CHAPTER

Reinforced Concrete Design — Fifth Edition

RECTANGULAR R/C CONCRETE BEAMS: TENSION STEEL ONLY

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Part I – Concrete Design and Analysis

2d

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ENCE 355 - Introduction to Structural Design

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Rectangular Beam Design for Moment (Tension Only)

- In a general sense, the design procedure for a rectangular cross section of a reinforced beam basically requires the determination of three quantities.
- The compressive strength of concrete f_c' and the yield strength f_y of steel are usually prescribed.

Rectangular Beam Design for Moment (Tension Only)

- The three quantities that need to be determined in a design problem for rectangular reinforced concrete beam are:
  - Beam Width, b
  - Beam Depth, d
  - Steel Area, A_s.

Rectangular Beam Design for Moment (Tension Only)

- Theoretically, a wide shallow beam may have the same ϕM_n as a narrow deep beam.
- However, practical considerations and code requirements will affect the final selection of these three quantities.
- There is no easy way to determine the best cross section, since economy depends on much more than simply the volume of concrete and amount of steel.

Rectangular Beam Design for Moment (Tension Only)

- Simplified Design Formulas
  - Using the internal couple method previously developed for beam analysis, modifications may be made whereby the design process may be simplified.
  - The resistance moment is given by

$$\phi M_n = \phi N_c Z = \phi N_T Z \qquad (1)$$

Rectangular Beam Design for Moment (Tension Only)

- Simplified Design Formulas

$$\phi M_n = \phi \left(0.85 f_c'\right) ba \left(d - \frac{a}{2}\right) \qquad (2)$$

where

$$a = \frac{A_s f_y}{\left(0.85 f_c'\right) b} \qquad (3)$$

The use of these formulas will now be simplified through the development of design constants, Which will eventually be tabulated.

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Rectangular Beam Design for Moment (Tension Only)

- Simplified Design Formulas

$$\rho = \frac{A_s}{bd} \quad \text{therefore } A_s = \rho b d \qquad (4)$$

Substituting Eq. 4 into Eq. 3, yields

$$a = \frac{A_s f_y}{(0.85 f_c')b} = \frac{\rho b d f_y}{(0.85 f_c')b} = \frac{\rho d f_y}{0.85 f'} \qquad (5)$$

Let's define the variable ω (omega) as

$$\omega = \rho \frac{f_y}{f_c'} \qquad (6)$$

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Rectangular Beam Design for Moment (Tension Only)

- Simplified Design Formulas

Substituting ω of Eq. 6 into Eq. 5, yields

$$a = \frac{\rho d f_y}{0.85 f'} = \frac{\omega d}{0.85} \qquad (7)$$

Substituting for a of Eq. 7 into Eq. 2, gives

$$\phi M_n = \phi(0.85 f_c')ba\left(d - \frac{a}{2}\right) = \phi(0.85 f_c')b\frac{\omega d}{0.85}\left[d - \frac{\omega d}{2(0.85)}\right] \qquad (8)$$

Rectangular Beam Design for Moment (Tension Only)

- Simplified Design Formulas

Eq. 8 can be simplified and rearranged to give

$$\phi M_n = \phi b d^2 f_c' \omega (1 - 0.59\omega) \qquad (9)$$

Let's define the coefficient of resistance $\bar{k}$ as

$$\bar{k} = f_c' \omega (1 - 0.59\omega) \qquad (10)$$

Tables A-7 through A-11 of the Textbook give the value of $\bar{k}$ in ksi for values of ρ (i.e., $0.75\rho_b$) and various combinations of f_c' and f_y.

Rectangular Beam Design for Moment (Tension Only)

- Sample Coefficient of Resistance Vs. Steel Ratio

$f_c' = 3\text{ ksi} \quad f_y = 40\text{ ksi}$

ρ	$\bar{k}$
0.0010	0.0397
0.0011	0.0436
0.0012	0.0475
0.0013	0.0515
0.0014	0.0554
0.0015	0.0593
0.0016	0.0632
0.0017	0.0671
0.0018	0.0710
0.0019	0.0749
0.0020	0.0787
0.0021	0.0826

$f_c' = 4\text{ ksi} \quad f_y = 60\text{ ksi}$

ρ	$\bar{k}$
0.0010	0.0595
0.0011	0.0654
0.0012	0.0712
0.0013	0.0771
0.0014	0.0830
0.0015	0.0888
0.0016	0.0946
0.0017	0.1005
0.0018	0.1063
0.0019	0.1121
0.0020	0.1179
0.0021	0.1237

Rectangular Beam Design for Moment (Tension Only)

- Simplified Design Formulas
  - The general analysis expression for ϕM_n may be written as

$$\phi M_n = M_u = \phi b d^2 \bar{k} \quad \text{(in.-kips)} \quad (11a)$$

or

$$\phi M_n = M_u = \frac{\phi b d^2 \bar{k}}{12} \quad \text{(ft-kips)} \quad (11b)$$

NOTE: Values of $\bar{k}$ are tabulated in ksi

Rectangular Beam Design for Moment (Tension Only)

- Note that Eq. 11 can also be used to simplify the analysis of a reinforced beam having a rectangular cross section.
- The following example was presented in Chapter 2c of the lecture notes (Ex. 1) and the beam was analyzed based on a lengthy procedure. However, now this beam will be analyzed based on Eq. 11.

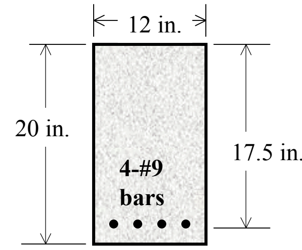
Rectangular Beam Design for Moment (Tension Only)

- Example 1

Find the nominal flexural strength and design strength of the beam shown.

$f'_c = 4{,}000 \text{ psi}$

$f_y = 60{,}000 \text{psi}$



Four No. 9 bars provide $A_s = 4.00 \text{ in}^2$

$$\rho = \frac{A_s}{bd} = \frac{4.00}{12(17.5)} = 0.0190$$

$$(\rho_{\min} = 0.0033) < (\rho = 0.0190) < (\rho_{\max} = 0.0214)$$ **OK**

Rectangular Beam Design for Moment (Tension Only)

Table A-5 Textbook

Table 1
Design Constants

f'_c (psi)	$\left[\frac{3\sqrt{f'_c}}{f_y} \geq \frac{200}{f_y}\right]$	$\rho_{\max} = 0.75\,\rho_b$	Recommended Design Values: ρ_b	Recommended Design Values: $\bar{k}$ (ksi)
F_y = 40,000 psi				
3,000	0.0050	0.0278	0.0135	0.4828
4,000	0.0050	0.0372	0.0180	0.6438
5,000	0.0053	0.0436	0.0225	0.8047
6,000	0.0058	0.0490	0.0270	0.9657
F_y = 50,000 psi				
3,000	0.0040	0.0206	0.0108	0.4828
4,000	0.0040	0.0275	0.0144	0.6438
5,000	0.0042	0.0324	0.0180	0.8047
6,000	0.0046	0.0364	0.0216	0.9657
F_y = 60,000 psi				
3,000	0.0033	0.0161	0.0090	0.4828
4,000	0.0033	0.0214	0.0120	0.6438
5,000	0.0035	0.0252	0.0150	0.8047
6,000	0.0039	0.0283	0.0180	0.9657
F_y = 75,000 psi				
3,000	0.0027	0.0116	0.0072	0.4828
4,000	0.0027	0.0155	0.0096	0.6438
5,000	0.0028	0.0182	0.0120	0.8047
6,000	0.0031	0.0206	0.0144	0.9657

Values used in the example.

Rectangular Beam Design for Moment (Tension Only)

- Example 1 (cont'd)
  - From Table 2 (Table A-10 , Text), with f_y = 60,000 psi, f'_c = 4,000 psi, and ρ = 0.0190, the value of $\bar{k}$ = 0.9489 ksi is found .
  - Using Eq. 11b, the nominal and design strengths are respectively

$$M_n = \frac{bd^2\bar{k}}{12} = \frac{12(17.5)^2(0.9489)}{12} = 291 \text{ ft - kips}$$

$$\phi M_n = 0.9(291) = 262 \text{ ft - kips}$$

Which are the same values obtained in the example of Ch.2c notes.

Rectangular Beam Design for Moment (Tension Only)

- Example 1 (cont'd)

Table 2
Part of Table A-10
of Textbook

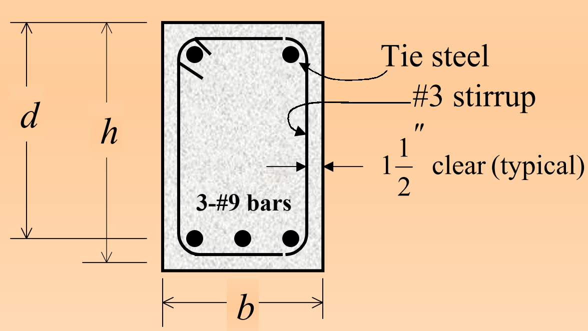
ρ	$\bar{k}$
0.0185	0.9283
0.0186	0.9323
0.0187	0.9363
0.0188	0.9403
0.0189	0.9443
0.0190	0.9489
0.0191	0.9523
0.0192	0.9563
0.0193	0.9602
0.0194	0.9642
0.0195	0.9681
0.0196	0.9720

Rectangular Beam Design for Moment (Tension Only)

- ACI Code Requirements for Concrete Protection for Reinforcement
  - For beams, girders, and columns not exposed to weather or in contact with the ground, the minimum concrete cover on any steel is 1.5 in.
  - For slabs, it is 0.75 in.
  - Clear space between bars in a single layer shall not be less than the bar diameter, but not less 1 in.

Rectangular Beam Design for Moment (Tension Only)

- Stirrups
  - Stirrups are special form of reinforcement that primarily resist shear forces that will be discussed later.



Procedure for Rectangular RC Beam Design for Moment

- A. Cross Section (b and h) Known; Find the Required A_s:

1. Convert the service loads or moments to design M_u (including the beam weight).
2. Based on knowing h, estimate d by using the relationship $d = h - 3$ in. (conservative for bars in a single layer). Calculate the required $\bar{k}$ from

$$\bar{k} = \frac{M_u}{\phi b d^2} \qquad (12)$$

Procedure for Rectangular RC Beam Design for Moment

3. From Tables A-7 through A-11 of your textbook, find the required steel ratio ρ.
4. Compute the required A_s:

$$A_s = \rho b d \qquad (13)$$

   Check $A_{s,\min}$ by using Table A-5 of textbook.
5. Select the bars. Check to see if the bars can fit into the beam in one layer (preferable). Check the actual effective depth and compare with the assumed effective depth. If the actual effective depth is slightly in excess of

Procedure for Rectangular RC Beam Design for Moment

the assumed effective depth, the design will be slightly conservative (on the safe side). If the actual effective depth is less than the assumed effective depth, the design is on the unconservative side and should be revised.

6. Sketch the design showing the details of the cross section and the reinforcement exact location, and the stirrups, including the tie bars.

Procedure for Rectangular RC Beam Design for Moment

- <u>B. Design for Cross Section and Required A_s:</u>

1. Convert the service loads or moments to design M_u. An estimated beam weight may be included in the dead load if desired. Make sure to apply the load factor to this additional dead load.
2. Select the desired steel ratio ρ. (see Table A-5 of textbook for recommended values. Use the ρ values from Table A-5 unless a small cross section or decreased steel is desired).

Procedure for Rectangular RC Beam Design for Moment

3. From Table A-5 of your textbook (or from Tables A-7 through A-11), find $\bar{k}$.
4. Assume b and compute the required d:

$$d = \sqrt{\frac{M_u}{\phi b \bar{k}}} \qquad (14)$$

If the d/b ratio is reasonable (1.5 to 2.2), use these values for the beam. If the d/b ratio is not reasonable, increase or decrease b and compute the new required d

Procedure for Rectangular RC Beam Design for Moment

5. Estimate h and compute the beam weight. Compare this with the estimated beam weight if an estimated beam weight was included.
6. Revise the design M_u to include the moment due to the beam's own weight using the latest weight determined. Note that at this point, one could revert to step 2 in the previous design procedure, where the cross section is known.
7. Using b and $\bar{k}$ previously determined along with the new total design M_u, find the new

Procedure for Rectangular RC Beam Design for Moment

Required d from

$$d = \sqrt{\frac{M_u}{\phi b \bar{k}}} \qquad (14)$$

Check to see if the d/b ratio is reasonable.

8. Find the required A_s:

$$A_s = \rho b d \qquad (15)$$

Check $A_{s,\min}$ using Table A-5 of textbook.

9. Select the bars and check to see if the bars can fit into a beam of width b in one layer (preferable).

Procedure for Rectangular RC Beam Design for Moment

10. Establish the final h, rounding this upward to the next 0.5 in. This will make the actual effective depth greater than the design effective depth, and the design will be slightly conservative (on the safe side).
11. Sketch the design showing the details of the cross section and the exact locations of the reinforcement and the stirrups, including the tie bars.

Beam Design Examples

- Example 2

  Design a rectangular reinforced concrete beam to carry a service dead load moment of 50 ft-kips (which includes the moment due to the weight of the beam) and a service live load moment of 100 ft-kips. Architectural considerations require the beam width to be 10 in. and the total depth h to be 25 in. Use f_c' = 3,000 psi and f_y = 60,000 psi.

Beam Design Examples

- Example 2 (cont'd)

  Following procedure A outlined earlier,

  1. The total design moment is

$$M_u = 1.4M_D + 1.7M_L$$
$$= 1.4(50) + 1.7(100) = 240 \text{ ft-kips}$$

  2. Estimate d:

$$d = h - 3 = 25 - 3 = 22 \text{ in.}$$

$$\text{required } \bar{k} = \frac{M_u}{\phi b d^2} = \frac{240(12)}{0.9(10)(22)^2} = 0.6612 \text{ ksi}$$

Beam Design Examples

- Example 2 (cont'd)

3. From Table 3 (Table A-8 Textbook), for $\bar{k}$ = 0.6612 and by interpolation,

$$\rho = 0.01301$$

From Table 1 (Table A-5 Textbook),

$$\rho_{\max} = 0.0161$$

4. Required $A_s = \rho bd = 0.01301(10)\,(22) = 2.86\ \text{in}^2$

Check $A_{s,\,\min}$. From Table 1 (Table A-5 Text),

$$A_{s,\min} = 0.0033 b_w d = 0.0033(10)(22) = 0.73\ \text{in}^2$$

Beam Design Examples

- Example 2 (cont'd)
  - By interpolation:

0.6608	0.0130
0.6612	ρ
0.6649	0.0131

0.6608	0.0130
0.6612	ρ
0.6649	0.0131

Therefore,

$$\frac{0.6612 - 0.6608}{0.6649 - 0.6608} = \frac{\rho - 0.0130}{0.0131 - 0.0130}$$

$$\rho = 0.01301$$

Table 3 (Table A-8 Textbook)

ρ	$\bar{k}$
0.0124	0.6355
0.0125	0.6398
0.0126	0.6440
0.0127	0.6482
0.0128	0.6524
0.0129	0.6566
0.013	0.6608
0.0131	0.6649
0.0132	0.6691
0.0133	0.6732
0.0134	0.6773
0.0135	0.6814

Beam Design Examples

Table A-5 Textbook

Table 1
Design Constants

f'_c (psi)	$\left[\frac{3\sqrt{f'_c}}{f_y} \geq \frac{200}{f_y}\right]$	$\rho_{max} = 0.75\,\rho_b$	Recommended Design Values	
			ρ_b	$\bar{k}$ (ksi)
F_y = **40,000 psi**				
3,000	0.0050	0.0278	0.0135	0.4828
4,000	0.0050	0.0372	0.0180	0.6438
5,000	0.0053	0.0436	0.0225	0.8047
6,000	0.0058	0.0490	0.0270	0.9657
F_y = **50,000 psi**				
3,000	0.0040	0.0206	0.0108	0.4828
4,000	0.0040	0.0275	0.0144	0.6438
5,000	0.0042	0.0324	0.0180	0.8047
6,000	0.0046	0.0364	0.0216	0.9657
F_y = **60,000 psi**				
3,000	0.0033	0.0161	0.0090	0.4828
4,000	0.0033	0.0214	0.0120	0.6438
5,000	0.0035	0.0252	0.0150	0.8047
6,000	0.0039	0.0283	0.0180	0.9657
F_y = **75,000 psi**				
3,000	0.0027	0.0116	0.0072	0.4828
4,000	0.0027	0.0155	0.0096	0.6438
5,000	0.0028	0.0182	0.0120	0.8047
6,000	0.0031	0.0206	0.0144	0.9657

Values used in the example.

Beam Design Examples

- Example 2 (cont'd)

5. Select the bars;

   In essence, the the bar or combination od bars that provide 2.86 in^2 of steel area will be satisfactory. From Table 4

   2 No. 11 bars: $A_s = 3.12$ in^2

   3 No. 9 bars: $A_s = 3.00$ in^2

   4 No. 8 bars: $A_s = 3.16$ in^2

   5 No. 7 bars: $A_s = 3.00$ in^2

Beam Design Examples

■ Example 2 (cont'd)

Table 4. Areas of Multiple of Reinforcing Bars (in^2)

Number of bars	#3	#4	$5	#6	#7	#8	#9	#10	#11
1	0.11	0.20	0.31	0.44	0.60	0.79	1.00	1.27	1.56
2	0.22	0.40	0.62	0.88	1.20	1.58	2.00	2.54	3.12
3	0.33	0.60	0.93	1.32	1.80	2.37	3.00	3.81	4.68
4	0.44	0.80	1.24	1.76	2.40	3.16	4.00	5.08	6.24
5	0.55	1.00	1.55	2.20	3.00	3.95	5.00	6.35	7.80
6	0.66	1.20	1.86	2.64	3.60	4.74	6.00	7.62	9.36
7	0.77	1.40	2.17	3.08	4.20	5.53	7.00	8.89	10.92
8	0.88	1.60	2.48	3.52	4.80	6.32	8.00	10.16	12.48
9	0.99	1.80	2.79	3.96	5.40	7.11	9.00	11.43	14.04
10	1.10	2.00	3.10	4.40	6.00	7.90	10.00	12.70	15.60

(Column heads are grouped under "Bar number".)

Table A-2 Textbook

Beam Design Examples

■ Example 2 (cont'd)

The width of beam required for 3 No. 9 bars is 9.5 in. (see Table 5), which is satisfactory. Note that beam width $b = 10$ in.

Check the actual effective depth d:

Actual $d = h$ – cover – stirrup – $d_b/2$

#9 bar. See Table A-1

#3 bar for stirrup. See Table A-1 for Diameter of bar.

$$25 - 1.5 - 0.38 - \frac{1.128}{2} = 22.6 \text{ in.}$$

The actual effective depth is slightly higher than the estimated one (22 in.). This will put the beam on The safe side (conservative).

Beam Design Examples

■ Example 2 (cont'd)

Table 5. Minimum Required Beam Width, *b* (in.)

Number of bars	Bar number # 3 and #4	$5	#6	#7	#8	#9	#10	#11
2	6.0	6.0	6.5	6.5	7.0	7.5	8.0	8.0
3	7.5	8.0	8.0	8.5	9.0	9.5	10.5	11.0
4	9.0	9.5	10.0	10.5	11.0	12.0	13.0	14.0
5	10.5	11.0	11.5	12.5	13.0	14.0	15.5	16.5
6	12.0	12.5	13.5	14.0	15.0	16.5	18.0	19.5
7	13.5	14.5	15.0	16.0	17.0	18.5	20.5	22.5
8	15.0	16.0	17.0	18.0	19.0	21.0	23.0	25.0
9	16.5	17.5	18.5	20.0	21.0	23.0	25.5	28.0
10	18.0	19.0	20.5	21.5	23.0	25.5	28.0	31.0

Table A-3 Textbook

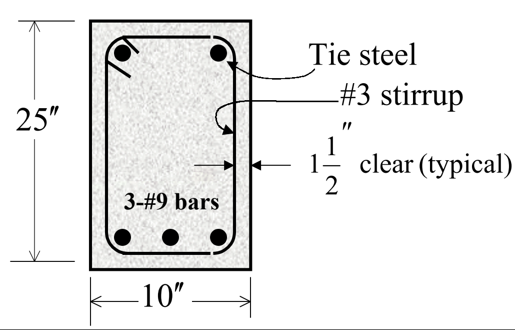
Beam Design Examples

■ Example 2 (cont'd)

Table 6. Reinforced Steel Properties Table A-1 Textbook

Bar number	3	4	5	6	7	8	9	10	11	14	18
Unit weight per foot (lb)	0.376	0.668	1.043	1.502	2.044	2.670	3.400	4.303	5.313	7.650	13.60
Diameter (in.)	0.375	0.500	0.625	0.750	0.875	1.000	1.128	1.270	1.410	1.693	2.257
Area (in^2)	0.11	0.20	0.31	0.44	0.60	0.79	1.00	1.27	1.56	2.25	4.00

6. Final Sketch



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Beam Design Examples

- Example 3

  Design a simply supported rectangular reinforced beam with tension steel only to carry a service load of 0.9 kip/ft and service live load of 2.0 kips/ft. (the dead load does not include the weight of the beam.) The span is 18 ft. Assume No. 3 stirrups. Use f_c' = 4,000 psi and f_y = 60,000 psi

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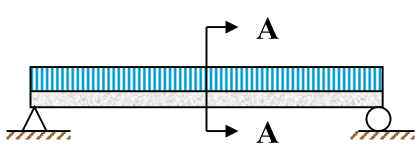
Beam Design Examples

- Example 3 (cont'd)



In this problem we have to determine h, b, and A_s. This is called "free design".

This problem can solved according to The outlines of Procedure B presented earlier. For complete solution for this problem, please see Example 2-8 of your Textbook.