CHAPTER

3a

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Reinforced Concrete Design Fifth Edition

REINFORCED CONCRETE BEAMS: T-BEAMS AND DOUBLY REINFORCED BEAMS

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Part I – Concrete Design and Analysis

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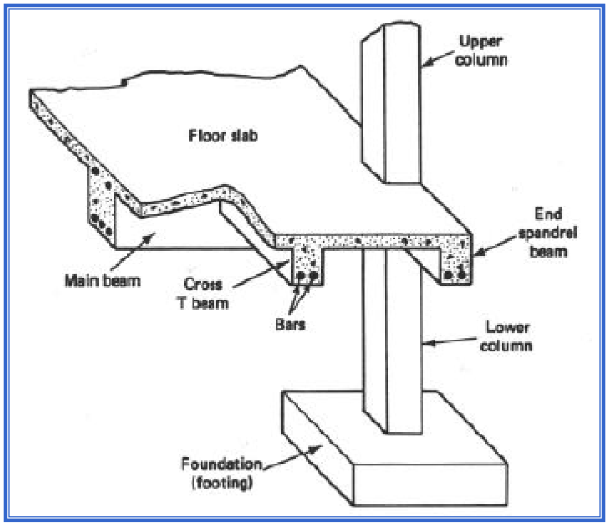
Introduction to T-Beams

- Reinforced concrete structural systems such as floors, roofs, decks, etc., are almost monolithic, except for precast systems.
- Forms are built for beam sides the underside of slabs, and the entire construction is poured at once, from the bottom of the deepest beam to the top of the slab.

Introduction to T-Beams

- Floor-Column Systems



Introduction to T-Beams

- Beam and Girder System
  - This system is composed of slab on supporting reinforced concrete beams and girder..
  - The beam and girder framework is, in turn, supported by columns.
  - In such a system, the beams and girders are placed monolithically with the slab.
  - The typical monolithic structural system is shown in Fig. 1.

Introduction to T-Beams

- Beam and Girder Floor System

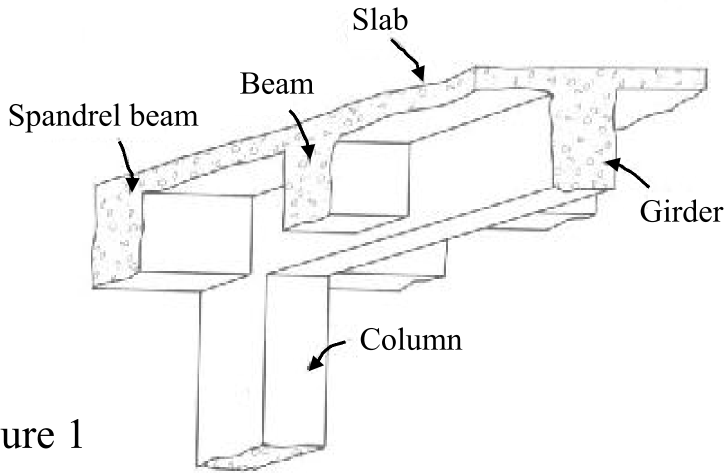


Figure 1

Introduction to T-Beams

- Common Beam and Girder Layout

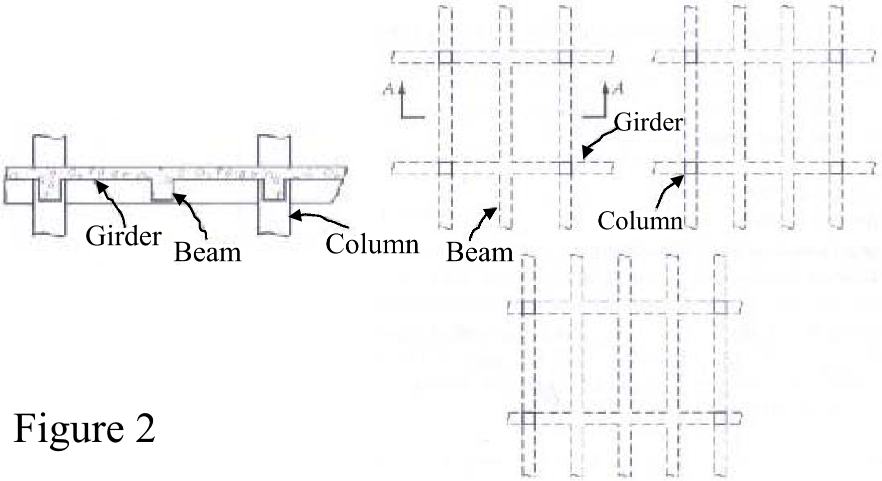


Figure 2

Introduction to T-Beams

- Positive Bending Moment
  - In the analysis and design of floor and roof systems, it is common practice to assume that the monolithically placed slab and supporting beam interact as a unit in resisting the positive bending moment.
  - As shown in Fig. 3, the slab becomes the compression **flange**, while the supporting beam becomes the **web** or stem.

Introduction to T-Beams

- T-Beam as Part of a Floor System

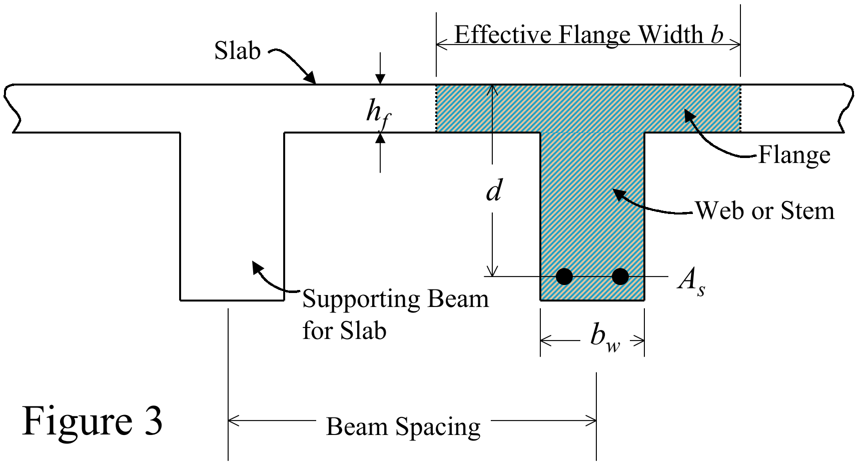


Figure 3

Introduction to T-Beams

- T-Beam
  - The interacting flange and web produce the cross section having the typical T-shape, thus the T-Beam gets its name.
- Negative Bending Moment
  - It should be noted that when the the T-Beam is subjected to negative moment, the slab at the top of the stem (web) will be in *tension* while the bottom of the stem is in compression. This usually occurs at interior support of continuous beam.

T-Beam Analysis

- ACI Code Provisions for T-Beams

1. The effective flange width must not exceed
   a. One-fourth the span length
   b. $b_w + 16h_f$
   c. Center-to-center spacing of the beam

   The smallest of the three values will control

2. For beam having a flange on one side only, the effective overhanging flange width must

T-Beam Analysis

- ACI Code Provisions for T-Beams

  Not exceed ***one-twelfth*** of the span length of the beam, nor ***six times*** the slab thickness, nor ***one-half*** of the clear distance to the next beam.

  3. For isolated beam in which the T-shape is used only for the purpose of providing additional compressive area, the flange thickness must not be less than **one-half** of the width of the web, and the total flange width must not be more than **four times** the web width.

T-Beam Analysis

- T-Beam Versus Rectangular Beam
  - The ductility requirements for T-beams are similar to those for rectangular beams.
  - The maximum steel ratio ρ shall not exceed $0.75\rho_b$.
  - However, this steel ratio is not the same value as that tabulated for rectangular beams because of the T-shaped compressive area.

T-Beam Analysis

- Formulas for Balanced T-Beam

  These formulas can be used to find A_{sb}. It will be illustrated in Example 1:

$$c_b = \frac{87{,}000}{f_y + 87{,}000} d$$

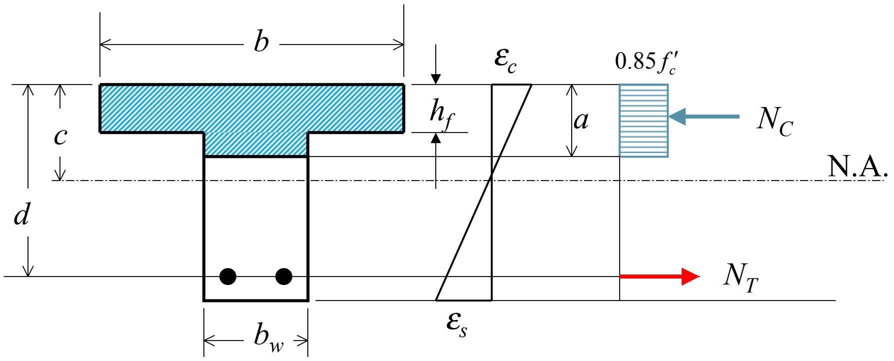
$$a_b = \beta_1 c_b \qquad (1)$$

$$N_{Cb} = 0.85 f'_c \left[b h_f + b_w \left(a_b - h_f \right) \right]$$

See Fig. 4 for definitions of variables

T-Beam Analysis

Figure 4



T-Beam Analysis

- Minimum Steel Ratio for T-Beams
  - The T-beam is subjected to positive moment:
    - The steel area shall not be less than that given by

$$A_{s,\,\min} = \frac{3\sqrt{f_c'}}{f_y} b_w d \geq \frac{200}{f_y} b_w d \qquad (2)$$

Note that the first expression controls if $f_c' > 4440$ psi

ACI Code

T-Beam Analysis

- Minimum Steel Ratio for T-Beams
  - The T-beam is subjected to negative moment:
    - The steel area A_s shall equal the smallest of the following expression:

$$A_{s,\,\min} = \text{smallest of } \frac{6\sqrt{f_c'}}{f_y} b_w d \quad \text{or} \quad \frac{3\sqrt{f_c'}}{f_y} b_w d \qquad (3)$$

ACI Code

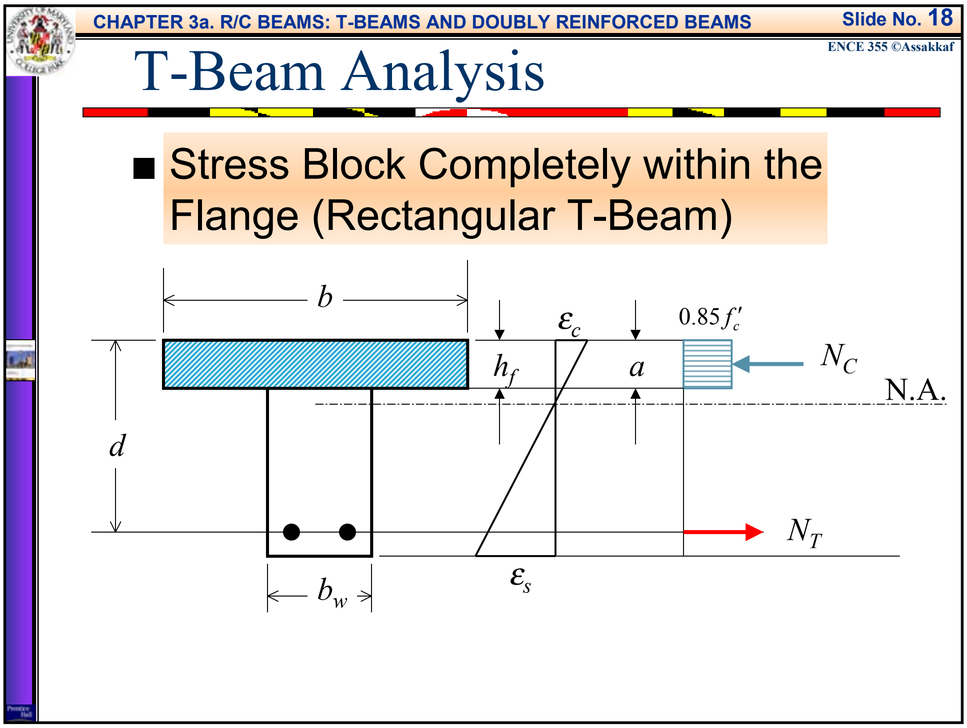
T-Beam Analysis

- Notes on the Analysis of T-Beams
  - Because of the large compressive in the flange of the T-beam, the moment strength is usually limited by the yielding of the tensile steel.
  - Therefore, it safe to assume that the tensile steel will yield before the concrete reaches its ultimate strain.
  - The ultimate tensile force may be found from

$$N_T = A_s f_y \qquad (4)$$

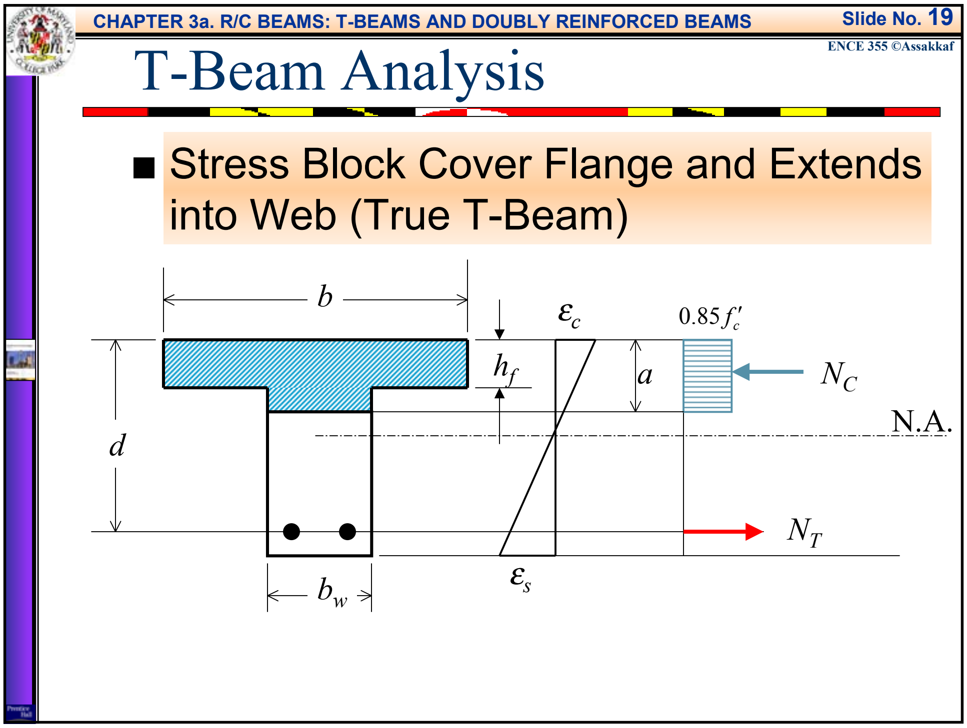
T-Beam Analysis

- Notes on the Analysis of T-Beams
  - In analyzing a T-beam, there might exist two conditions:
    1. The stress block may be completely within the flange.
    2. The stress block may cover the flange and extend into the web.
  - These two conditions will result in what are termed: ***a rectangular T-beam*** and a ***true T-beam***, respectively.



T-Beam Analysis

- Stress Block Completely within the Flange (Rectangular T-Beam)



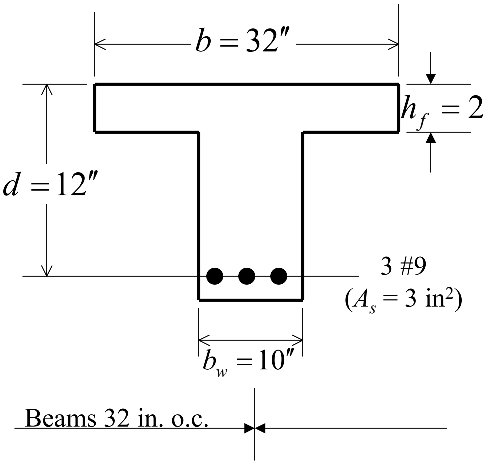
T-Beam Analysis

- Stress Block Cover Flange and Extends into Web (True T-Beam)

T-Beam Analysis

- Example 1

The T-beam shown in the figure is part of a floor system. Determine the practical moment strength ϕM_n if f_y = 60,000 psi (A615 grade 60) and f'_c = 3,000 psi.



T-Beam Analysis

- Example 1 (cont'd)

Since the span length is not given, we determine the flange width in terms of the flange thickness and beam spacing:

$$b_w + 16h_f = 10 + 16(2) = 42 \text{ in.}$$

$$\text{Beam spacing} = 32 \text{ in. o.c.}$$

Therefore,

Use $b = 32$ in. (smallest of the two)

T-Beam Analysis

- Example 1 (cont'd)

Find N_T assuming that the steel has yielded:

$$N_T = A_s f_y = 3(60) = 180 \text{ kips}$$

If the flange alone is stressed to $0.85f_c'$, then the total compressive force would be

$$N_T = 0.85 f_c' h_f b = 0.85(3)(2)(32) = 163.2 \text{ kips}$$

Since 180 > 163, the beam should be analyzed as true T-beam, and the stress block will extend into the web (Fig. 5)

T-Beam Analysis

- Example 1 (cont'd)

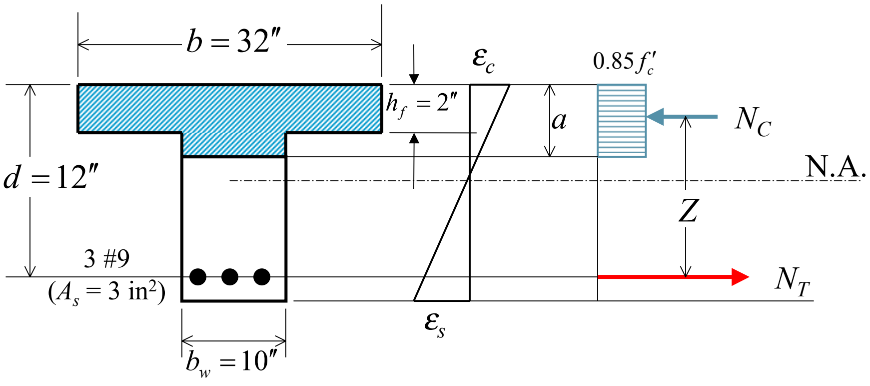


Figure 5

T-Beam Analysis

- Example 1 (cont'd)

  The remaining compression is therefore

$$\text{Remaining Compression} = N_T - N_{Cf}$$

$$N_T - N_{Cf} = 0.85 f'_c b_w \left(a - h_f\right)$$

$$a - h_f = \frac{N_T - N_{Cf}}{0.85 f'_c b_w}$$

$$a = \frac{N_T - N_{Cf}}{0.85 f'_c b_w} + h_f = \frac{180 - 163.2}{0.85(3)(10)} + 2 = 2.66\ \text{in.}$$

T-Beam Analysis

- Example 1 (cont'd)

  Check $A_{s,\text{ min}}$ using Eq. 3 or Table 1

$$\text{From Table 1 (also Table A - 5 Text):}$$

$$A_{s,\min} = 0.0033 b_w d = 0.0033(10)(12) = 0.40\ \text{in}^2$$

$$\left(A_s = 3.0\ \text{in}^2\right) > \left(A_{s,\min} = 0.4\ \text{in}^2\right) \qquad \text{OK}$$

  – In order to find the internal couple, we have to find the couple arm *Z*:

$$\bar{y} = \frac{\sum Ay}{\sum A}$$

T-Beam Analysis

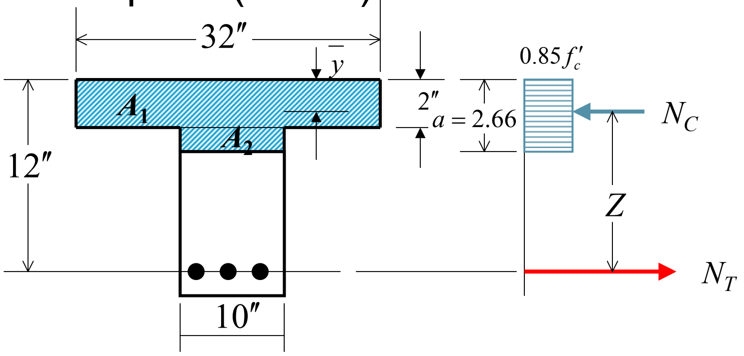
(Table A-5 Text)

Table 1.
Design Constants

f'_c (psi)	$\left[\frac{3\sqrt{f'_c}}{f_y} \geq \frac{200}{f_y}\right]$	$\rho_{max} = 0.75\,\rho_b$	Recommended Design Values	
			ρ_b	$\bar{k}$ (ksi)
		F_y = 40,000 psi		
3,000	0.0050	0.0278	0.0135	0.4828
4,000	0.0050	0.0372	0.0180	0.6438
5,000	0.0053	0.0436	0.0225	0.8047
6,000	0.0058	0.0490	0.0270	0.9657
		F_y = 50,000 psi		
3,000	0.0040	0.0206	0.0108	0.4828
4,000	0.0040	0.0275	0.0144	0.6438
5,000	0.0042	0.0324	0.0180	0.8047
6,000	0.0046	0.0364	0.0216	0.9657
		F_y = 60,000 psi		
3,000	0.0033	0.0161	0.0090	0.4828
4,000	0.0033	0.0214	0.0120	0.6438
5,000	0.0035	0.0252	0.0150	0.8047
6,000	0.0039	0.0283	0.0180	0.9657
		F_y = 75,000 psi		
3,000	0.0027	0.0116	0.0072	0.4828
4,000	0.0027	0.0155	0.0096	0.6438
5,000	0.0028	0.0182	0.0120	0.8047
6,000	0.0031	0.0206	0.0144	0.9657

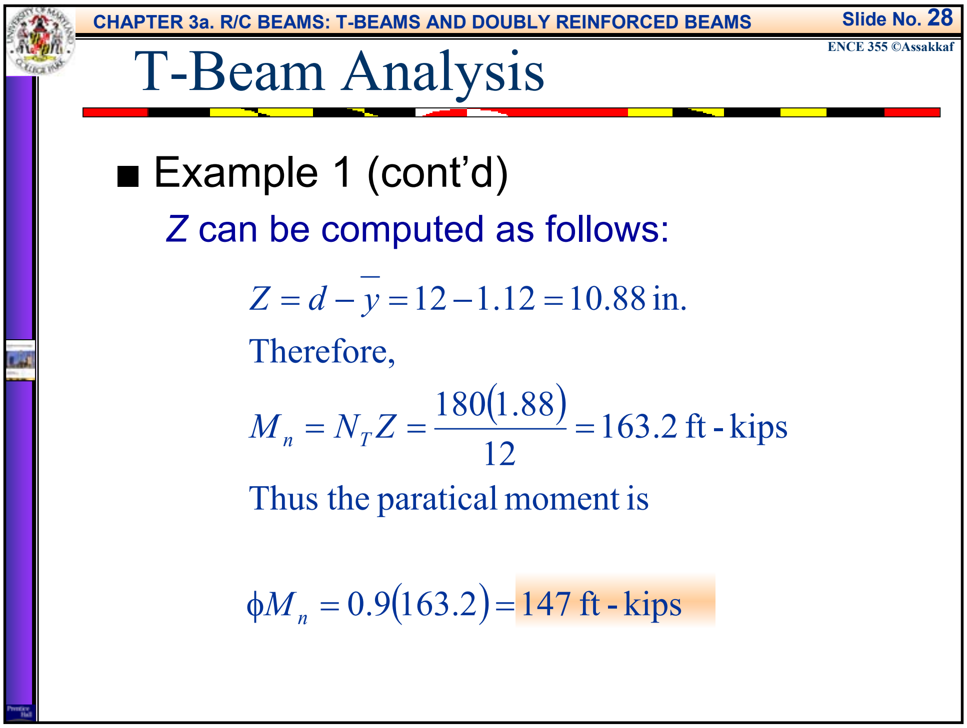
T-Beam Analysis

- Example 1 (cont'd)



Using a reference axis at the top:

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{[32(2)](1)+[10(0.66)](2+0.33)}{32(2)+10(0.66)} = 1.12 \text{ in}$$



T-Beam Analysis

- Example 1 (cont'd)

  Z can be computed as follows:

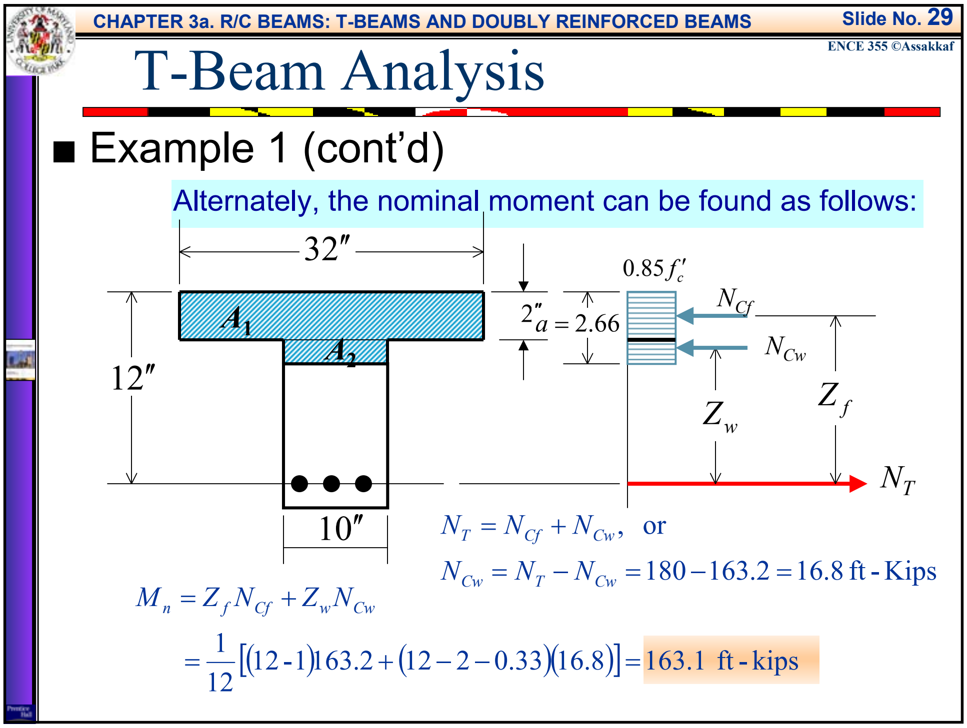
$$Z = d - \bar{y} = 12 - 1.12 = 10.88 \text{ in.}$$

Therefore,

$$M_n = N_T Z = \frac{180(1.88)}{12} = 163.2 \text{ ft-kips}$$

Thus the paratical moment is

$$\phi M_n = 0.9(163.2) = 147 \text{ ft-kips}$$



T-Beam Analysis

- Example 1 (cont'd)

  Alternately, the nominal moment can be found as follows:

$$N_T = N_{Cf} + N_{Cw}, \quad \text{or}$$

$$N_{Cw} = N_T - N_{Cw} = 180 - 163.2 = 16.8 \text{ ft-Kips}$$

$$M_n = Z_f N_{Cf} + Z_w N_{Cw}$$

$$= \frac{1}{12}\left[(12-1)163.2 + (12-2-0.33)(16.8)\right] = 163.1 \text{ ft-kips}$$

T-Beam Analysis

- Example 1 (cont'd)

Check assumption for ductile failure:

From Eq. 1

$$c_b = \frac{87{,}000}{f_y + 87{,}000} d = \frac{87}{60+87}(12) = 7.10\text{ in.}$$

$$a_b = \beta_1 c_b = 0.85(7.1) = 6.035\text{ in.}$$

$$\begin{aligned} N_{Cb} &= 0.85 f'_c\left[bh_f + b_w\left(a_b - h_f\right)\right] \\ &= 0.85(3)\left[32(2) + 10(6.035-2)\right] \\ &= 266.09\text{ kips} = N_{Tb} \end{aligned}$$

T-Beam Analysis

- Example 1 (cont'd)

$$A_{sb} = \frac{N_{Tb}}{f_y} = \frac{266.09}{60} = 4.44\text{ in}^2$$

$$\begin{aligned} A_{s,\max} &= 0.75 A_{sb} \\ &= 0.75(4.44) \\ &= 0.33 \end{aligned}$$

$$\left(A_s = 3.0\text{ in}^2\right) < \left(A_{s,\max} = 4.44\text{ in}^2\right)$$

OK