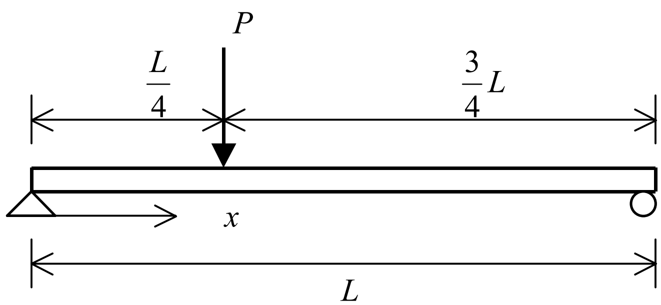
Homework #1b Solution
ENCE 203 - Spring 2001
Due 2/9, F

For the simply supported beam and loading condition shown below, perform spreadsheet calculations to estimate the maximum value of the deflection $y_{\max}$ and its location $x_{\max}$ along the length of the beam. Verify your results using analytical methods. Tabulate and plot the relationship of y and θ with x in separate graphs. Also, plot y vs. θ in a third graph.



The slope and deflection at any point x along the beam is given by

(1) For $x \le L/4$:

$$\theta = \frac{1}{EI}\left(\frac{3}{8}Px^2 - \frac{7PL^2}{128}\right)$$

$$y = \frac{1}{EI}\left(\frac{1}{8}Px^3 - \frac{7PL^2}{128}x\right)$$

(2) For $x > L/4$:

$$\theta = \frac{1}{EI}\left(-\frac{1}{8}Px^2 + \frac{1}{4}PLx - \frac{11PL^2}{128}\right)$$

$$y = \frac{1}{EI}\left(-\frac{Px^3}{24} + \frac{1}{8}PLx^2 - \frac{11PL^2}{128}x + \frac{PL^3}{384}\right)$$

where, P = concentrated load (lb), L = length of the beam (in), E = modulus of elasticity (lb/in^2), I = moment of inertia (in^4), θ = slope, and y = deflection at any point x along the length of the beam. Use the following values for P, L, E, and I:

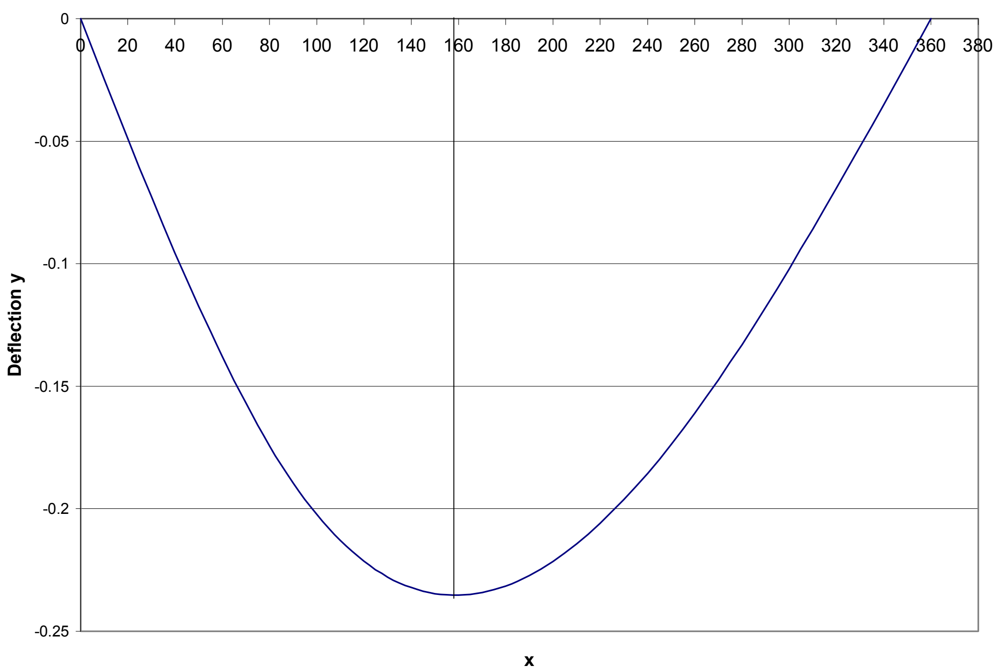
P = 2,000 lb
L = 360 in
E = 29,000,000 lb/in^2
I = 199 in^4.

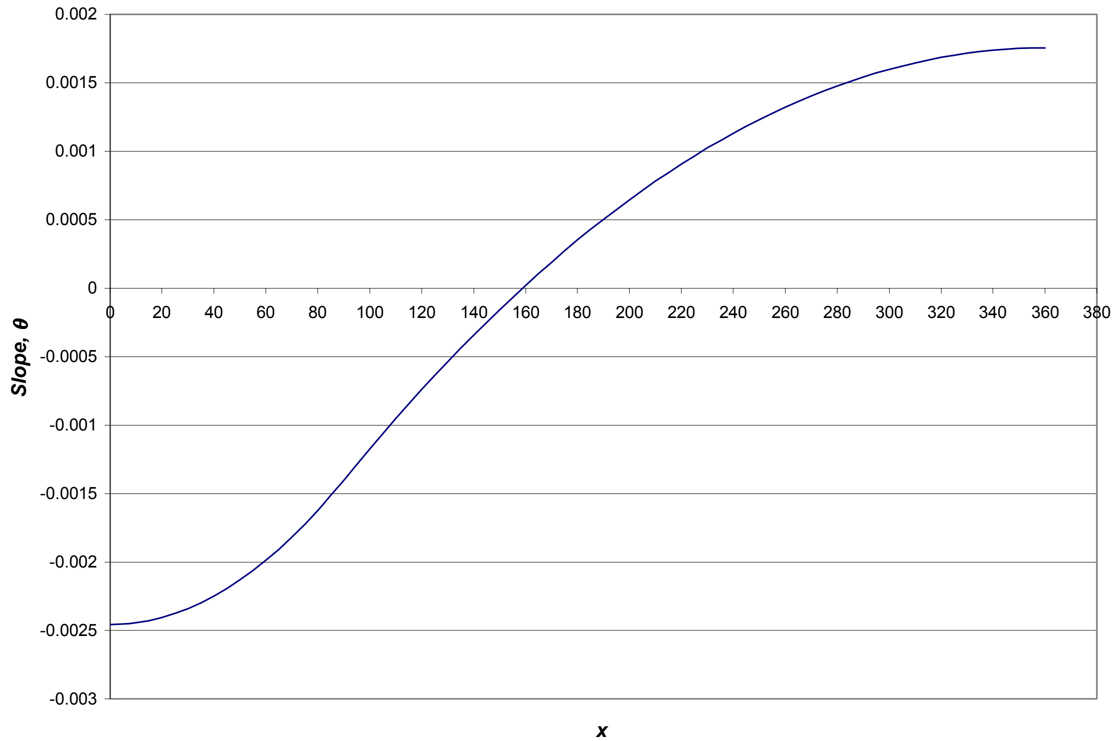
(**HINT**: y will be a maximum when θ is zero and $x > L/4$)

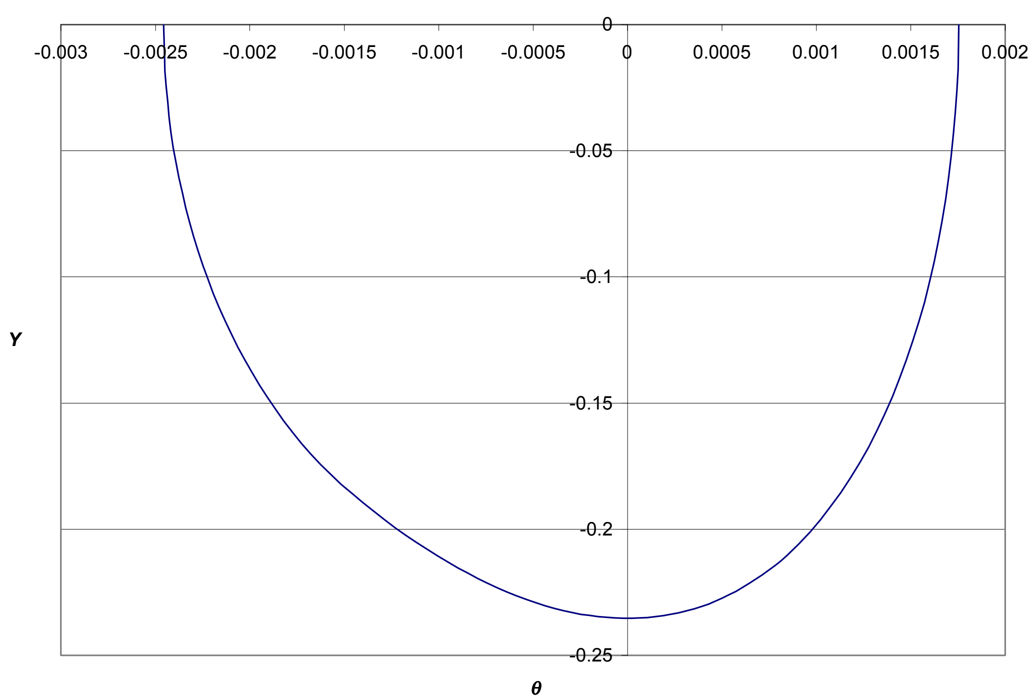
*** SOLUTION ****

x	θ	*y*	*x*	θ	*y*
0	-0.00246	0	190	0.000503	-0.22731
10	-0.00244	-0.02452	200	0.000645	-0.22157
20	-0.0024	-0.04878	210	0.00078	-0.21443
30	-0.00234	-0.07252	220	0.000905	-0.206
40	-0.00225	-0.09548	230	0.001022	-0.19636
50	-0.00213	-0.1174	240	0.001131	-0.18558
60	-0.00199	-0.13802	250	0.00123	-0.17377
70	-0.00182	-0.15708	260	0.001321	-0.16101
80	-0.00162	-0.17432	270	0.001404	-0.14737
90	-0.0014	-0.18948	280	0.001477	-0.13296
100	-0.00117	-0.20236	290	0.001542	-0.11786
110	-0.00095	-0.21299	300	0.001599	-0.10215
120	-0.00074	-0.22145	310	0.001646	-0.08592
130	-0.00054	-0.22783	320	0.001685	-0.06925
140	-0.00034	-0.23222	330	0.001715	-0.05224
150	-0.00016	-0.23471	340	0.001737	-0.03497
160	2.17E-05	-0.23537	350	0.00175	-0.01753
170	0.000191	-0.2343	360	0.001754	0
180	0.000351	-0.23159			

Numerical Solution:
From the Table and graphs, the maximum deflection occurs at approximately $x = 160$ with a value of –0.23537.




0.002
0.0015
0.001
0.0005
0
-0.0005
-0.001
-0.0015
-0.002
-0.0025
-0.003
0
20
40
60
80
100
120
140
160
180
200
220
240
260
280
300
320
340
360
380
Slope, θ
x


-0.003
-0.0025
-0.002
-0.0015
-0.001
-0.0005
0
0.0005
0.001
0.0015
0.002
0
-0.05
-0.1
-0.15
-0.2
-0.25
Y
θ

Analytical Solution:

$$\theta = \frac{1}{EI}\left(-\frac{1}{8}Px^2 + \frac{1}{4}PLx - \frac{11PL^2}{128}\right) = 0$$

$$\left(-\frac{1}{8}Px^2 + \frac{1}{4}PLx - \frac{11PL^2}{128}\right) = 0$$

$$\frac{-2000}{8}x^2 + \frac{(2000)(360)}{4}x - \frac{11(2000)(360)^2}{128} = 0$$

$$x^2 - 720x + 89100 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-720 \pm \sqrt{(720)^2 - 4(1)(89100)}}{2} = -158.75, \quad -561.24$$

Take x = -158.75. Note that x = -561.24 is out of the physical range of the problem.
Hence, the maximum deflection y_m:

$$y_m\big|_{x=-158.75} = \frac{1}{EI}\left(-\frac{Px^3}{24} + \frac{1}{8}PLx^2 - \frac{11PL^2}{128}x + \frac{PL^3}{384}\right) =$$

$$= \frac{1}{(29\times10^6)(199)}\left(-\frac{(2000)}{24}(158.75)^3 + \frac{2000(360)(258.75)^2}{8} - \frac{11(2000)(360)^2}{128}(158.75) + \frac{2000(360)^3}{384}\right)$$

$$= -\frac{1,358,411.295}{(29\times10^6)(199)} = -0.23539 \text{ in.}$$

Therefore:

$$y_{\max} = -0.23539 \text{ in at } x = 158.75$$