Homework #9 Solution
ENCE 203 - Spring 2001
Due W, 4/25

Problem1:

Textbook: 7-2

Forward: $\frac{df(x)}{dx} = f'(3) \approx \frac{f(4) - f(3)}{4 - 3} = \frac{1676 - 455}{1} = 1221$

Backward: $\frac{df(x)}{dx} = f'(3) \approx \frac{f(3) - f(2)}{3 - 2} = \frac{455 - 152}{1} = 303$

Two-Step: $\frac{df(x)}{dx} = f'(54) \approx \frac{f(4) - f(2)}{4 - 2} = \frac{1676 - 152}{2} = 762$

True Value: $f'(x) = f'(3) = 533$

The accuracy of the solution is shown in the following table:

	Forward	Backward	Two-Step	TRUE
$f'(x)$	1221	303	762	533
% Error	129.1	43.2	43.0	---

Problem 2:

Textbook: 7-7

t	D	ΔD	$\Delta^2 D$	$\Delta^3 D$	$\Delta^4 D$	$\Delta^5 D$
0	0					
		10				
2	10		30			
		40		30		
4	50		60		-10	
		100		20		10
6	150		80		0	
		180		20		
8	330		100			
		280				
10	610					

(a) $V(6) = \left.\frac{dD}{dt}\right|_{t=6} \approx \left.\frac{\Delta D}{\Delta t}\right|_{t=6} = \frac{180 + 100}{8 - 4} = 70 \ \frac{\text{ft}}{\text{sec}}$

(b) $V(8) = \left.\frac{dD}{dt}\right|_{t=8} \approx \left.\frac{\Delta D}{\Delta t}\right|_{t=8} = \frac{280 + 180}{10 - 6} = 115 \ \frac{\text{ft}}{\text{sec}}$

$V(10) = \left.\frac{dD}{dt}\right|_{t=10} \approx \left.\frac{\Delta D}{\Delta t}\right|_{t=10} = \frac{280}{10 - 8} = 140 \ \frac{\text{ft}}{\text{sec}}$

By linear interpolation between t = 8 sec and t = 10 sec,

$$V(8.7) = 123.75 \ \frac{\text{ft}}{\text{sec}}$$

(c) $a(2) = \left.\frac{d^2 D}{dt^2}\right|_{t=2} \approx \left.\frac{\Delta^2 D}{\Delta t^2}\right|_{t=2} = \frac{30}{(2)^2} = 7.5 \ \frac{\text{ft}}{\text{sec}^2}$

By linear interpolation between $t = 0$ sec and $t = 2$ sec,

$$a(1) = 3.75 \ \frac{\text{ft}}{\text{sec}^2}$$

(d) $a(6) = \left.\frac{d^2 D}{dt^2}\right|_{t=6} \approx \left.\frac{\Delta^2 D}{\Delta t^2}\right|_{t=6} = \frac{80}{(2)^2} = 20 \ \frac{\text{ft}}{\text{sec}^2}$

$a(8) = \left.\frac{d^2 D}{dt^2}\right|_{t=8} \approx \left.\frac{\Delta^2 D}{\Delta t^2}\right|_{t=8} = \frac{100}{(2)^2} = 25 \ \frac{\text{ft}}{\text{sec}^2}$

By linear interpolation between $t = 6$ sec and $t = 8$ sec,

$$a(6.3) = 20.75 \ \frac{\text{ft}}{\text{sec}^2}$$

Problem 3:

Textbook: 7-10

By the method of undetermined coefficients, the interpolating polynomial and its first derivative are:

$f(x) = 1100 - 2540x + 2011x^2 - 643x^3 + 77x^4$

$f'(x) = -2540 + 4022x - 1929x^2 + 308x^3$

Estimations of the first derivative at $x = 3$ and $x = 5.5$:

(a) $f'(3) = 481$

(b) $f'(5.5) = 12472.25$

The following table compares the estimates with the true values:

x	$f'(x)$	$f'(x)_{\text{True}}$	Error(%)
3	481	533	9.8
5.5	12472.3	14138	11.8

Problem 4:

Textbook: 7-16

The data table is re-produced here to show θ in radians.

θ(rad)	0	0.2618	0.5236	0.7854	1.0472	1.3090	1.5708
sinθ	0	0.2588	0.50000	0.70701	0.8660	0.9659	1.0000

$$\int_0^{1.5708} \sin\theta d\theta \approx \frac{0.2618}{2}\left[0 + 2(0.2588) + 2(0.5) + 2(0.70701) + 2(0.8660) + 2(0.9659) + 1.000\right] = 0.99424$$

True value = 1.0, % error = $\frac{1 - 0.99424}{1}$x100 = 0.576%

Problem 5:

Textbook: 7-19

Interval width = 1

x	0	1	2	3
$f(x)$	0	1	1.2599	1.4422

$$\int_0^3 f(x)\mathrm{d}x \approx \frac{(1)}{2}\left[0 + 2(1) + 2(1.2599) + 1.4422\right] = 2.9810$$

Interval width = 0.5

x	0	0.5	1.0	1.5	2.0	2.5	3.0
$f(x)$	0	0.7937	1	1.1447	1.2599	1.3572	1.4422

$$\int_0^3 f(x)\mathrm{d}x \approx \frac{(0.5)}{2}\begin{bmatrix} 0 + 2(0.7937) + 2(1) + 2(1.1447) + 2(1.2599) + 2(1.3572) \\ +1.4422 \end{bmatrix} = 3.1383$$

Interval width = 0.25

x	$f(x)$
0	0.0000
0.25	0.6300
0.5	0.7937
0.75	0.9086
1	1.0000
1.25	1.0772
1.5	1.1447
1.75	1.2051
2	1.2599
2.25	1.3104
2.5	1.3572
2.75	1.4010
3	1.4422

$$\int_0^3 f(x)\mathrm{d}x \approx \frac{(0.25)}{2}\begin{bmatrix} 0 + 2(0.63) + 2(0.7937) + 2(0.9086) + 2(1) + 2(1.0772) \\ +2(1.1447) + 2(1.2051) + 2(1.2599) + 2(1.3104) \\ +2(1.3572) + 2(1.4010) + 1.4422 \end{bmatrix} = 3.2022$$

The following tables compares the estimates with the true value:

Interval Width	Integral	%error	True Value
1	2.981	8.1	3.2451
0.5	3.1383	3.3	
0.25	3.2022	1.3	

Problem 6:

Textbook: 7-27

Number of intervals = 4

x	1	1.75	2.5	3.25	4
$f(x)$	0.11853	0.06892	0.03654	0.01832	0.00891

$$\int_1^4 \frac{\mathrm{d}x}{\left(3 + 2e^x\right)} \approx \frac{(0.75)}{3}\begin{bmatrix} (0.11853 + 4(0.06892) + 0.03654) \\ +(0.03654 + 4(0.01832) + 0.00891) \end{bmatrix} = 0.13737$$

Number of intervals = 8

x	$f(x)$
1	0.11853
1.375	0.09166
1.75	0.06892
2.125	0.05064
2.5	0.03654
2.875	0.02601
3.25	0.01832
3.625	0.01281
4	0.00891

$$\int_1^4 \frac{\mathrm{d}x}{\left(3+2e^x\right)} \approx \frac{(0.375)}{3}\begin{bmatrix}(0.11853+4(0.09166)+0.06892) \\ +(0.06892+4(0.05064)+0.03654) \\ +(0.03654+4(0.02601)+0.01832) \\ +(0.01832+4(0.01281)+0.00891)\end{bmatrix} = 0.13744$$

Number of intervals = 10

x	$f(x)$
1	0.11853
1.3	0.09672
1.6	0.07748
1.9	0.06108
2.2	0.04751
2.5	0.03654
2.8	0.02786
3.1	0.0211
3.4	0.01589
3.7	0.01192
4	0.00891

$$\int_1^4 \frac{\mathrm{d}x}{\left(3+2e^x\right)} \approx \frac{(0.3)}{3}\begin{bmatrix}(0.11853+4(0.09672)+0.07748) \\ +(0.07748+4(0.06108)+0.04751) \\ +(0.04751+4(0.03654)+0.02786) \\ +(0.02786+4(0.0211)+0.01589) \\ +(0.01589+4(0.01192)+0.00891)\end{bmatrix} = 0.13744$$