31.5(12)}{4} = 94.5\text{ in.}$$

$$16h_f + b_w = 16(5.5) + 15 = 103\text{ in.}$$

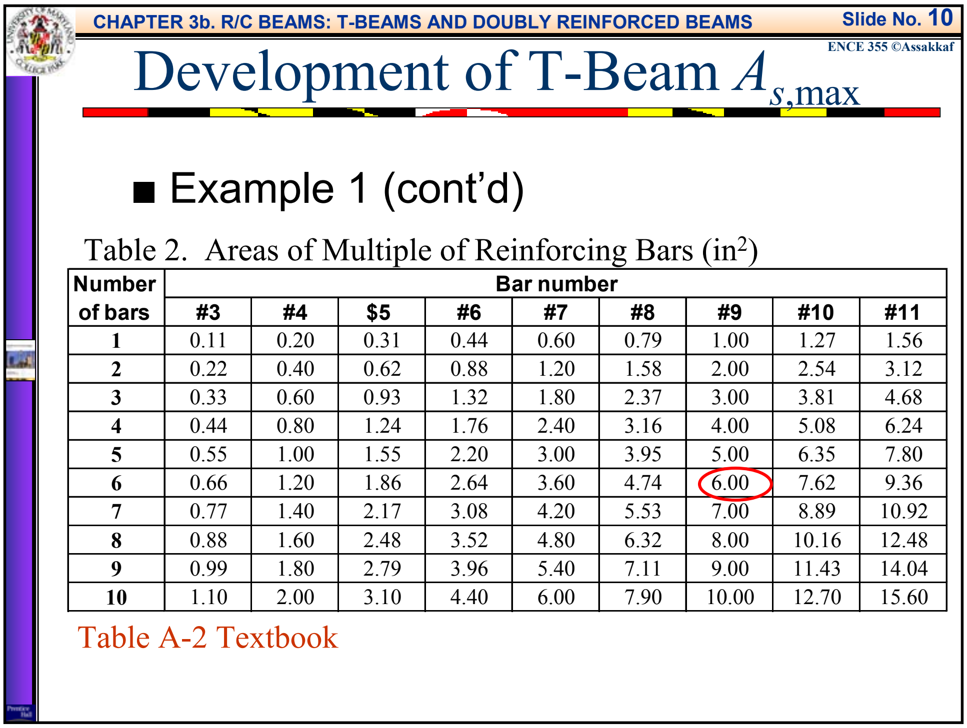
$$\text{Beam spacing} = 8(12) = 96\text{ in.}$$

$$\text{Therefore, use } b = 94.5\text{ in.}$$

For $A_{s,\min}$, CHECK:

$$A_{s,\min} = 0.0033(b_w)(d) = 0.0033(15)(32) = 1.58\text{ in}^2 < 6.0\text{ in}^2 \quad \textbf{OK}$$

See Table 3



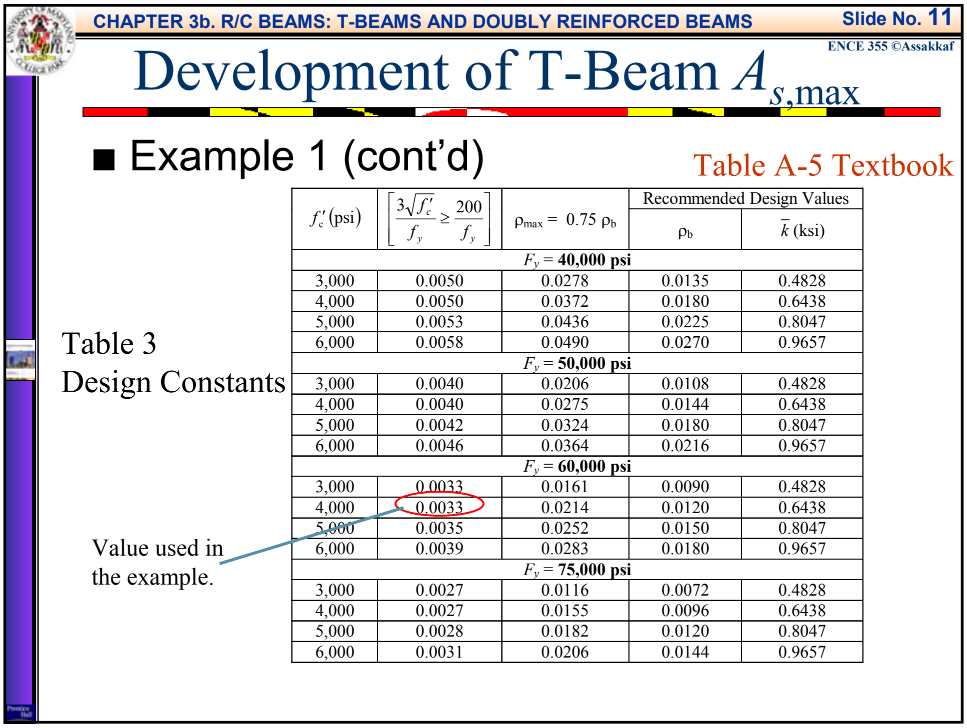
Development of T-Beam $A_{s,\max}$

- Example 1 (cont'd)

Table 2. Areas of Multiple of Reinforcing Bars (in^2)

Number of bars	Bar number								
	#3	#4	$5	#6	#7	#8	#9	#10	#11
1	0.11	0.20	0.31	0.44	0.60	0.79	1.00	1.27	1.56
2	0.22	0.40	0.62	0.88	1.20	1.58	2.00	2.54	3.12
3	0.33	0.60	0.93	1.32	1.80	2.37	3.00	3.81	4.68
4	0.44	0.80	1.24	1.76	2.40	3.16	4.00	5.08	6.24
5	0.55	1.00	1.55	2.20	3.00	3.95	5.00	6.35	7.80
6	0.66	1.20	1.86	2.64	3.60	4.74	6.00	7.62	9.36
7	0.77	1.40	2.17	3.08	4.20	5.53	7.00	8.89	10.92
8	0.88	1.60	2.48	3.52	4.80	6.32	8.00	10.16	12.48
9	0.99	1.80	2.79	3.96	5.40	7.11	9.00	11.43	14.04
10	1.10	2.00	3.10	4.40	6.00	7.90	10.00	12.70	15.60

Table A-2 Textbook



Development of T-Beam $A_{s,\max}$

- Example 1 (cont'd)

Table A-5 Textbook

Table 3
Design Constants

f'_c (psi)	$\left[\frac{3\sqrt{f'_c}}{f_y} \geq \frac{200}{f_y}\right]$	$\rho_{\max} = 0.75\,\rho_b$	Recommended Design Values	
			ρ_b	$\bar{k}$ (ksi)
F_y = **40,000 psi**				
3,000	0.0050	0.0278	0.0135	0.4828
4,000	0.0050	0.0372	0.0180	0.6438
5,000	0.0053	0.0436	0.0225	0.8047
6,000	0.0058	0.0490	0.0270	0.9657
F_y = **50,000 psi**				
3,000	0.0040	0.0206	0.0108	0.4828
4,000	0.0040	0.0275	0.0144	0.6438
5,000	0.0042	0.0324	0.0180	0.8047
6,000	0.0046	0.0364	0.0216	0.9657
F_y = **60,000 psi**				
3,000	0.0033	0.0161	0.0090	0.4828
4,000	0.0033	0.0214	0.0120	0.6438
5,000	0.0035	0.0252	0.0150	0.8047
6,000	0.0039	0.0283	0.0180	0.9657
F_y = **75,000 psi**				
3,000	0.0027	0.0116	0.0072	0.4828
4,000	0.0027	0.0155	0.0096	0.6438
5,000	0.0028	0.0182	0.0120	0.8047
6,000	0.0031	0.0206	0.0144	0.9657

Value used in
the example.

Development of T-Beam $A_{s,\max}$

- Example 1 (cont'd)

Check beam ductility by comparing $A_{s,\max}$ with actual A_s:

From Table 1 (Table 3-1, Text)

$$A_{s,\max} = 0.0425 h_f \left\{ b + b_w \left[\frac{0.503}{h_f} - 1 \right] \right\}$$

$$= 0.0425(5.5)\left\{ 94.5 + 15\left[\frac{0.503}{5.5}(32) - 1 \right] \right\} = 28.8\ \text{in}^2$$

Since (A_s = 6 in^2) < $A_{s,\max}$ =28.8 in^2 **OK**

The beam meets the ductility requirements, and the steel yields at the ultimate moment.

Development of T-Beam $A_{s,\max}$

- Example 1 (cont'd)

Determine if the beam can be analyzed as a rectangular T-beam or true T-beam:

$$N_T = A_s f_y = 6(60) = 360\ \text{kips}$$

$$N_{Cf} = 0.85 f_c' b h_f = 0.85(4)(94.5)(5.5) = 1{,}767.2\ \text{kips}$$

Since (N_{Cf} = 1,767.2 k) > (N_T = 360 k), the beam can be analyzed as a rectangular T-beam (simple analysis).

For flexure:

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{6(60)}{0.85(4)(94.5)} = 1.12\ \text{in.}$$

$$Z = d - \frac{a}{2} = 32 - \frac{1.12}{2} = 31.44\ \text{in.}$$

Development of T-Beam $A_{s,\max}$

- Example 1 (cont'd)

$$\phi M_n = \phi A_s f_y Z = \frac{(0.9)(6)(60)(31.44)}{12} = 849 \text{ ft - kips}$$

Alternative method for finding ϕM_n:

$$\rho = \frac{A_s}{bd} = \frac{6}{94.5(32)} = 0.002$$

For ρ= 0.002, go to Table 4 (Table A-10, Text) and find the Required $\bar{k}$:

$$\text{required } \bar{k} = 0.1179$$

$$\phi M_n = \phi b d^2 \bar{k} = \frac{0.9(94.5)(32)^2(0.1179)}{12} = 856 \text{ ft - kips}$$

Development of T-Beam $A_{s,\max}$

- Example 1 (cont'd)

Table 4.
Coefficient of Resistance

Table A-10 Textbook

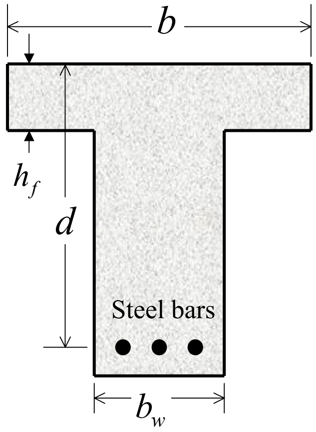
ρ	$\bar{k}$
0.0010	0.0595
0.0011	0.0654
0.0012	0.0712
0.0013	0.0771
0.0014	0.0830
0.0015	0.0888
0.0016	0.0946
0.0017	0.1005
0.0018	0.1063
0.0019	0.1121
0.0020	0.1179
0.0021	0.1237

Value used in the example.

T-Beam Design (For Moment)

- Quantities that need to be determined in the design of a T-beam are:

1. Flange Dimensions:
   - Effective Width, b
   - Thickness, h_f
2. Web Dimensions:
   - Width, b_w
   - Height
3. Area of Tension Steel, A_s



T-Beam Design (For Moment)

- In normal situations, the flange thickness is determined by the design of the slab, and the web size is determined by the shear and moment requirements at the end of the supports for continuous beam.
- Column size sometimes dictate web width.

T-Beam Design (For Moment)

- ACI code dictates permissible effective flange width, b.
- The flange itself generally provides more than sufficient compression area; therefore the stress block usually lies completely in the flange.
- Thus, most T-beam are only wide rectangular beams with respect to flexural behavior.

T-Beam Design (For Moment)

- Design Method
  - The recommended design method depends whether the beam behaves as a *rectangular T-beam* or a *true T-beam*.
  - For rectangular-T-Beam behavior, the design procedure is the same as for the tensile reinforced rectangular beam.
  - For true-T-beam behavior, the design proceeds by designing a flange component and a web components and combining the two.

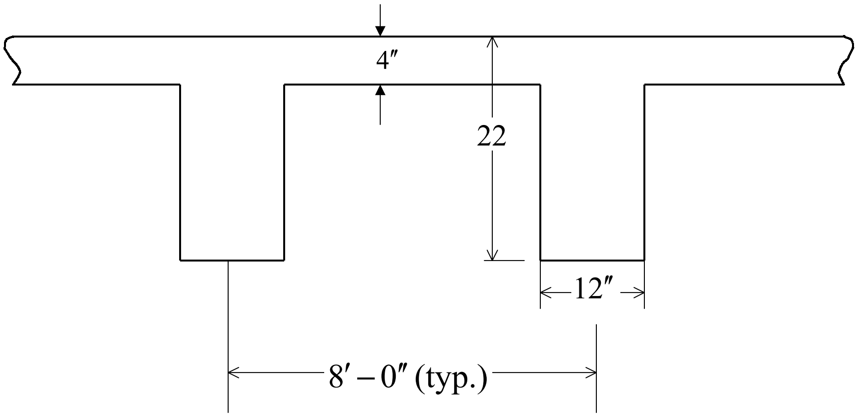
T-Beam Design (For Moment)

- Example 2

  Design the T-beam for the floor system shown in the figure. The floor has a 4-in. slab supported by 22-ft-span-length beams cast monolithically with the slab. Beams are 8 ft-0 in. on center and have a web width of 12 in. and a total depth = 22 in.; f_y = 60,000 psi (A615 grade 60) and f_c' =3000 psi. Service loads are 0.125 ksf live load and 0.2 ksf dead load. The given dead load does not include the weight of the floor system.

T-Beam Design (For Moment)

- Example 2 (cont'd)



T-Beam Design (For Moment)

- Example 2 (cont'd)

Determine the Design Moment M_u:

$$\text{slab weight} = \frac{(8\times 12)(4)}{144}(0.150) = 0.4\ \text{k/ft}$$

$$\text{Stem (or web) weight} = \frac{(12)(22-4)}{144}(0.150) = 0.225\ \text{k/ft}$$

$$\text{Total} = 0.625\ \text{k/ft}$$

$$\text{service DL} = (8)(0.2) = 1.6\ \text{k/ft}$$

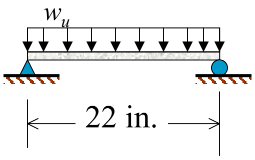
$$\text{service LL} = (8)(0.125) = 1.0\ \text{k/ft}$$

$$U = 1.4D + 1.7L \qquad \text{ACICode}$$

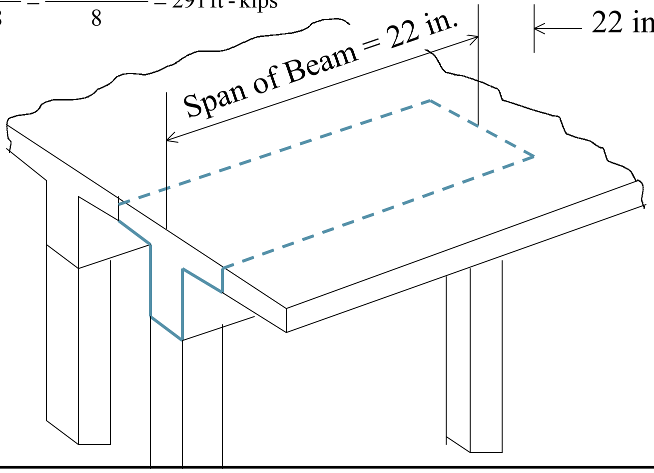
$$w_u = 1.4(0.625 + 1.6) + 1.7(1) = 4.815\ \text{k/ft}$$

T-Beam Design (For Moment)

- Example 2 (cont'd)



$$M_u = \frac{w_u L^2}{8} = \frac{4.815(22)^2}{8} = 291\ \text{ft-kips}$$



T-Beam Design (For Moment)

- Example 2 (cont'd)

  Assume an effective depth $d = h - 3$

  $$d = 22 - 3 = 19 \text{ in.}$$

  Find the effective flange width, b:

  $$\frac{1}{4}\text{span length} = \frac{1}{4}(22 \times 12) = 66 \text{ in.} \quad \longleftarrow \text{Controls}$$

  $$b_w + 16h_f = 12 + 16(4) = 76 \text{ in.}$$

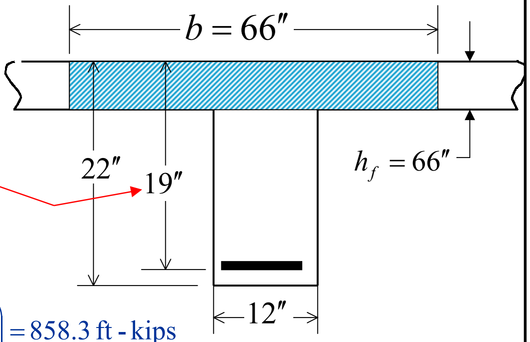
  $$\text{beam spacing} = 8 \times 12 = 96 \text{ in.}$$

  Therefore, use $b = 66$ in. (smallest)

T-Beam Design (For Moment)

- Example 2 (cont'd)

  Find out what type of beam to be used for design analysis, i.e., Is it a rectangular T-beam or a true T-beam?



$$\phi M_{nf} = \phi(0.85 f_c')bh_f\left(d - \frac{h_f}{2}\right)$$

$$= \frac{0.9(0.85)(3)(66)(4)}{12}\left(19 - \frac{4}{2}\right) = 858.3 \text{ ft - kips}$$

T-Beam Design (For Moment)

- Example 2 (cont'd)

Because (ϕM_{nf} = 858.3 ft-k) > (M_u = 291 ft-k), therefore $a < h_f$, and the total effective flange need not be completely used in compression.

The beam can be analyzed as rectangular T-beam

Design a rectangular beam:

$$\text{required } \bar{k} = \frac{M_u}{\phi b d^2} = \frac{291 \times 12}{0.9(66)(19)^2} = 0.1628 \text{ ksi}$$

$$\text{required } \rho = 0.0028 \text{ From Table 5}$$

T-Beam Design (For Moment)

- Example 2 (cont'd)

Table A-8 Textbook

Table 5.
Coefficient of Resistance

ρ	$\bar{k}$
0.0020	0.1172
0.0021	0.1229
0.0022	0.1286
0.0023	0.1343
0.0024	0.1399
0.0025	0.1456
0.0026	0.1512
0.0027	0.1569
0.0028	0.1625
0.0029	0.1681
0.0030	0.1736
0.0031	0.1792

Value used in the example.

T-Beam Design (For Moment)

- Example 2 (cont'd)

Calculate the required steel area:

$$\text{required } A_s = \rho bd = 0.0028(66)(19) = 3.51\,\text{in}^2$$

Select the steel bars:

Use 3 #10 bars ($A_s = 3.81\,\text{in}^2$) From Table 2

Table 6

Minimum $b_w = 10.5\,\text{in.} < 66\,\text{in.}$ OK

Check the effective depth, *d*:

Diameter of #10 bar
See Table 7

Diameter of #3 Stirrup
See Table 7

$$d = 22 - 1.5 - 0.375 - \frac{1.27}{2} = 19.49\,\text{in.}$$

$19.49\,\text{in.} > 19\,\text{in.}$ OK

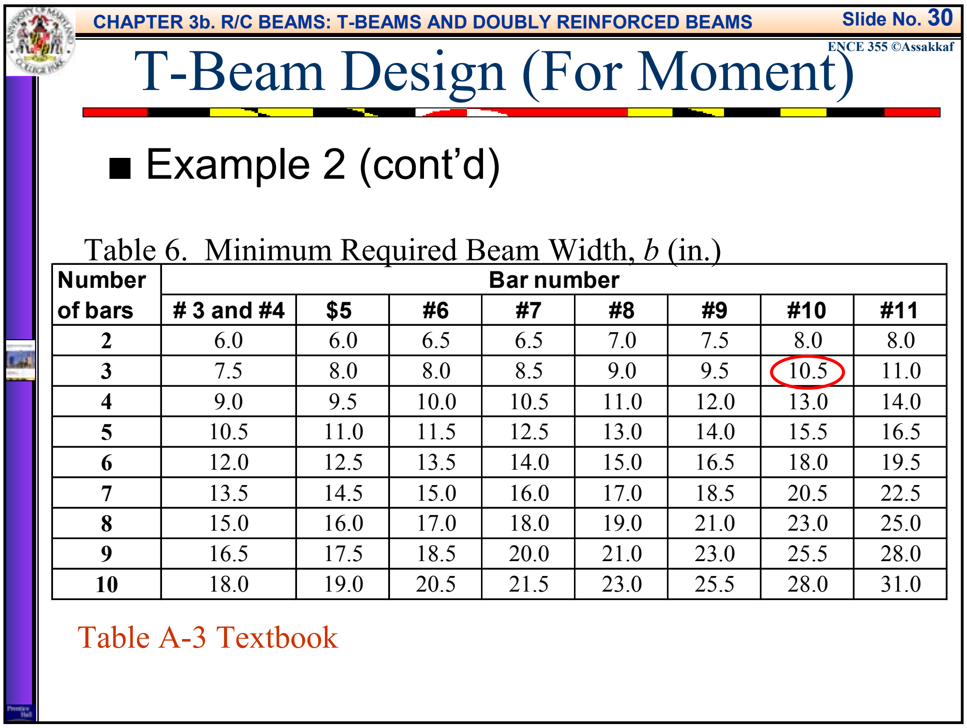
T-Beam Design (For Moment)

- Example 2 (cont'd)

Table 2. Areas of Multiple of Reinforcing Bars (in^2)

Number	Bar number								
of bars	#3	#4	$5	#6	#7	#8	#9	#10	#11
1	0.11	0.20	0.31	0.44	0.60	0.79	1.00	1.27	1.56
2	0.22	0.40	0.62	0.88	1.20	1.58	2.00	2.54	3.12
3	0.33	0.60	0.93	1.32	1.80	2.37	3.00	3.81	4.68
4	0.44	0.80	1.24	1.76	2.40	3.16	4.00	5.08	6.24
5	0.55	1.00	1.55	2.20	3.00	3.95	5.00	6.35	7.80
6	0.66	1.20	1.86	2.64	3.60	4.74	6.00	7.62	9.36
7	0.77	1.40	2.17	3.08	4.20	5.53	7.00	8.89	10.92
8	0.88	1.60	2.48	3.52	4.80	6.32	8.00	10.16	12.48
9	0.99	1.80	2.79	3.96	5.40	7.11	9.00	11.43	14.04
10	1.10	2.00	3.10	4.40	6.00	7.90	10.00	12.70	15.60

Table A-2 Textbook



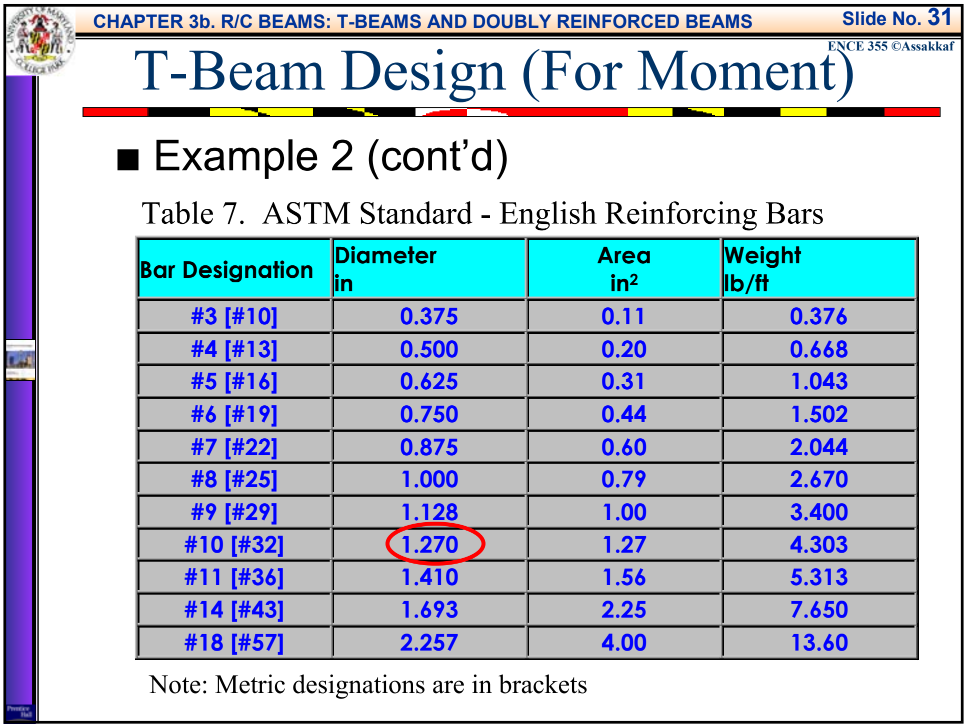
T-Beam Design (For Moment)

- Example 2 (cont'd)

Table 6. Minimum Required Beam Width, *b* (in.)

Number	Bar number							
of bars	**# 3 and #4**	**$5**	**#6**	**#7**	**#8**	**#9**	**#10**	**#11**
2	6.0	6.0	6.5	6.5	7.0	7.5	8.0	8.0
3	7.5	8.0	8.0	8.5	9.0	9.5	10.5	11.0
4	9.0	9.5	10.0	10.5	11.0	12.0	13.0	14.0
5	10.5	11.0	11.5	12.5	13.0	14.0	15.5	16.5
6	12.0	12.5	13.5	14.0	15.0	16.5	18.0	19.5
7	13.5	14.5	15.0	16.0	17.0	18.5	20.5	22.5
8	15.0	16.0	17.0	18.0	19.0	21.0	23.0	25.0
9	16.5	17.5	18.5	20.0	21.0	23.0	25.5	28.0
10	18.0	19.0	20.5	21.5	23.0	25.5	28.0	31.0

Table A-3 Textbook



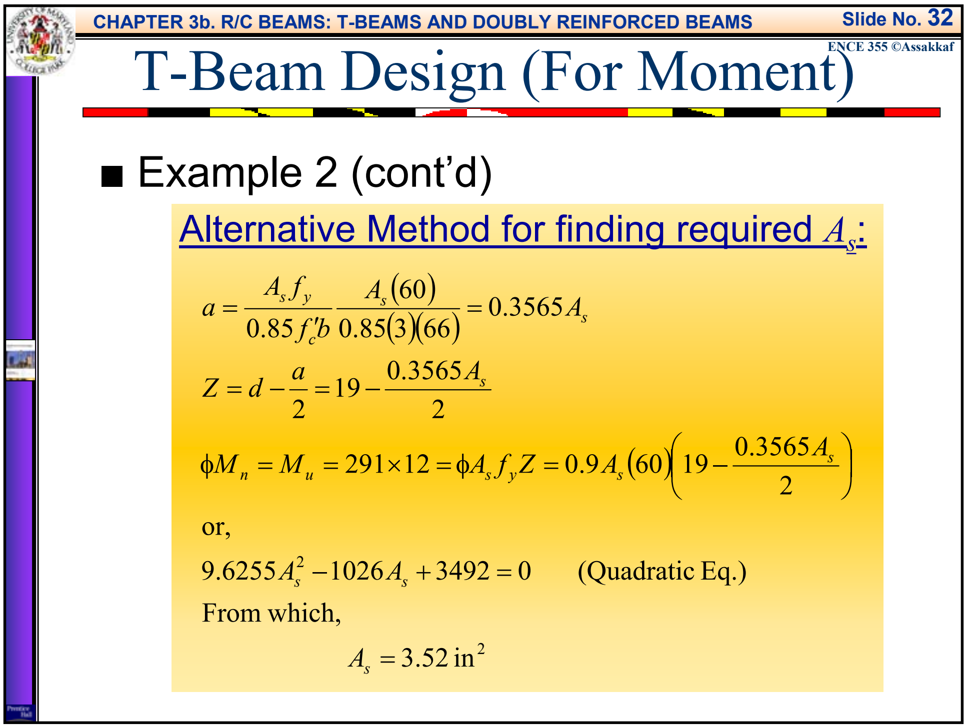
T-Beam Design (For Moment)

- Example 2 (cont'd)

Table 7. ASTM Standard - English Reinforcing Bars

Bar Designation	Diameter in	Area in^2	Weight lb/ft
#3 [#10]	0.375	0.11	0.376
#4 [#13]	0.500	0.20	0.668
#5 [#16]	0.625	0.31	1.043
#6 [#19]	0.750	0.44	1.502
#7 [#22]	0.875	0.60	2.044
#8 [#25]	1.000	0.79	2.670
#9 [#29]	1.128	1.00	3.400
#10 [#32]	1.270	1.27	4.303
#11 [#36]	1.410	1.56	5.313
#14 [#43]	1.693	2.25	7.650
#18 [#57]	2.257	4.00	13.60

Note: Metric designations are in brackets



T-Beam Design (For Moment)

- Example 2 (cont'd)

Alternative Method for finding required A_s:

$$a = \frac{A_s f_y}{0.85 f'_c b} \quad \frac{A_s(60)}{0.85(3)(66)} = 0.3565 A_s$$

$$Z = d - \frac{a}{2} = 19 - \frac{0.3565 A_s}{2}$$

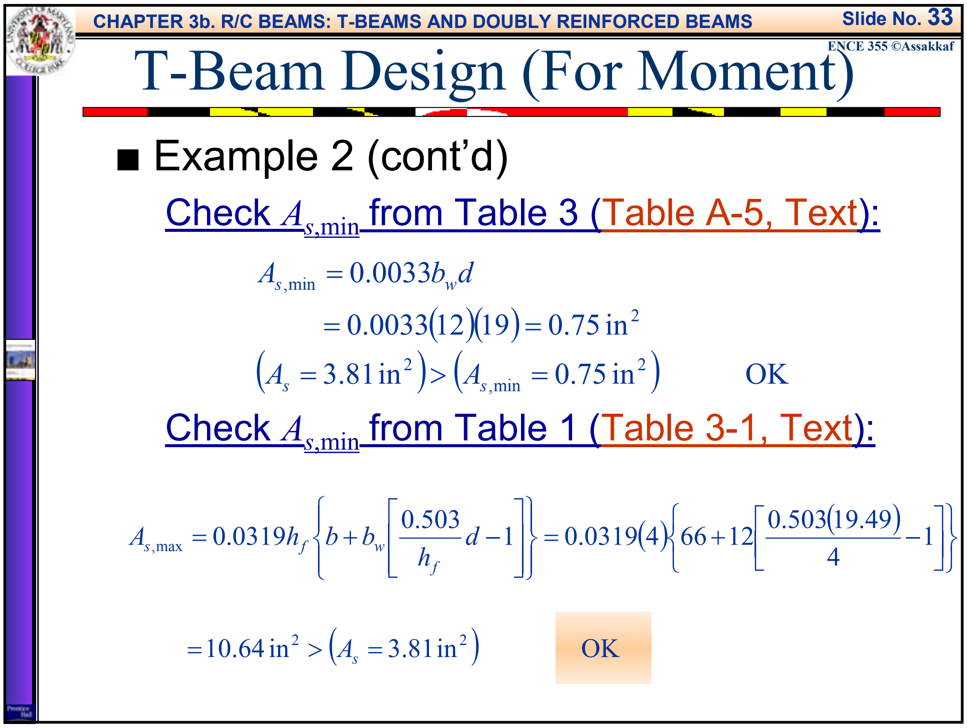
$$\phi M_n = M_u = 291 \times 12 = \phi A_s f_y Z = 0.9 A_s (60)\left(19 - \frac{0.3565 A_s}{2}\right)$$

or,

$$9.6255 A_s^2 - 1026 A_s + 3492 = 0 \qquad \text{(Quadratic Eq.)}$$

From which,

$$A_s = 3.52 \text{ in}^2$$



T-Beam Design (For Moment)

- Example 2 (cont'd)

Check $A_{s,\min}$ from Table 3 (Table A-5, Text):

$$A_{s,\min} = 0.0033 b_w d$$

$$= 0.0033(12)(19) = 0.75 \text{ in}^2$$

$$\left(A_s = 3.81 \text{ in}^2\right) > \left(A_{s,\min} = 0.75 \text{ in}^2\right) \qquad \text{OK}$$

Check $A_{s,\min}$ from Table 1 (Table 3-1, Text):

$$A_{s,\max} = 0.0319 h_f \left\{ b + b_w \left[\frac{0.503}{h_f} d - 1 \right] \right\} = 0.0319(4)\left\{ 66 + 12 \left[\frac{0.503(19.49)}{4} - 1 \right] \right\}$$

$$= 10.64 \text{ in}^2 > \left(A_s = 3.81 \text{ in}^2\right) \qquad \text{OK}$$

T-Beam Design (For Moment)

- Example 1 (cont'd)

Table A-5 Textbook

Table 3
Design Constants

f_c' (psi)	$\left[\frac{3\sqrt{f_c'}}{f_y} \geq \frac{200}{f_y}\right]$	$\rho_{max} = 0.75\ \rho_b$	Recommended Design Values	
			ρ_b	$\bar{k}$ (ksi)
		F_y = 40,000 psi		
3,000	0.0050	0.0278	0.0135	0.4828
4,000	0.0050	0.0372	0.0180	0.6438
5,000	0.0053	0.0436	0.0225	0.8047
6,000	0.0058	0.0490	0.0270	0.9657
		F_y = 50,000 psi		
3,000	0.0040	0.0206	0.0108	0.4828
4,000	0.0040	0.0275	0.0144	0.6438
5,000	0.0042	0.0324	0.0180	0.8047
6,000	0.0046	0.0364	0.0216	0.9657
		F_y = 60,000 psi		
3,000	0.0033	0.0161	0.0090	0.4828
4,000	0.0033	0.0214	0.0120	0.6438
5,000	0.0035	0.0252	0.0150	0.8047
6,000	0.0039	0.0283	0.0180	0.9657
		F_y = 75,000 psi		
3,000	0.0027	0.0116	0.0072	0.4828
4,000	0.0027	0.0155	0.0096	0.6438
5,000	0.0028	0.0182	0.0120	0.8047
6,000	0.0031	0.0206	0.0144	0.9657

Value used in the example.

T-Beam Design (For Moment)

Table 1. Expressions for $A_{s,\max}$ (T-Beams)

f_c' (psi)	f_y (psi)	$A_{s,\max}$ (in^2)
3,000	40,000	$0.0478h_f\left\{b+b_w\left[\frac{0.582}{h_f}d-1\right]\right\}$
	60,000	$0.0319h_f\left\{b+b_w\left[\frac{0.503}{h_f}d-1\right]\right\}$
4,000	40,000	$0.0638h_f\left\{b+b_w\left[\frac{0.582}{h_f}d-1\right]\right\}$
	60,000	$0.0425h_f\left\{b+b_w\left[\frac{0.503}{h_f}d-1\right]\right\}$

T-Beam Design (For Moment)

- Example 1 (cont'd)

Final Detailed Sketch of the Design: