Homework #7 Solution
ENCE 203 - Spring 2001
Due M, 4/9

Problem1:

Textbook: 5-30

$$X_1 = \frac{-10.8 + 2X_2 - 3X_3}{4}$$

$$\begin{aligned} 4X_1 - 2X_2 + 3X_3 &= -10.8 \\ 3X_1 + 5X_2 - 2X_3 &= 11.9 \\ -2X_1 - 2X_2 + 5X_3 &= -0.2 \end{aligned} \quad \Rightarrow \quad X_2 = \frac{11.9 - 3X_1 + 2X_3}{5}$$

$$X_3 = \frac{-0.2 + 2X_1 + 2X_2}{5}$$

The following tables shows the results for both the Jacobi and Gauss-Seidel iterations:

JACOBI ITERATION

Iteration	X_1	$\lvert\Delta X_1\rvert$	X_2	$\lvert\Delta X_2\rvert$	X_3	$\lvert\Delta X_3\rvert$
0	1.000	-----	1.000	-----	1.000	-----
1	-2.950	3.950	2.180	1.180	0.760	0.240
2	-2.180	0.770	4.454	2.274	-0.348	1.108
3	-0.212	1.968	3.549	0.905	0.870	1.218
4	-1.578	1.366	2.855	0.694	1.295	0.425
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.	.	.	.	.	.	.
35	-1.500	0.001	3.600	0.000	0.800	0.000
36	-1.500	0.000	3.600	0.001	0.800	0.000
37	-1.500	0.001	3.600	0.000	0.800	0.000
38	-1.500	0.000	3.600	0.000	0.800	0.000

GAUSS-SEIDEL ITERATION

Iteration	X_1	$\lvert\Delta X_1\rvert$	X_2	$\lvert\Delta X_2\rvert$	X_3	$\lvert\Delta X_3\rvert$
0	1.000	-----	1.000	-----	1.000	-----
1	-2.950	3.950	4.550	3.550	0.600	0.400
2	-0.875	2.075	3.145	1.405	0.868	0.268
3	-1.779	0.904	3.794	0.649	0.766	0.102
4	-1.378	0.401	3.513	0.281	0.814	0.048
.	.	.	.	.	.	.
.	.	.	.	.	.	.
10	-1.499	0.003	3.599	0.002	0.800	0.000
11	-1.500	0.001	3.600	0.001	0.800	0.000
12	-1.500	0.001	3.600	0.000	0.800	0.000
13	-1.500	0.000	3.600	0.000	0.800	0.000

Comparing the results of the two tables, the Gauss-Seidel method has a faster rate of convergence than that of the Jacobi method. The solution for the system of equations is

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} -1.500 \\ 3.600 \\ 0.800 \end{bmatrix}$$

<u>Problem 2:</u>

Textbook: 5-35

$$|\boldsymbol{A}| = \begin{vmatrix} 2 & -3 & 1 & -2 \\ 3 & 4 & -5 & 1 \\ -2 & -2 & 0 & 4 \\ 1 & 1 & -3 & 1 \end{vmatrix} = (1)\begin{vmatrix} 3 & 4 & 1 \\ -2 & -2 & 4 \\ 1 & 1 & 1 \end{vmatrix} + (5)\begin{vmatrix} 2 & -3 & -2 \\ -2 & -2 & 4 \\ 1 & 1 & 1 \end{vmatrix} + (3)\begin{vmatrix} 2 & -3 & -2 \\ 3 & 4 & 1 \\ -2 & -2 & 4 \end{vmatrix}$$

$$= (1)(6) + (5)(-30) + (3)(74) = 78$$

$$|\boldsymbol{A}_1| = \begin{vmatrix} -10.3 & -3 & 1 & -2 \\ 4.3 & 4 & -5 & 1 \\ -5.4 & -2 & 0 & 4 \\ -1.8 & 1 & -3 & 1 \end{vmatrix} = -46.8$$

$$|\boldsymbol{A}_2| = \begin{vmatrix} 2 & -10.3 & 1 & -2 \\ 3 & 4.3 & -5 & 1 \\ -2 & -5.4 & 0 & 4 \\ 1 & -1.8 & -3 & 1 \end{vmatrix} = 273$$

$$|\boldsymbol{A}_3| = \begin{vmatrix} 2 & -3 & -10.3 & -2 \\ 3 & 4 & 4.3 & 1 \\ -2 & -2 & -5.4 & 4 \\ 1 & 1 & -1.8 & 1 \end{vmatrix} = 124.8$$

$$|\boldsymbol{A}_4| = \begin{vmatrix} 2 & -3 & 1 & -10.3 \\ 3 & 4 & -5 & 4.3 \\ -2 & -2 & 0 & -5.4 \\ 1 & 1 & -3 & -1.8 \end{vmatrix} = 7.8$$

Thus, the solution is

$$X_1 = \frac{|\boldsymbol{A}_1|}{|\boldsymbol{A}|} = \frac{-46.8}{78} = -0.6$$

$$X_2 = \frac{|\boldsymbol{A}_2|}{|\boldsymbol{A}|} = \frac{273}{78} = 3.5$$

$$X_3 = \frac{|\boldsymbol{A}_3|}{|\boldsymbol{A}|} = \frac{124.8}{78} = 1.6$$

$$X_4 = \frac{|\boldsymbol{A}_4|}{|\boldsymbol{A}|} = \frac{7.8}{78} = 0.1$$

Problem 3:

Textbook: 5-41

$n = 5 \qquad \sum X_i = 15 \qquad \sum X_i^2 = 59 \qquad \sum Y_i = 11 \qquad \sum X_i Y_i = 40$

Hence, the set of the two simultaneous equations and its solution are given by

$$\begin{aligned} 5a + 15b &= 11 \\ 15a + 59b &= 40 \end{aligned} \quad \Rightarrow \quad \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0.700 \\ 0.500 \end{bmatrix}$$

Problem 4:

Textbook: 6-5

$$\begin{bmatrix} 1 & 0.4 & 0.16 & 0.064 \\ 1 & 0.5 & 0.25 & 0.125 \\ 1 & 0.6 & 0.36 & 0.216 \\ 1 & 0.7 & 0.49 & 0.343 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0.064 \\ 0.125 \\ 0.216 \\ 0.343 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Thus, the interpolating polynomial is

$f(x) \approx x^3$

and

$f(0.47) \approx (0.58)^3 = 0.195$

Problem 5:

Textbook: 6-9

The interpolating polynomial can be formed by applying Eq. 6-10 of the textbook for each pair of values in increasing order of x. The calculations are summarized to four significant figures in the following table:

i	x	$f(x)$	Eq. 6-10 (Textbook)	a_i
1	0.50	0.4621	$0.4621=a_1$	0.4621
2	0.55	0.5005	$0.5005=a_1+a_2(0.55-0.50)$	0.7680
3	0.60	0.5370	$0.5370=a_1+a_2(0.60-0.50)+a_3(0.60-0.50)(0.60-0.55)$	-0.3800
4	0.65	0.5717	$0.5717=a_1+a_2(0.65-0.50)+a_3(0.65-0.50)(0.65-0.55)+$. $a_4(0.65-0.50)(0.65-0.55)(0.65-0.60)$	0.13333

This yields the following interpolating polynomial:

$$f(x) \approx 0.4621 + 0.7680(x-0.5) - 0.3800(x-0.5)(x-0.55) + 0.1333(x-0.5)(x-0.55)(x-0.60)$$

From which

$f(0.475) = 0.4422$

$f(0.525) = 0.4815$ $\Rightarrow$ To four significant figure, there is no error generated as compared with true values.