CHAPTER

2c

Prentice Hall

Reinforced Concrete Design Fifth Edition

RECTANGULAR R/C CONCRETE BEAMS: TENSION STEEL ONLY

A. J. Clark School of Engineering •Department of Civil and Environmental Engineering

Part I – Concrete Design and Analysis

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ENCE 355 - Introduction to Structural Design

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Strength Requirements

- The basic criterion for strength design may be expressed as

$$\textit{Strength furnished} \geq \textit{Strength required} \quad (1)$$

- All members and all sections of members must be proportioned to meet this criterion.
- Eq. 1 can be thought of as a supply and a demand.

Strength Requirements

- The supply is considered as the strength furnished, while the demand as the strength required.
- The required strength may be expressed in the forms of design loads or their related moments, shears, and forces.
- Design loads may be defined as service loads multiplied by their appropriate factors.

Strength Requirements

- Eq. 1 can be expressed in a more compact general form as

$$\phi R_n \geq \sum_{i=1}^{m} \gamma_i L_{ni} \qquad (2)$$

Where
ϕ = strength reduction factor
γ_i = load factor for the i^{th} load component out of n components
R_n = nominal or design strength (stress, moment, force, etc.)
L_{ni} = nominal (or design) value for the i^{th} load component out of m components

Strength Requirements

- Eq. 2 is the basis for Load and Resistance Factor Design (LRFD) for Structural Members.
- This equation uses different partial safety factors for the strength and the load effects.
- The load factors are usually amplifying factors (>1), while the strength factors are called reduction factors (<1).

Strength Requirements

- Strength Factor
  - The strength reduction factor ϕ provide for the possibility that small adverse variation in material strength, workmanship, and dimensions may combine to result in undercapacity.
- Load Factors
  - The load factors γ's attempt to assess the possibility that prescribed service loads may be exceeded. Obviously, a live load is more apt to be exceeded than a dead load, which is largely fixed by the weight.

Strength Requirements

- ACI Code Provisions
  - In assigning strength reduction factors, the degree of ductility and the importance of the member as well as the degree of accuracy with which the strength of the member can be established are considered.
  - The ACI Code provides for these variables by using the following ϕ factors as provided in Table 1.

Strength Requirements

Table 1. Strength Reduction Factors

Type of Loading	ϕ
Bending	0.90
Shear and Torsion	0.85
Compression members (spirally reinforced)	0.75
Compression Members (tied)	0.70
Bearing on Concrete	0.70

Strength Requirements

- ACI Code Provisions
  - When word design is used throughout the ACI Code, it indicates that the load factors are included.
  - The subscript u is used to indicate design loads, moments, shears, and forces.
  - For example, the design load $w_u = 1.4w_{DL} + 1.7w_{LL}$ and the required or design moment strength for dead and live loads is

$$M_u = 1.4M_{DL} + 1.7M_{LL}$$

where 1.4 and 1.7 are the load factors.

Strength Requirements

- ACI Requirements for Dead and Live Loads
  - For dead and live loads, the ACI Code specifies design loads, design shears, and design moments be obtained from service loads by the using the relation

$$U = 1.4D + 1.7L \qquad (3)$$

Strength Requirements

- ACI Requirements for Strength
  - The ACI Code stipulates that the strength (moment, shear, force) furnished shall meet the following requirements

$$\phi R_n \geq 1.4D + 1.7L \qquad (4)$$

Where
ϕ = strength reduction factor as provided in Table 1
R_n = nominal or design strength (stress, moment, force, etc.)

Rectangular Beam Analysis for Moment

- The analysis of a reinforced concrete beam implies that we know precisely what comprises the section of the beam.
- The following data are known:

1. Tension bar size or number (or A_s).
2. Beam width (b).
3. Effective depth (d) or total depth (h).
4. Compressive strength of concrete (f'_c).
5. Yield strength of steel (f_y).

Rectangular Beam Analysis for Moment

- Variables that need to be found or answered include the following:
  1. Find the strength ϕM_n.
  2. Check the adequacy of a given beam, or
  3. Find an allowable load that the beam can carry.

Rectangular Beam Analysis for Moment

- Example 1

  Determine if the simply supported beam shown in Fig. 1 is adequate as governed by the ACI Code. The prescribed loads are as follows:

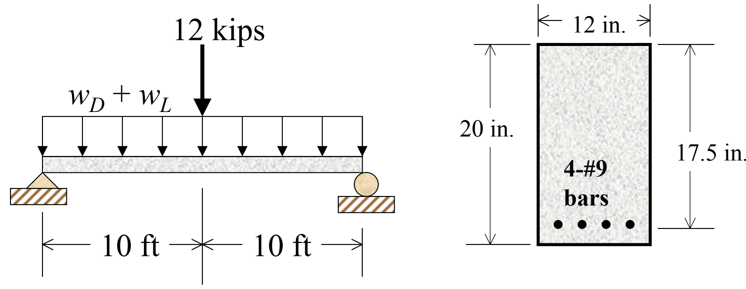
  w_D = 0.80 kip/ft (excludes beam weight)

  w_L = 0.80 kip/ft

  Assume that the compressive strength of concrete is 4,000 psi, while the yield strength of steel is 60,000 psi.

Rectangular Beam Analysis for Moment

- Example 1 (cont'd)



Rectangular Beam Analysis for Moment

- Example 1 (cont'd)

Area for No. 9 bar = 1.00 in^2 (see Table 2 or Table A - 2 Text)

Therefore, $A_s = 4(1.00) = 4.00 \text{ in}^2$

Next we need to find the maximum and minimum Reinforcement for this beam as specified by the ACI .

$$\rho_{\max} = 0.75\rho_b$$
$$A_{s_{\max}} = 0.75 A_{sb}$$

ACI Code

$$\rho_b = \frac{0.85 f_c' \beta_1}{f_y}\left(\frac{87{,}000}{f_y + 87{,}000}\right) = \frac{0.85(4)(0.85)}{60}\left(\frac{87}{60+87}\right) = 0.02851$$

Therefore, $\rho_{\max} = 0.75\rho_b = 0.75(0.02851) = 0.0214$

Rectangular Beam Analysis for Moment

Table 2. ASTM Standard - English Reinforcing Bars

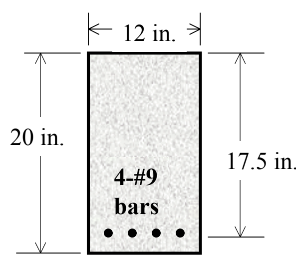
Bar Designation	Diameter in	Area in²	Weight lb/ft
#3 [#10]	0.375	0.11	0.376
#4 [#13]	0.500	0.20	0.668
#5 [#16]	0.625	0.31	1.043
#6 [#19]	0.750	0.44	1.502
#7 [#22]	0.875	0.60	2.044
#8 [#25]	1.000	0.79	2.670
#9 [#29]	1.128	1.00	3.400
#10 [#32]	1.270	1.27	4.303
#11 [#36]	1.410	1.56	5.313
#14 [#43]	1.693	2.25	7.650
#18 [#57]	2.257	4.00	13.60

Note: Metric designations are in brackets

Rectangular Beam Analysis for Moment

- Example 1 (cont'd)

$$A_{s,\min} = \frac{3\sqrt{f'_c}}{f_y} b_w d \geq \frac{200}{f_y} b_w d \qquad \text{ACI}$$



$$A_{s,\min} = \frac{3\sqrt{4{,}000}}{60{,}000}(12)(17.5) \geq \frac{200}{60{,}000}(12)(17.5)$$

$$A_{s,\min} = 0.664 \geq 0.700$$

Therefore, take $A_{s,\min} = 0.70 \text{ in}^2$

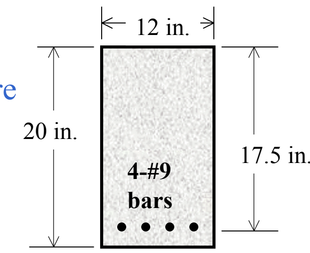
Calculate the steel ratio ρ for this beam:

$$\rho = \frac{A_s}{bd} = \frac{4}{12(17.5)} = 0.0191$$

Rectangular Beam Analysis for Moment

- Example 1 (cont'd)

Since $\rho = 0.0191 < \rho_{max} = 0.0214$, failure by yielding is assured.
Also, $A_s = 4.00\ \text{in}^2 > 0.70\ \text{in}^2$ OK



Note that $\rho_{max} = 0.75\ \rho_b = 0.0214$ can be obtained directly from from Table 3 (Table A-5 Text).

Also note that $A_{s,\min}$ can be obtained from Table 3 (Table A-5 Text) as follows

$$A_{s,\min} = 0.0033\ b\ d = 0.0033\ (12)(17.5) = 0.693 \approx .70\ \text{in}^2$$

Rectangular Beam Analysis for Moment

Table A-5 Textbook

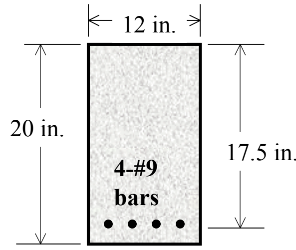
Table 3
Design Constants

f'_c (psi)	$\left[\frac{3\sqrt{f'_c}}{f_y} \geq \frac{200}{f_y}\right]$	$\rho_{max} = 0.75\ \rho_b$	Recommended Design Values ρ_b	$\bar{k}$ (ksi)
		F_y = 40,000 psi		
3,000	0.0050	0.0278	0.0135	0.4828
4,000	0.0050	0.0372	0.0180	0.6438
5,000	0.0053	0.0436	0.0225	0.8047
6,000	0.0058	0.0490	0.0270	0.9657
		F_y = 50,000 psi		
3,000	0.0040	0.0206	0.0108	0.4828
4,000	0.0040	0.0275	0.0144	0.6438
5,000	0.0042	0.0324	0.0180	0.8047
6,000	0.0046	0.0364	0.0216	0.9657
		F_y = 60,000 psi		
3,000	0.0033	0.0161	0.0090	0.4828
4,000	0.0033	0.0214	0.0120	0.6438
5,000	0.0035	0.0252	0.0150	0.8047
6,000	0.0039	0.0283	0.0180	0.9657
		F_y = 75,000 psi		
3,000	0.0027	0.0116	0.0072	0.4828
4,000	0.0027	0.0155	0.0096	0.6438
5,000	0.0028	0.0182	0.0120	0.8047
6,000	0.0031	0.0206	0.0144	0.9657

Values used in the example.

Rectangular Beam Analysis for Moment

■ Example 1 (cont'd)



$$N_C = N_T$$

$$0.85 f_c' ba = A_s f_y$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{4(60)}{0.85(4)(12)} = 5.88 \text{ in.}$$

$$Z = d - \frac{a}{2} = 17.5 - \frac{5.88}{2} = 14.6 \text{ in.}$$

Therefore, $M_n = A_s f_y Z = 0.85 f_c' baZ$

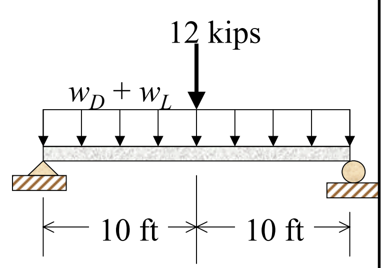
$$M_n = 4(60)(14.6) = 3{,}504 \text{ in - kips} = \frac{3{,}504}{12} \text{ ft - kips}$$

Hence, $M_n = \boxed{292 \text{ ft - kips (based on Steel)}}$

Rectangular Beam Analysis for Moment

■ Example 1 (cont'd)

– Service Loads:



The beam weight is to be calculated:

Beam weight = Volume × 0.150 kip/ft

$$\text{Weight} = \left(\frac{20}{12}\text{ft} \times \frac{12}{12}\text{ft} \times 1\text{ft}\right)\left(0.15\frac{\text{kip}}{\text{ft}}\right) = 0.25 \text{ kip /ft}$$

Total uniform dead load, $w_D = 0.25 + 0.80 = 1.05$ kips/ft
Total uniform dead load, $w_L = 0.80 =$ kips/ft

Using Eq. 3 $\quad U = 1.4D + 1.7L \quad$ ACI Code

Rectangular Beam Analysis for Moment

- Example 1 (cont'd)

ACI Code

$$\phi R_n \geq 1.4D + 1.7L$$

$$w_u = 1.4w_D + 1.7w_L$$
$$= 1.4(1.05) + 1.7(0.80) = 2.83 \text{ kips/ft}$$

$$P_u = 1.7P_L = 1.7(12) = 20.4 \text{ kips}$$

$$M_u = \frac{w_u L^2}{8} + \frac{P_u L}{4} = \frac{2.83(20)^2}{8} + \frac{12(20)}{4} = 243.5 \text{ ft-kips}$$

Check ACI Code Requirement:

$$\phi R_n \geq (1.4M_D + 1.7M_L = M_u)$$
$$[0.9M_n = 0.9(292) = 262.8 \text{ ft-kips}] > [M_u = 243.5 \text{ ft-kips}]$$

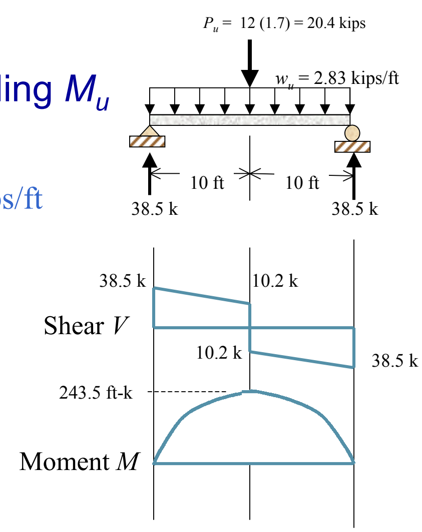
OK

Therefore the beam is adequate

Rectangular Beam Analysis for Moment

- Example 1 (cont'd)
  - Alternative way for finding M_u

$$w_u = 1.4w_D + 1.7w_L$$
$$= 1.4(1.05) + 1.7(0.80) = 2.83 \text{ kips/ft}$$



The factored maximum moment can be obtained from the moment diagram directly:

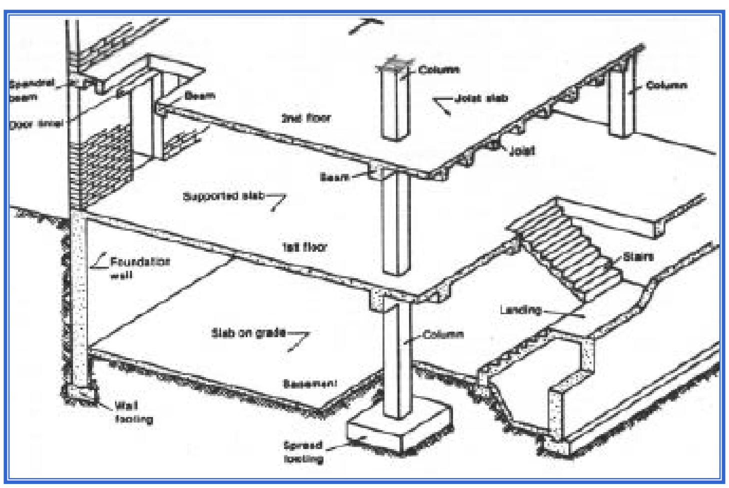
$$M_u = 243.5 \text{ ft-kips}$$

Introduction to Slabs

- Slabs are considered specialized type of bending members.
- They are used both in structural steel and reinforced concrete construction.
- Types of Slabs:
  - One-way Slab
  - Two-way Slab
    - Flat Slab

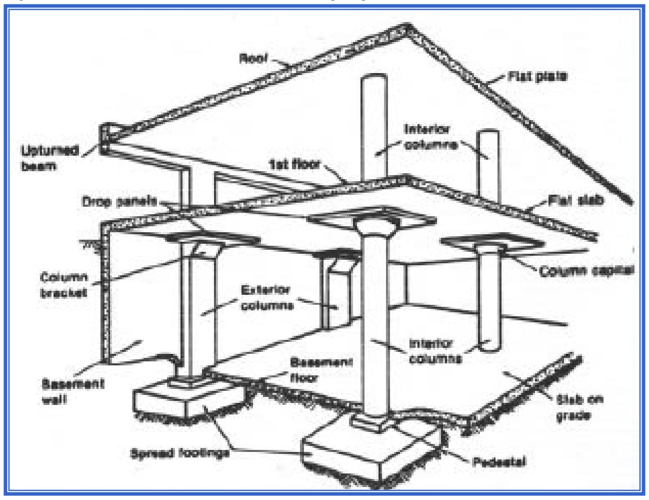
Introduction to Slabs

- Typical Structure (1)



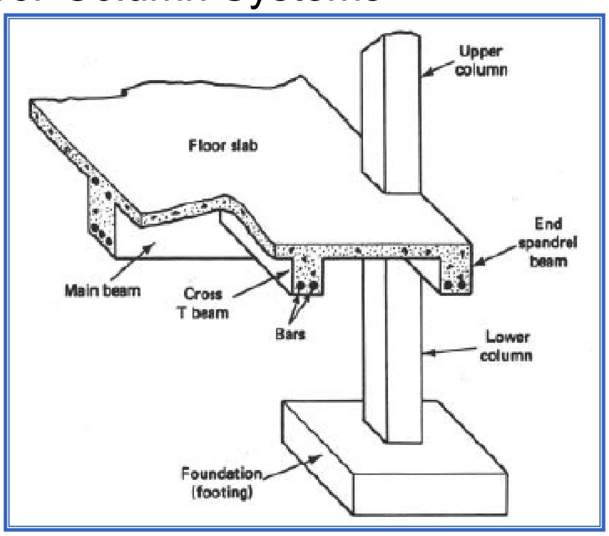
Introduction to Slabs

- Typical Structure (2)



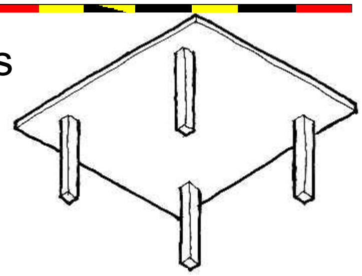
Introduction to Slabs

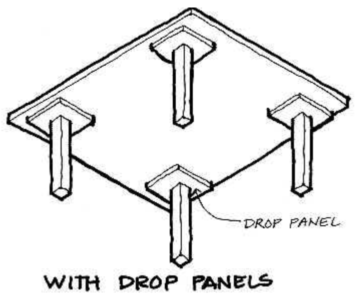
- Floor-Column Systems

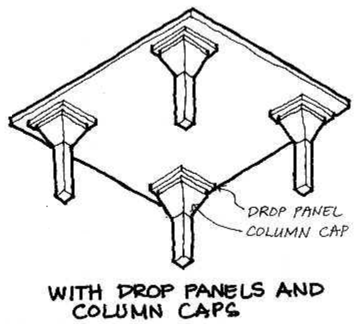


Introduction to Slabs

- Floor-Column Systems





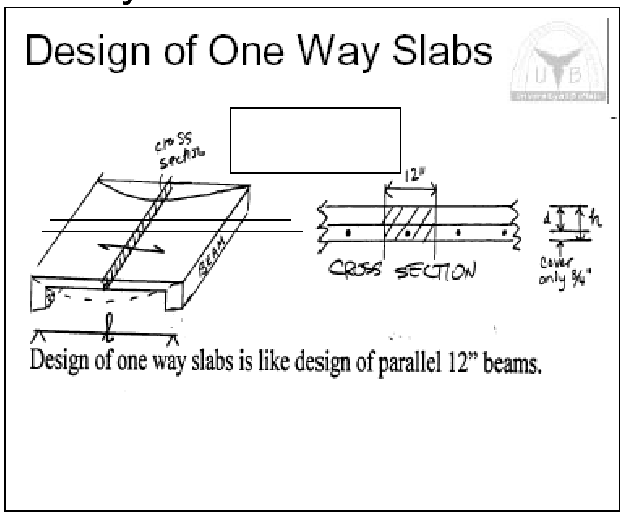


Introduction to Slabs

- One-Way Slab
  - A one-way slab can be defined as a structural reinforced concrete slab supported on two opposite sides so that the bending occurs in one direction only, that is, perpendicular to the supported edges.

Introduction to Slabs

- One-Way Slab

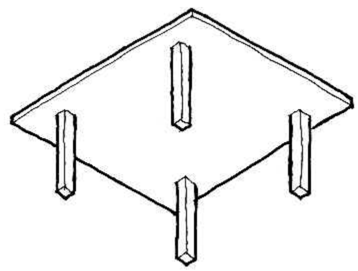


Introduction to Slabs

- Two-Way Slab
  - A two-way slab can be defined as a structural reinforced concrete slab supported along four edges so that the bending occurs in two directions perpendicular to each other.
  - However, If the ratio of the lengths of the two perpendicular sides is in excess of 2, the slab may be assumed to act as a one-way slab with bending primarily occurring in the short direction.

Introduction to Slabs

- Flat Slab
  - A specific type of two-way slab is categorized as a flat slab. A flat slab may be defined as a concrete slab reinforced in two or more directions, generally without beams or girders to transfer the loads to the supporting members.



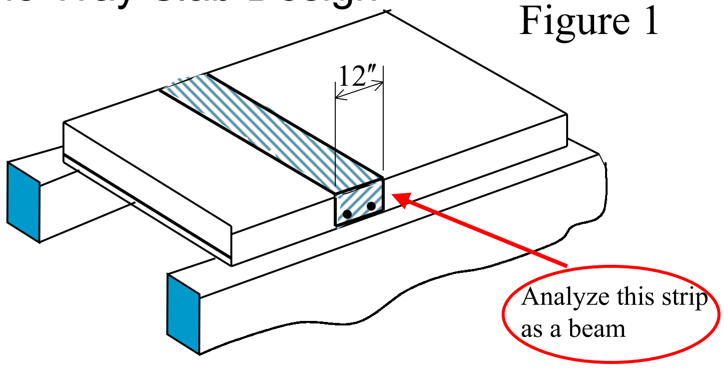
One-Way Slab: Analysis for Moment

- In this course, we are concerned primarily with one-way slab that is assumed to be a rectangular beam with width b = 12 in. as shown in Fig. 1.
- When loaded with uniformly distributed load, the slabs deflects so that it has curvature, and therefore bending moment, in only one direction (Fig. 1).

One-Way Slab: Analysis for Moment

- One-Way Slab Design

Figure 1



The procedure for finding ϕM_n for one-way slab is almost identical to that of a beam.

One-Way Slab: Analysis for Moment

- ACI Code Requirements for Slabs
  - Minimum Steel Area, $A_{s,min}$:
    - For grade 40 or 50 steel:

$$A_s = 0.0020bh \qquad (5a)$$

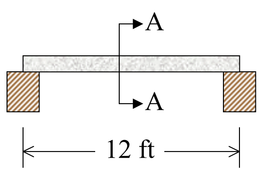
    - For grade 60 steel:

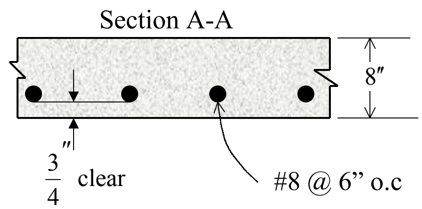
$$A_s = 0.0018bh \qquad (5b)$$

  - Concrete protection:
    - Concrete protection for reinforcement in slabs must be not less than 0.75 in.
    - For surfaces exposed to weather and ground, min. protection is 2 in (#6 to #18) and 1.5 in (#5)

One-Way Slab: Analysis for Moment

- Example 2
  - The one-way slab shown spans 12 ft from center of the support to the center of support. Calculate ϕM_n and determine the service live load (psf) that the slab may carry. Use $f_c' = 3{,}000$ psi and $f_y = 40{,}000$ psi.





One-Way Slab: Analysis for Moment

- Example 2 (cont'd)
  - Analyze a 12-in wide strip of slab:
  - For $f_c' = 3{,}000$ psi and $f_y = 40{,}000$ psi

$$\rho_{max} = 0.0278 \quad \text{from Table 3 (Table A-5 Text)}$$

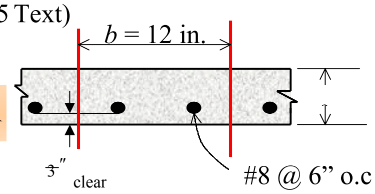
$$A_{s,\,min} = 0.0020(12)(8) = 0.19\ \text{in}^2$$

$$A_s = 2(0.79) = 1.58\ \text{in}^2 > 0.19\ \text{in}^2 \quad \text{OK}$$

$$d = 8 - 0.75 - 0.5 = 6.75\ \text{in.}$$

$$\rho = \frac{A_s}{bd} = \frac{1.58}{12(6.75)} = 0.0195 < 0.0278 \quad \text{OK}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{1.58(40)}{0.85(3)(12)} = 2.07\ \text{in.}$$



One-Way Slab: Analysis for Moment

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One-Way Slab: Analysis for Moment

Table A-5 Textbook

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Design Constants

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F_y = **50,000 psi**				
3,000	0.0040	0.0206	0.0108	0.4828
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6,000	0.0046	0.0364	0.0216	0.9657
F_y = **60,000 psi**				
3,000	0.0033	0.0161	0.0090	0.4828
4,000	0.0033	0.0214	0.0120	0.6438
5,000	0.0035	0.0252	0.0150	0.8047
6,000	0.0039	0.0283	0.0180	0.9657
F_y = **75,000 psi**				
3,000	0.0027	0.0116	0.0072	0.4828
4,000	0.0027	0.0155	0.0096	0.6438
5,000	0.0028	0.0182	0.0120	0.8047
6,000	0.0031	0.0206	0.0144	0.9657

Values used in the example.

One-Way Slab: Analysis for Moment

- Example 2 (cont'd)

$$Z = d - \frac{a}{2} = 6.75 - \frac{2.07}{2} = 5.72 \text{ in.}$$

$$M_n = A_s f_y Z = \frac{1.58(40)(5.72)}{12} = 30.13 \text{ ft-kips}$$

Therefore,

$$\phi M_n = 0.9(30.13) = 27.1 \text{ ft-kips}$$

$$M_u = \phi M_n = 27.1 = \frac{w_u L^2}{8}$$

$$w_u = \frac{27.1(8)}{L^2} = \frac{27.1(8)}{(12)^2} = 1.51 \text{ k/ft}$$

One-Way Slab: Analysis for Moment

- Example 2 (cont'd)

ACI Code

$$\phi R_n \geq 1.4D + 1.7L$$

$$w_u = 1.4 w_D + 1.7 w_L$$

$$w_D = \text{weight of slab} = \frac{8(12)}{144}(0.150) = 0.10 \text{ k/ft}$$

$$1.51 = 1.4(0.10) + 1.7 w_L$$

$$1.7 w_L = 1.51 - 1.4(0.10)$$

Hence,

$$w_L = \frac{1.51 - 1.4(0.1)}{1.7} = 0.806 \text{ k/ft} = 806 \text{ psf}$$