Homework #6 Solution
ENCE 203 - Spring 2001
Due M, 4/2

Problem1:

Textbook: 5-1

Forward elimination: Step 1

$$\begin{bmatrix} 1 & 2 & -1 & 3.1 \\ -3 & 1 & 1 & 1.4 \\ -1 & -1 & 4 & 7.3 \end{bmatrix} \quad \begin{aligned} R_1' &= R_1 \\ R_2' &= R_2/(-3) - R_1' \\ R_3' &= R_3 + R_1' \end{aligned} \quad \begin{bmatrix} 1 & 2 & -1 & 3.1 \\ 0 & -2.3333 & 0.6667 & -3.5667 \\ 0 & 1 & 3 & 10.4 \end{bmatrix}$$

Forward elimination: Step 2

$$\begin{bmatrix} 1 & 2 & -1 & 3.1 \\ 0 & -2.3333 & 0.6667 & -3.5667 \\ 0 & 1 & 3 & 10.4 \end{bmatrix} \quad \begin{aligned} R_1' &= R_1 \\ R_2' &= R_2/(-2.3333) \\ R_3' &= R_3 - R_2' \end{aligned} \quad \begin{bmatrix} 1 & 2 & -1 & 3.1 \\ 0 & 1 & -0.285714 & 1.528571 \\ 0 & 0 & 3.285714 & 8.87144 \end{bmatrix}$$

Forward elimination: Step 3

$$\begin{bmatrix} 1 & 2 & -1 & 3.1 \\ 0 & 1 & -0.285714 & 1.528571 \\ 0 & 0 & 3.285714 & 8.87144 \end{bmatrix} \quad \begin{aligned} R_1' &= R_1 \\ R_2' &= R_2 \\ R_3' &= R_3/(3.285714) \end{aligned} \quad \begin{bmatrix} 1 & 2 & -1 & 3.1 \\ 0 & 1 & -0.285714 & 1.528571 \\ 0 & 0 & 1 & 2.700000 \end{bmatrix}$$

The last matrix represents the following set of equations:

$$\begin{aligned} X_1 + 2X_2 - X_3 &= 3.100000 \\ X_2 - 0.285714X_3 &= 1.528571 \\ X_3 &= 2.700000 \end{aligned}$$

The backward substitution can be used to obtain values for the unknowns:

Backward Substitution: Step 1

$$\begin{bmatrix} 1 & 2 & -1 & 3.1 \\ 0 & 1 & -0.285714 & 1.528571 \\ 0 & 0 & 1 & 2.700000 \end{bmatrix} \quad \begin{aligned} R_1' &= R_1 + R_3' \\ R_2' &= R_2 + 0.285714R_3' \\ R_3' &= R_3 \end{aligned} \quad \begin{bmatrix} 1 & 2 & 0 & 5.8 \\ 0 & 1 & 0 & 2.3 \\ 0 & 0 & 1 & 2.7 \end{bmatrix}$$

Backward Substitution: Step 2

$$\begin{bmatrix} 1 & 2 & 0 & 5.8 \\ 0 & 1 & 0 & 2.3 \\ 0 & 0 & 1 & 2.7 \end{bmatrix} \quad \begin{aligned} R_1' &= R_1 - 2R_2' \\ R_2' &= R_2 \\ R_3' &= R_3 \end{aligned} \quad \begin{bmatrix} 1 & 0 & 0 & 1.2 \\ 0 & 1 & 0 & 2.3 \\ 0 & 0 & 1 & 2.7 \end{bmatrix}$$

The last matrix contains the solution:

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 1.2 \\ 2.3 \\ 2.7 \end{bmatrix}$$

Problem 2:

Textbook: 5-7

Step 1

$$\begin{bmatrix} 3 & 3 & -2 & 7.6 \\ 2 & -4 & 1 & 1.4 \\ -1 & -2 & 5 & -6.3 \end{bmatrix} \qquad \begin{aligned} R_1' &= R_1 / (3) \\ R_2' &= R_2 / (2) - R_1' \\ R_3' &= R_3 + R_1' \end{aligned} \qquad \begin{bmatrix} 1 & 1 & -0.6667 & 2.5333 \\ 0 & -3 & 1.1667 & -1.8222 \\ 0 & 1 & -4.3333 & 3.7667 \end{bmatrix}$$

Step 2

$$\begin{bmatrix} 1 & 1 & -0.6667 & 2.5333 \\ 0 & -3 & 1.1667 & -1.8333 \\ 0 & 1 & -4.3333 & 3.7667 \end{bmatrix} \qquad \begin{aligned} R_1' &= -(R_1 - R_2') \\ R_2' &= R_2 / (-3) \\ R_3' &= R_3 - R_2' \end{aligned} \qquad \begin{bmatrix} 1 & 0 & -0.2778 & 1.9222 \\ 0 & 1 & -0.3889 & 0.6111 \\ 0 & 0 & -3.9444 & 3.1556 \end{bmatrix}$$

Step 3

$$\begin{bmatrix} 1 & 0 & -0.2778 & 1.9222 \\ 0 & 1 & -0.3889 & 0.6111 \\ 0 & 0 & -3.9444 & 3.1556 \end{bmatrix} \qquad \begin{aligned} R_1' &= R_1 + 0.2778 R_3' \\ R_2' &= R_2 + 0.3889 R_3' \\ R_3' &= R_3 / (-3.9444) \end{aligned} \qquad \begin{bmatrix} 1 & 0 & 0 & 1.7 \\ 0 & 1 & 0 & 0.3 \\ 0 & 0 & 1 & -0.8 \end{bmatrix}$$

The last matrix contains the following solution:

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 1.7 \\ 0.3 \\ -0.8 \end{bmatrix}$$

Problem 3:

Textbook: 5-8

There is an infinite number of solutions for this system of equations. The first and the third equation are not unique, since one equation can be reproduced by the other by multiplying it by a constant.

Problem 4:

Textbook: 5-16

$$\boldsymbol{A} = \begin{bmatrix} -2 & 3 & -1 \\ 3 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix} \qquad \boldsymbol{C} = \begin{bmatrix} -0.6 \\ -3.3 \\ 1.9 \end{bmatrix}$$

Using Eqs. 5-52a of the textbook, the following results

$$l_{11} = a_{11} = -2$$
$$l_{21} = a_{21} = 3$$
$$l_{31} = a_{31} = 1$$

Using Eq. 5-52b of the textbook, the following results

$$u_{12} = \frac{a_{12}}{l_{11}} = \frac{3}{-2} = -1.5$$

$$u_{13} = \frac{a_{13}}{l_{11}} = \frac{-1}{-2} = 0.5$$

Using Eq. 5-52c of the textbook, the following results

$$l_{22} = a_{22} - \sum_{k=1}^{2-1} l_{2k} u_{k2} = a_{22} - l_{21}u_{12} = 1-(3)(-1.5) = 5.5$$

$$l_{32} = a_{32} - \sum_{k=1}^{2-1} l_{3k} u_{k2} = a_{32} - l_{31}u_{12} = 2-(1)(-1.5) = 3.5$$

Using Eq. 5-52d of the textbook, the following results

$$u_{23} = \frac{a_{23} - \sum_{k=1}^{2-1} l_{2k}u_{k3}}{l_{22}} = \frac{a_{23} - l_{21}u_{13}}{l_{22}} = \frac{-2-(3)(0.5)}{5.5} = -0.636364$$

Using Eq. 5-52e of the textbook, the following results

$$l_{33} = a_{33} - \sum_{k=1}^{3-1} l_{3k} u_{k3} = -a_{33} - l_{31}u_{13} - l_{32}u_{23} = 1-(1)(0.5)-(3.5)(-0.636364) = 2.727274$$

Therefore, the $\boldsymbol{L}$ and $\boldsymbol{U}$ matrices are

$$\boldsymbol{L} = \begin{bmatrix} -2 & 0 & 0 \\ 3 & 5.5 & 0 \\ 1 & 3.5 & 2.727274 \end{bmatrix}$$

$$\boldsymbol{U} = \begin{bmatrix} 1 & -1.5 & 0.5 \\ 0 & 1 & -0.636364 \\ 0 & 0 & 1 \end{bmatrix}$$

Equations 5-53 and 5-54 of the textbook result in

$$e_1 = \frac{C_1}{l_{11}} = \frac{-0.6}{-2} = 0.3$$

$$e_2 = \frac{C_2 - \sum_{j=1}^{2-1} l_{2j}e_j}{l_{22}} = \frac{C_2 - l_{21}e_1}{l_{22}} = \frac{-3.3-(3)(0.3)}{5.5} = -0.763636$$

$$e_3 = \frac{C_3 - \sum_{j=1}^{3-1} l_{3j}e_j}{l_{33}} = \frac{C_3 - l_{31}e_1 - l_{32}e_2}{l_{33}} = \frac{1.9-(1)(0.3)-(3.5)(-0.763636)}{2.727274} = 1.566665$$

$$X_3 = e_3 = 1.566665$$

$$X_2 = e_2 - \sum_{j=2+1}^{3} u_{2j}X_j = e_2 - u_{23}X_3 = -0.763636 - (-0.636364)(1.566665) = 0.233333$$

$$X_1 = e_1 - \sum_{j=1+1}^{3} u_{1j}X_j = e_1 - u_{12}X_2 - u_{13}X_3 = 0.3-(-1.5)(0.233333)-(0.5)(1.566665) = -0.133333$$

Summary of the solution:

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} -0.133333 \\ 0.233333 \\ 1.566665 \end{bmatrix}$$