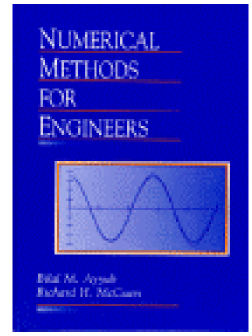
University of Maryland at College Park

Department of Civil and Environmental Engineering

ENCE 203 – Computation Methods in Civil Engineering II



EXAM I SOLUTION

(Closed Book & Notes, 8.5''x11" sheet is permitted)

Monday, March 12, 2001
10:00 am – 10:50 am, EGR 3106

Instructor: Dr. I. Assakkaf

"***Show your work & state all your assumptions***"

Student Name:__________SAMPLE_______________

SSN:_123-45-6789

Grade:___100___☺_

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Problem 1 (30 points)

(a) Identify the following matrices by their type:

$$\begin{bmatrix} 0 & t & 6 & 1 \\ 0 & 0 & x & 8 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 5 & 3 & t & 0 & 1 \\ 3 & 4 & 8 & x & 3 \\ t & 8 & 6 & 10 & 6 \\ 0 & x & 10 & 7 & y \\ 1 & 3 & 6 & y & 9 \end{bmatrix}$$

1. Strictly upper triangular__   2. Unit or identity matrix_____   3. Symmetric matrix______

$$\begin{bmatrix} 0 & -4 & 10 \\ 4 & 0 & -3 \\ -10 & 3 & 0 \end{bmatrix} \qquad \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & -4 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix} \qquad \begin{bmatrix} t^2 & \cos t & 0 & 0 & 0 \\ \sin t & 5 & \sin t & 0 & 0 \\ 0 & \cos t & 8 & -6 & 0 \\ 0 & 0 & 6 & 10 & t \\ 0 & 0 & 0 & -t & 2 \end{bmatrix}$$

4. Skew symmetric matrix______   5. Diagonal matrix___________   6. Tridiagonal banded matrix____

(b) For the following two matrices, compute:

1. the trace,
2. the determinant,
3. the transpose,
4. the eigenvalues (only for matrix $\boldsymbol{B}$), and
5. the inverse using $B^{-1} = \dfrac{B^a}{|B|}$ (only for matrix $\boldsymbol{B}$)
6. Is the inverse defined for matrix $\boldsymbol{A}$? Why?

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 14 & 5 \\ 2 & 4 & 6 \end{bmatrix} \text{ and } \boldsymbol{B} = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$$

*** SOLUTION ***

1.
tr($\boldsymbol{A}$) = 1 + 14 + 6 = 21,   tr($\boldsymbol{B}$) = 1 + 1 = 2

2.
det($\boldsymbol{A}$) = |$\boldsymbol{A}$| = 0 (the first row is proportional to the third),   det($\boldsymbol{B}$) = |$\boldsymbol{B}$| = (1)(1)-0.5(0.5) = 0.75

3.

$$A^T = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 14 & 4 \\ 3 & 5 & 6 \end{bmatrix} \qquad B^T = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$$

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4.

$$|B-\lambda I| = \left|\begin{bmatrix}1 & 0.5\\ 0.5 & 1\end{bmatrix} - \begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix}\right| = \begin{vmatrix}1-\lambda & 0.5\\ 0.5 & 1-\lambda\end{vmatrix} = 0$$

or

$$(1-\lambda)^2 - 0.25 = 0$$

$$1-\lambda = \pm 0.5 \Rightarrow \quad \lambda_1 = 0.5 \text{ and } \lambda_2 = 1.5$$

5.

$$B^c = \begin{bmatrix}1 & -0.5\\ -0.5 & 1\end{bmatrix}, \qquad (B^c)^T = B^a = \begin{bmatrix}1 & -0.5\\ -0.5 & 1\end{bmatrix}$$

$$\det(B) = |B| = 0.75 = \frac{3}{4}$$

$$B^{-1} = \frac{B^a}{|B|} = \frac{\begin{bmatrix}1 & -0.5\\ -0.5 & 1\end{bmatrix}}{3/4} = \begin{bmatrix}4/3 & -4/6\\ -4/6 & 4/3\end{bmatrix} = \begin{bmatrix}1.333 & -0.667\\ -0.667 & 1.333\end{bmatrix}$$

6. No, the inverse is not defined for matrix ***A*** because $|\boldsymbol{A}| = 0$.

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Problem 2 (20 points)

Develop a Taylor series expansion of the following function for three terms:

$$f(x) = x^3 - 3x^2 + 5x + 10$$

Use $x_0 = 1$ as the starting (or base) point. Evaluate the series for $x = 1.2$ and 1.8, and compare your results with the true value for both cases (i.e., find the relative true error (%) for both cases). Explain what happens when h decreases. Show your work to four significant digits.

*** SOLUTION ***

$$f(x) = x^3 - 3x^2 + 5x + 10 \quad \Rightarrow f(1) = 13$$

$$f'(x) = 3x^2 - 6x + 5 \quad \Rightarrow f'(1) = 2$$

$$f''(x) = 6x - 6 \quad \Rightarrow f''(1) = 0$$

$$f(x) = f(x_0 + h) \approx f(x_0) + hf'(x_0) + \frac{h^2}{2!} f''(x_0)$$

$$f(x_0 + h) \approx 13 + 2h + 0$$

For $x = 1.2$:

$$f(1.2) \approx f(1 + 0.2) = 13 + 2(0.2) = 13.400 \qquad \text{true } f(1.2) = 13.408$$

$$\varepsilon_r = \left| \frac{13.408 - 13.400}{13.408} \right| \times 100 = 0.05967\%$$

For $x = 1.8$:

$$f(1.8) \approx f(1 + 0.8) = 13 + 2(0.8) = 14.600 \qquad \text{true } f(1.8) = 15.112$$

$$\varepsilon_r = \left| \frac{15.112 - 14.600}{15.112} \right| \times 100 = 3.388\%$$

As h decreases, the Taylor series approximation gets closer to the true value.

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Problem 3 (25 points)

(a) Use the Newton-Raphson method with two iterations to find the root of the following equation with an initial guess of 1:

$$e^x - 2x - 1 = 0$$

Compute the relative error (%) at the second iteration. Show all your results to 4 significant figures.

(b) For the third-order polynomial $f(x) = x^3 - 3x^2 - x + 3 = 0$, a good estimate of the root was found to be 3 based on the secant method. Reduce the above polynomial to second-order polynomial using polynomial reduction technique, and find the rest of the roots using any method of your choice (including analytical methods).

*** SOLUTION ***

(a)

$x_0 = 1$

$$f(x) = e^x - 2x - 1$$

$$f'(x) = e^x - 2$$

Newton-Raphson Iteration:

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} = x_i - \frac{e^{x_i} - 2x_i - 1}{e^{x_i} - 2}$$

For $i = 0, x_0 = 1$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{e^1 - 2(1) - 1}{e^1 - 2} = 1 - (-0.3922) = 1.3922$$

For $i = 1, x_1 = 1.3922$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.3922 - \frac{e^{1.3922} - 2(1.3922) - 1}{e^{1.3922} - 2} = 1.3922 - 0.1182 = 1.2740$$

Relative error at second iteration:

$$\varepsilon_r = \left|\frac{x_{i+1} - x_i}{x_{i+1}}\right| \times 100 = \left|\frac{x_2 - x_1}{x_2}\right| \times 100 = \left|\frac{1.2740 - 1.3922}{1.2740}\right| = 9\%$$

(b)

$$\begin{array}{r|l}
 & x^2 - 1 \\
\hline
x - 3 & x^3 - 3x^2 - x + 3 \\
 & \underline{x^3 - 3x^2} \\
 & \quad - x + 3 \\
 & \quad \underline{- x + 3} \\
 & \quad 0 = \text{error}
\end{array}$$

$x^2 - 1 = 0 \Rightarrow x_1 = 1, \text{and } x_2 = -1$

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Problem 4(25 points)

(a) Given the following two row vectors $\boldsymbol{V}_1$ and $\boldsymbol{V}_2$:

$$\boldsymbol{V}_1 = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & -\frac{2}{3} \end{bmatrix} \quad \text{and} \quad \boldsymbol{V}_2 = \begin{bmatrix} \frac{2}{7} & -\frac{3}{7} & \frac{6}{7} \end{bmatrix}$$

1. Are the row vectors normalized? Why? If they are not normalized, normalize the row vectors.
2. Are the row vectors orthonormal? Why?
3. What is the transpose ($\boldsymbol{V}_1^{\mathrm{T}}$) of $\boldsymbol{V}_1$?
4. Is the vector product $\boldsymbol{V}_1^{\mathrm{T}}\,\boldsymbol{V}_2$ defined? Why?

(b) Errors, in general, can be classified as (1) numerical errors and (2) non-numerical errors. List at least two examples for each. Briefly define the term “accuracy” in numerical context.

*** SOLUTION ***

(a)

1. Both are normalized because their lengths equal to one:

$$|V_1| = \sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(-\frac{2}{3}\right)^2} = 1$$

$$|V_2| = \sqrt{\left(\frac{2}{7}\right)^2 + \left(-\frac{3}{7}\right)^2 + \left(\frac{6}{7}\right)^2} = 1$$

2. No, they are not orthonormal, because

$$\begin{bmatrix} \frac{1}{3} & \frac{2}{3} & -\frac{2}{3} \end{bmatrix} \begin{bmatrix} 2/7 \\ -3/7 \\ 6/7 \end{bmatrix} = \frac{1}{3}\left(\frac{2}{7}\right) + \frac{2}{3}\left(-\frac{3}{7}\right) - \frac{2}{3}\left(\frac{6}{7}\right) = \frac{2}{21} - \frac{6}{21} - \frac{12}{21} = -\frac{16}{21} \neq 0$$

3.

$$V_1^T = \begin{bmatrix} 1/3 \\ 2/3 \\ -2/3 \end{bmatrix}$$

5. The vector product $\boldsymbol{V}_1^{\mathrm{T}}\,\boldsymbol{V}_2$ is defined because $\boldsymbol{V}_1^{\mathrm{T}}$ has one column and $\boldsymbol{V}_2$ has one row as shown.

$$V_1^T V_2 = \begin{bmatrix} 1/3 \\ 2/3 \\ -2/3 \end{bmatrix} \begin{bmatrix} \frac{2}{7} & -\frac{3}{7} & \frac{6}{7} \end{bmatrix}$$

(3×1) (1×3)

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(b)

Non-numerical errors examples:

1. Modeling error
2. Blunders and mistakes
3. Uncertainty in information and data

Numerical errors examples:

1. Round-off errors
2. Truncation errors (i.e., Taylor series)
3. Propagation errors
4. Mathematical-approximation errors

<u>Accuracy</u> can be defined as the closeness or nearness of the measurements (or computed values) to the true or actual value of the quantity being measured or evaluated. Bias and precision are elements of accuracy.