|  | ( Making Hard Decision $\begin{gathered}\text { Duxbury } \\ \text { Thomson } \\ \text { Learning }\end{gathered}$ Third Edition |
| :---: | :---: |
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## Introduction

- How important is it to deal with uncertainty in a careful and systematic way?
- Subjective assessments of uncertainty are an important element of decision analysis.
- A basic tenet of modern decision analysis is that subjective judgments of uncertainty can be made in terms of probability. Is it worthwhile to develop more rigorous approach to measure uncertainty?


## Introduction

- It is not clear that it is worthwhile to develop a more rigorous approach to measure the uncertainty that we feel.
- Question

How important is it to deal with uncertainty in a careful and systematic way?

## Uncertainty and Public Policy

- Because of the potential losses, care in assessing probabilities is important.
- Examples:

1. Earthquake Prediction: Survey published a report that estimated a 0.60 probability of a major earthquake (7.5-8 on the Richter scale) occurring in Southern California along the southern portion of the San Andreas Fault within the next 30 years.

## Uncertainty and Public Policy

- Examples (cont'd)


## 2. Environmental Impact Statements:

 Assessments of the risks associated with proposed projects. These risk assessments often are based on the probabilities of various hazards occurring.
## Uncertainty and Public Policy

- Examples (cont'd)

3. Public Policy and Scientific Research: The possible presence of conditions that may require action by the government. But action sometimes must be taken without absolute certainty that a condition exists.

## Uncertainty and Public Policy

- Examples (cont'd)

4. Medical Diagnosis: A complex computer system known as APACHE III (Acute Physiology, Age, and Chronic Health Evaluation). Evaluates the patient's risk as a probability of dying either in the ICU or later in the hospital.

Because of the high stakes involved in these examples and others, it is important for policy makers to exercise care in assessing the uncertainties they face.

## Probability: A Subjective Interpretation

- Many introductory textbooks present probability in terms of long-run frequency.
- In many cases, however, it does not make sense to think about probabilities as long-run frequencies.


## Probability: A Subjective Interpretation

## - Example

In assessing the probability that the California condor will be extinct by the year 2010 or the probability of a major nuclear power plant failure in the next 10 years, thinking in terms of long-run frequencies or averages is not responsible because we cannot rerun the "experiment" many times to find out what proportion of the times the condor becomes extinct or a power plant fails. We often hear references to the chance that a catastrophic nuclear holocaust will destroy life on the planet. Let us not even consider the idea of a long-run frequency in this case!

## Probability: A Subjective Interpretation

## - Example (cont'd)

Note:

1. Unless you know the answer, you are uncertain.
2. We can view uncertainty in a way that is different from the traditional long-run frequency approach.
3. You are uncertain about the outcome because you do not know what the outcome was; the uncertainty is in your mind.

## Probability: A Subjective Interpretation

- Example (cont'd)


## Note:

1. The uncertainty lies in your own brain cells.
2. When we think of uncertainty and probability in this way, we are adopting a subjective interpretation, with a probability representing an individual's degree of belief that a particular outcome will occur.
3. Decision analysis requires numbers for probabilities, not phrases such as "common," "unusual," "toss-up," or "rare."

## Probability: A Subjective Interpretation

- Two main ways of looking at probability
- Dice example
- Crop experiments
- Probability as subjective likelihood of occurrence
- Nuclear power plant failure in next 10 years (don't want to repeat this)
- Major California earthquake before 2020 (need to quantify this type of uncertainty).


# It all depends on your degree of belief in the subject at hand. 

- There are three basic methods for assessing probabilities:


## Method \#1:

- The decision maker should assess the probability directly by asking:
"What is your belief regarding the probability that even such and such will occur?"


## Methods for Assessing Discrete

## Probabilities

## Method \#2:

- Ask about the bets that the decision maker would be willing to place.
- The idea is to find a specific amount to win or lose such that the decision maker is indifferent about which side of the bet to take.
- If person is indifferent about which side to bet, then the expected value of the bet must be the same regardless of which is taken. Given these conditions, we can then solve for the probability.



## Methods for Assessing Discrete

## Probabilities

- Example: College Basketball
- Suppose that UMD are playing the Duke in the NCAA finals this year.
- We are interested in finding the decision maker's probability that the UMD will win the championship. The decision maker is willing to take either of the following two bets (on the next page):


## Methods for Assessing Discrete

## Probabilities

The Alternatives and Their Outcomes
UMD vs. Duke in the NCAA finals want $P$ (UMD wins the championship)

Bet1 (Bet for UMD)
Win \$X if UMD wins Lose \$Y if UMD loses

Bet2 (Bet against UMD)


Lose \$X if UMD wins
Win \$Y if UMD loses


## Methods for Assessing Discrete

## Probabilities

- Example: College Basketball (cont'd)
- The assessor's problem is to find $X$ and $Y$ so that he or she is indifferent about betting for or against the UMD.
- If decision maker is indifferent between bets 1 and 2 then:

- Their Expected values are equal
- The computation is carried as follows:


## Methods for Assessing Discrete

## Probabilities

- Example: College Basketball (cont'd)

Computation

```
XP(UMD Win) - Y[1 - P(UMD Win)]
= -XP(UMD Win) + Y[1 - P(UMD Win)]
2{X P(UMD Win) - Y[1 - P(UMD Win)]} = 0
XP(UMD Win) - Y + YP(UMD Win) = 0
(X+Y)P(UMD Win) - Y=0
P(UMD Win) }=\frac{Y}{X+Y
```


## Methods for Assessing Discrete

## Probabilities

- Example: College Basketball (cont'd)

Now put in \$ amounts
Say you are indifferent if:
Win $\$ 2.50$ if UMD wins, $(X)$
Lose $\$ 3.80$ if UMD loses, $(Y)$
$\Rightarrow \mathrm{P}($ UMD Wins $)=\frac{3.80}{2.50+3.80}=0.603$

- Therefore there is $60.3 \%$ chance of winning

Your subjective probability that UMD wins is implied by your bedding behavior

## Methods for Assessing Discrete

## Probabilities

- Notes on Method \#2:
- The betting approach to assessing probabilities appears straightforward enough, but it does suffer from a number of problems:
- Many people simply do not like the idea of betting (even though most investments can be framed as a bet of some kind).
- Most people also dislike the prospect of losing money; they are risk-averse. Risk-averse people had to make the bets small enough to rule out risk-aversion.


## Methods for Assessing Discrete

## Probabilities

- Notes on Method \#2 (cont'd):
- The betting approach also presumes that the individual making the bet cannot make any other bets on the specific event (or even related events).


## Methods for Assessing Discrete

## Probabilities

## Method \#3:

- Adopt a thought experiment strategy in which the decision maker compares two lottery-like games, each of which can result in a Prize (A or B).


Prize A



Prize B

## Methods for Assessing Discrete

## Probabilities

- Method \#3 (cont'd)


## Reference Lottery



- $2^{\text {nd }}$ lottery is called the "reference lottery" for which the probability mechanism must be specified.


## Methods for Assessing Discrete

## Probabilities

## A Typical Lottery Mechanism is:

## 1. Equivalent Urn (EQU):

it involves drawing a ball randomly from an urn full of 100 red and white balls in which the proportion of red balls is known to be $P$ while the white balls are known with fraction (1-P). Drawing a red ball results in winning prize $A$ while drawing a white ball results in winning prize $B$. Drawing a colored ball from an urn where \% of colored ball is $p$.

## Methods for Assessing Discrete

## Probabilities

A Typical Lottery Mechanism is (cont'd):

## 2. Wheel of Fortune:

Another Lottery Mechanism is the Wheel of fortune with known area that represents "win"

If the wheel is spun and if spinner (pointer) lands in the "win" area.
$\Rightarrow$ you get Prize A (when UMD wins)

## Methods for Assessing Discrete

## Probabilities

- Method \#3 (cont'd)
- Once the mechanism is understood by the decision maker, you adjust the probability of winning in the reference lottery until the decision maker is indifferent between the 2 lotteries.
- The trick is to adjust the probability of winning in the reference lottery until the decision maker is indifferent between the two lotteries.


## Methods for Assessing Discrete

## Probabilities

- Method \#3 (cont'd)
- Indifference in this case means that the decision maker has no preference between the two lotteries, but slightly changing probability $p$ makes one or the other lottery clearly preferable.
- For UMD example: if Indifference $\Rightarrow$ $P($ UMD wins $)=p$


## Methods for Assessing Discrete

## Probabilities

- The Question: How do we find the $p$ that makes the decision maker indifferent?


## Methods for Assessing Discrete

## Probabilities

■ The Answer

- The basic idea is to start with some $p_{1}$ and ask which lottery the decision maker prefers.
- If she/he prefers the reference lottery, then $p_{1}$ must be too high; she/he perceives that the chance of winning in the reference lottery is high.


## Methods for Assessing Discrete

## Probabilities

- The Answer (cont'd)
- In this case, choose $p_{2}$ less than $p_{1}$ and ask her/his preference again.
- Continue adjusting the probability in the reference lottery until the indifference point is found.
- It is important to begin with extremely wide brackets and to converge on the indifference probability slowly.
- The Answer (cont'd)
- Going slowly allows the decision maker plenty of time to think hard about the assessment, and the result will probably be much better.


## Methods for Assessing Discrete

## Probabilities

- The Wheel of Fortune
- The Wheel of Fortune is a particularly useful way to assess probabilities.
- By changing the setting of the wheel to represent the probability of winning in the reference lottery, it is possible to find the decision maker's indifferent point quite easily.



## Methods for Assessing Discrete

## Probabilities

- The Wheel of Fortune (cont'd)
- The use of the wheel avoids the bias that can occur from using only "even probabilities ( $0.1,0.2,0.3$, and so on).
- With the wheel, a probability can be any value between 0 and 1 .


## Methods for Assessing Discrete

## Probabilities

## - The Wheel of Fortune - Example 1

| Situation: You may bet that the price of oil will be $\$ 30$ or less in five years or that the spinner will fall on the dark portion of the probability wheel afterspinning the wheel |  |  |
| :---: | :---: | :---: |
| Appearance of Wheel | Response | Implication |
|  | Prefer Spinner | P(price $\leq \$ 30)<0.5$ |
|  | Prefer Spinner | $P($ price $\leq \$ 30)<0.25$ |
|  | Indifferent | $P($ price $=\$ 30)=0.125$ |

Subjective Probability Example Using The Probability Wheel Mechanism [ Source: Buffa and Dyer, 1981]

## Methods for Assessing Discrete

## Probabilities

- The Wheel of Fortune - Example 2
- Probability assessment wheel for the Texaco reaction node.
- The user can change the proportion of the wheel that corresponds to any of the vents. Clicking on the "OK" button returns the user to the screen with appropriate probabilities entered on the branches of the chance node.


## Methods for Assessing Discrete

## Probabilities

- The Wheel of Fortune - Example 2

Accept $\$ 5 \mathrm{~B}=17 \%$


## CHAPTER 8. SUBJECTIVE PROBABILITY

## Methods for Assessing Discrete

## Probabilities

- The Wheel of Fortune: Shortcoming

The lottery-base approach to probability assessment is not without its own shortcoming:
A. Some people have a difficult time grasping the hypothetical game that they are asked to envision, and as a result they have trouble making assessments.
B. Others dislike the idea of a lottery or carnival-like game.
C. In some cases it may be better to recast the assessment procedure in terms of risks that are similar to the kinds of financial risks an individual might take.

## Methods for Assessing Discrete

## Probabilities

- Check for Consistency
- The last step in assessing probabilities is to check for consistency.
- Many problems will require the decision maker to assess several interrelated probabilities.
- It is important that these probabilities be consistent among themselves; they should obey the probability laws.


## Methods for Assessing Discrete

## Probabilities

- Example

If $P(A), P(B \mid A)$, and $P(A$ and $B)$ were all assessed, then it should be the case that:

$$
\mathrm{P}(A) \mathrm{P}(B \mid A)=\mathrm{P}(A \text { and } B)
$$

- If a set of assessed probabilities is found to be inconsistent, then the decision maker should reconsider and modify the assessments as necessary to achieve consistency.


## Assessing Continuous Probabilities

- It is always possible to model a decision maker's uncertainty using probabilities.
- How would this be done in the case of an uncertain but continuous quantity?
- Two strategies for assessing a subjective CDF.


## Assessing Continuous Probabilities

- Two strategies for assessing a subjective CDF:

1. Using a Reference Lottery and Probability Wheel:
(Adjusting the Probability in the reference lottery to assess probability of uncertain value in the upper lottery)
2. Using the Fractile method:
(Fixing the Probability in the reference lottery at a defined probability value and changing the uncertain value in the upper lottery)

## Assessing Continuous Probabilities

- Example: Movie Star Age
- The problem is to derive a probability distribution representing a probability assessor's uncertainty regarding a particular movie star's age. Several probabilities were found, and these were transformed into cumulative probabilities.


## Assessing Continuous Probabilities

- Example: Movie Star Age
- A typical cumulative assessment would be to assess $P($ Age < a), where $a$ is a particular value. For example, consider $P$ (Age $\leq 46$ ). A probability wheel can be used to assess the value of $p$ in the reference lottery of the following decision tree, until the decision maker is indifferent at a value of $p$. From the graph in the next two slides this would be $p=0.55$. Therefore, $\mathrm{P}($ Age $\leq 46)=0.55$


## Assessing Continuous Probabilities

## Example (cont'd)



## Assessing Continuous Probabilities

- Example (cont'd)
- Using this technique to find a CDF amounts to assessing the cumulative probability for a number of points, plotting them, and drawing a smooth curve through the plotted points.
- Suppose the following assessments were made:
$\mathrm{P}($ Age $\leq 29)=0.00$
$P($ Age $\leq 40)=0.05$
$P($ Age $\leq 44)=0.50$
$P($ Age $\leq 50)=0.85$
$P($ Age $\leq 65)=1.00$

- The CDF for Movie

Star's Age

## Assessing Continuous Probabilities

- The Fractile Method
- Decision tree for assessing the 0.35 fractile of a continuous distribution for $X$.
- The decision maker's task is to find $x$ in Lottery A that results in indifference between the two lotteries where the 0.35 value is fixed in the reference lottery (the 0.35 fractile). The 0.35 fractile is approximately 42 years. The remaining fractiles can be seen in the CDF shown in the previous slide.


## Assessing Continuous Probabilities

- The Fractile Method

- The Median in the Fractile method
- Decision tree for assessing the Median of the distribution for the movie star's age. The assessment task is to adjust the number of years a in Lottery A to achieve indifference. The median of 0.5 fractile is at age of 44 years.


## Assessing Continuous Probabilities

- The Median in the Fractile method

| B | $\text { Age } \leq a$ | Hawaiian Trip |
| :---: | :---: | :---: |
|  | Age $\geq a$ | Beer |
|  | (0.50) | Hawaiian Trip |
|  | (0.50) | Beer |

## Assessing Continuous Probabilities

- The Quartiles in the Fractile Method
- A subjectively assessed CDF for pretzel demand.
- 0.05 fractile for demand $=5,000$
-0.95 fractile for demand $=45,000$
- Demand is just likely to be above 13,000 as below or equal to 23,000
- There is a 0.25 chance that demand will be below 16,000
- There is a 0.75 chance that demand will be below 31,000


## Assessing Continuous Probabilities

- The Quartiles in the Fractile Method
- This means that:
- 0.05 fractile for demand $=5,000$
- First quartile (0.25)=16,000
- Median (0.5)=23,000
- Third quartile (0.75)=31,000
- 0.95 fractile for demand $=45,000$


## Assessing Continuous Probabilities

- The Quartiles in the Fractile Method


CDF for pretzel demand.

## Assessing Continuous Probabilities

- Three-Branch Discrete Values in the Fractile Method
- Replacing a continuous distribution with a three-branch discrete uncertainty node in a decision tree. This is called Pearson-Tukey approximation.
- $P(X=5000)=0.185$
- $\mathrm{P}(X=23,000)=0.63$
- $\mathrm{P}(X=45,000)=0.185$


## Assessing Continuous Probabilities

- Three-Branch Discrete Values in the Fractile Method



## Assessing Continuous Probabilities

## Bracket Medians in the Fractile Method

- Finding the bracket median for the interval between a and b . The cumulative probabilities $p$ and $q$ correspond to $a$ and $b$, respectively. Bracket median $\mathrm{m}^{*}$ is associated with a cumulative probability that is halfway between $p$ and $q$. That is to say that: $P\left(X \leq m^{*}\right)=(p+q) / 2$
- Bracket Medians in the Fractile Method


Shows relation at "bracket median" to underlying CDF.

## Assessing Continuous Probabilities

## Example: Bracket Medians

- Finding bracket medians for the pretzel demand distribution:

$$
\begin{array}{ll}
\mathrm{P}\left(X=m_{1}=8\right)=0.20, & \mathrm{P}\left(X=m_{2}=18\right)=0.20 \\
\mathrm{P}\left(X=m_{3}=23\right)=0.20 & \mathrm{P}\left(X=m_{4}=29\right)=0.20 \\
\mathrm{P}\left(X=m_{5}=39\right)=0.20 &
\end{array}
$$

## Assessing Continuous Probabilities

- Example (cont'd): Bracket Medians



## Assessing Continuous Probabilities

- Representing The Bracket Medians on Decision Trees



## Pitfalls: Heuristics and Biases

- Pitfalls: Heuristics (used by people to make probability judgment) and Biases.
- We tend to use rather primitive cognitive techniques to make our probability assessments. Tversky and Kahneman (1974) have labeled these techniques heuristics.
- Heuristics can be thought of as rule of thumb for accomplishing tasks.
- Heuristics tend to be simple, are easy to perform, and usually do not give optimal answers.
- Heuristics for assessing probabilities:

1. They are easy and intuitive ways to deal with uncertain situations.
2. They tend to result in probability assessments that are biased in different ways depending on the heuristics used.

## Representativeness

- Making a judgment based on how similar the description of a person or thing is to your own preconceived notions of the kind of people or things that you know to find in the field of study or a situation under consideration.
- The representativeness heuristic is used to judge the probability that someone or something belongs to a particular category.


## Representativeness

- The judgment is made by comparing the information known about the person or thing with the stereotypical member of the category.
- Misunderstanding of random processes is another phenomenon attributed to the representativeness heuristic.


## Representativeness

- The representativeness heuristic surfaces in many different situations and can lead to a variety of different biases:

1. Insensitivity to base rates or prior probabilities.
2. Replying on old and unreliable information to make predictions.

## Representativeness

3. Insensitivity to sample size is another possible result of the representativeness heuristic. Sometimes termed the law of small numbers, people (even scientists!) draw conclusions from highly representative small samples even though small samples are subject to considerably more statistical error than are large samples.

## Representativeness

4. Making equally precise predictions regardless of the inherent uncertainty in a situation.

## Misunderstanding of random processes is another phenomenon attributed to the representativeness heuristic

- We judge the probability that an event will occur according to the ease with which we can retrieve similar events from memory.
- External events and influences
- Example:
- They have a substantial effect on the availability of similar incidents. Seeing a traffic accident can increase one's estimate of the chance of being in an accident.
- Differential attention by the news media to different kinds of incidents can result in availability bias.
- Example:
- suppose the local newspaper plays up death resulting from homicide but plays down traffic deaths. To some extend, the unbalanced reporting can affect readers' judgments of the relative incident of homicides and traffic fatalities, thus affecting the community's overall perception.
- Some situations are simply easier to imagine than others.
- In other cases, it may be difficult to recall context in which a particular event occurs.
- Another situation involves illusory correlation.
- If a pair of events is perceived as happening together frequently, this perception can lead to an incorrect judgment regarding the strength of the relationship between the two events


## Anchoring-and-Adjusting

- Refers to the notion that in making estimates we often choose an initial anchor and then adjust that anchor based on our knowledge of the specific event in question.
- Example:
- Many people make sales forecasts by considering the sales figures for the most recent period and then adjusting those values based on new circumstances. The problem is that the adjustment usually is insufficient.


## Anchoring-and-Adjusting

Note:

1. The anchor-and-adjust heuristic affects the assessment of probability distributions for continuous uncertain quantities more than it affects discrete assessments.
2. Because of the tendency to underadjust, most subjectively assessed probability distributions are too narrow, inadequately reflecting the inherent variability in the uncertain quantity.
3. Subjective CDF using the median and quartiles is subject to overconfidence from anchoring and adjusting.

## Motivational Bias

- Incentives often exist that lead people to report probabilities or forecasts that do not entirely reflect their true beliefs
- Examples:

1. A Salesperson asked for a sales forecast, may be inclined to forecast low so that he will look good (and perhaps receive a bonus) when she/he sells more than the amount forecasted.

## Motivational Bias

2. Some evidence suggests that weather forecasters, in assessing the probability of precipitation, persistently err on the high side; they tend to overstate the probability of rain. Perhaps they would rather people were prepared for bad weather (and were pleasantly surprised by sunshine) instead of expecting good weather and being unpleasantly surprised.

## Heuristic and Biases: Implications

1. Some evidence suggests that individuals can learn to become good at assessing probabilities.
2. Awareness of the heuristics and biases may help individuals make better probability assessments. Knowing about some of the effects you now may be able to recognize them when they occur.
3. Assessing probabilities involve thinking about lotteries and chances in a structured way.

- By thinking hard about probabilities using these methods, it may be possible to avoid some heuristic reasoning and attendant biases.
- Thinking about lotteries provides a new perspective in the assessment process.

4. Some problems simply cannot be addressed well in the form in which they are presented.

## Decomposition and Probability

## Assessment



## Decomposition and Probability

## Assessment

- It is possible to break a probability assessment into smaller and more manageable chunks.
- This process is known as decomposition.
- There are at least three different scenarios in which decomposition of a probability assessment may be appropriate.


## Decomposition and Probability

## Assessment

- Scenario 1:

Thinking about how the event of interest is related to other events.

## Example:

- Assessing the probability that a given stock price increases. Instead of considering only the stock itself, we might think about its relationship to the market as a whole


## Decomposition and Probability

## Assessment

## Scenario 2:

Thinking about what kinds of uncertain outcomes could eventually lead to the outcome in question.

- Example:
- Suppose that you are an engineer in a nuclear power plant. Your boss calls you into his office and explains that the Nuclear Regulatory Commission (NRC) has requested safety information.


## Decomposition and Probability

## Assessment

- Example (cont'd)
- One item that the commission has requested is an assessment of the probability of an accident resulting in the release of radioactive material into the environment.
- Your boss knows that you have had a course in decision analysis, and so you are given the job of assessing the probability.


## Decomposition and Probability

## Assessment

- Example (cont'd)


Simple influence diagram
for assessing the probability of a nuclear power plant accident

## Decomposition and Probability

## Assessment

- Example (cont'd)
$\checkmark$ The four conditional probabilities we must assess for Outcome $A$ are: $\mathbf{P}(\mathbf{A} \mid \mathbf{L}, \mathbf{N}), \mathbf{P}(\mathbf{A} \mid \mathbf{L}, \mathbf{N}), \mathbf{P}(\mathbf{A} \mid \mathbf{L}, \mathbf{N}), \mathbf{P}(\mathbf{A} \mid \mathrm{L}, \mathbf{N})$. For the cooling system node, probabilities $P(L \mid E)$ and $P(L \mid E)$ must be assessed.
$\checkmark$ Likewise, for the control system node $\mathbf{P}(\mathbf{N} \mid \mathbf{E})$ and $\mathbf{P}(\mathbf{N} \mid \overline{\mathbf{E}})$ must be assessed. Finally, P(E) must be assessed for the electrical system node.

$$
\begin{aligned}
\mathrm{P}(A)= & \mathrm{P}(A \mid L, N) \mathrm{P}(L, N)+\mathrm{P}(A \mid \bar{L}, N) \mathrm{P}(\bar{L}, N) \\
& +\mathrm{P}(A \mid L, \bar{N}) \mathrm{P}(L, \bar{N})+\mathrm{P}(A \mid \bar{L}, \bar{N}) \mathrm{P}(\bar{L}, \bar{N})
\end{aligned}
$$

## Decomposition and Probability

## Assessment

## Scenario 3:

The third scenario is related to the second. It is a matter of thinking through all of the different events that must happen before the outcome in questions occurs. The second and third scenarios can be combined. There may be alternative paths to a failure, each requiring that certain individual outcomes occur.

## Note:

1. As with many decision-analysis techniques, there may be more than one-way to decompose a probability assessment.
2. The whole reason to use decomposition is to make the assessment process easier. The best decomposition to use is the one that is easiest to think about and that gives the clearest view of the uncertainty in the decision problem.
3. 3. Decomposition in decision-analysis assessment of probabilities is important because it permits the development of large and complex models of uncertainty.

## Experts and Probability Assessment:

## Pulling It All Together

- In complex problems, expert risk assessment plays a major role in the decision-making process.
- The process by which the expert information was acquired must stand up to professional scrutiny, and thus policy makers who acquire and use expert information must be able to document the assessment process.

Experts and Probability Assessment: Pulling It All Together

- The assessment of expert judgments must also adhere to standards. The standards for experts are different from those for data collection.


## Experts and Probability Assessment:

Pulling It All Together
Note:

1. The policy makers must be able to document and justify the expert-selection process, just as the data-collecting scientist must be able to document and justify the process by which specific data points were selected.
2. If judgments from multiple experts are combined to obtain a single probability distribution, then issues of relative expertise and redundancy among the experts must be taken into account.

## Experts and Probability Assessment

## Protocols

1. Background: Identify those variables for which expert assessment is needed.
2. Identification and Recruitment of Experts: Need appropriate experts.
3. Motivating Experts: Need to motivate experts to express their opinions.

## Experts and Probability Assessment

 Protocols4. Structuring and Decomposition: (Knowledge of exploration) Develop a general model (expressed, for example, as an influence diagram) that reflects the experts' thinking about the relationships among the variables.

## Experts and Probability Assessment

## Protocols

5. Probability-Assessment Training:
a. Explain the principles of assessment, to provide information on the inherent biases in the process and ways to counteract those biases, and
b. Give the experts an opportunity to practice making probability assessments.

## Experts and Probability Assessment

## Protocols

6. Probability Elicitation and Verification: The expert's assessments are checked to be sure they are consistent. Encouraging a thorough examination of the expert's knowledge base can help to counteract the biases associated with the psychological heuristics of availability, anchoring, and representativeness.

## 7. Aggregation of Experts' Probability

 Distribution: Ask the expert themselves to generate a consensus distribution.
## Methods for Group Decision-Making

 and/or Probability Assessment
## - Brainstorming:

- The purpose of the group brainstorming is to generate not to evaluate ideas
- Brainstorming encourages creative and new ideas by Groups.
- Group members are expected to state any extreme ideas which should not be ignored or states as ridiculous.
- Each idea presented belongs to the group and not to the individual who stated it.


## Methods for Group Decision-Making

 and/or Probability AssessmentDelphi Technique:

- It is an aggregated scheme which is used to obtain the opinion of a group of decisionmakers.
- It involves bringing those who have considerable experience about the domain under study to give their expectations and forecast of the proposed case.


## Methods for Group Decision-Making and/or Probability Assessment

- Delphi Technique (cont'd):
- This is done through getting anonymous judgment by mail questionnaires.
- Its advantage appears in removing the bias which can occur in face-to-face interaction.
- It is done in several rounds practically not less than four.


## Methods for Group Decision-Making

 and/or Probability Assessment
## - Delphi Technique (cont'd):

- In the first round, the members answer the questions sent to them by generating their ideas about the case and return them to the manager in charge who summarize these answers and follow them by another set of questionnaire for reassessment.
- In the second round the members independently evaluate their earlier inputs about the problem and return their answers.


## Methods for Group Decision-Making

and/or Probability Assessment

- Delphi Technique (cont'd):
- In the third round the members are offered another chance to revise their opinion about the problem.
- In the fourth round an average estimate is taken as the final response for the problem.
- Thus in summary the Delphi Technique promotes creativity and imagination by anonymous judgment of ideas to reach consensus decisions.


## Methods for Group Decision-Making

 and/or Probability Assessment- Nominal Group Technique (NGT):
- Its name is derived from bringing experts together but without allowing them to communicate verbally. The collection of the group is 'nominally' or 'in name only'.
- It is a structured group meeting that involves 7 to 10 individuals sitting around a table but do not speak to one another.
- In the first phase, each person expresses her/his beliefs about an opinion by generating an idea in writing.
- After five minutes, a structured sharing of ideas takes place.
- The person who records the meeting then displays all ideas on a flip chart in full view of the entire group without discussion.


## Methods for Group Decision-Making

 and/or Probability Assessment- Nominal Group Technique (NGT):
- In the second phase, the ideas that can reach 20 to 25 are listed and a structured discussion is allowed where every idea receives attention and a vote is taken by specifying the degree of support to every idea proposed.
- In the third phase every individual has to express his rank for every idea presented privately.


## Methods for Group Decision-Making

and/or Probability Assessment

- Nominal Group Technique (NGT):
- The resulting group decision is the mathematically pooled outcome of the individual votes.
- Thus in summary the NGT promotes creativity and imagination by bringing people together in a very structured meeting that does not required verbal communication.


## Constructing Distribution Using

## RISKview or @RISK

- Riskview is an Excel-based program designed to construct probability distribution based on assessed probabilities.
- This program can be used to fit a piecewise linear distribution to your probability assessment.


## Constructing Distribution Using

## RISKview or @RISK

- Once the fitted distribution has been constructed, one can calculate probability of events, draw density function, cumulative distribution function, and modify the distribution by revising the input data of the assessment.
- The best way to understand RISKview is by the following demonstrative example.


## Constructing Distribution Using

## RISKview or @RISK

- Example: Marketing Soft Pretzels
- Suppose that you have developed a new soft pretzels that you are thinking about marketing through some agent. Your assessment for the monthly demand of the pretzels is shown in the following viewgraph:

Please, refer to the steps provided in pages 329 to 335 of the textbook.

- Example: Marketing Soft Pretzels (cont'd)

1. Construct a Density Mass Function.
2. Construct a CDF.
3. Find

$$
\mathrm{P}(10,000 \leq x \leq 40,000)
$$

4. Find $\mathrm{P}(x>40,000)$

| Monthly <br> Demand, $X$ | CDF <br> $\mathrm{P}(D \leq X)$ |
| :---: | :---: |
| 5,000 | 0.05 |
| 16,000 | 0.25 |
| 23,000 | 0.50 |
| 31,000 | 0.75 |
| 45,000 | 0.95 |

## Constructing Distribution Using

## RISKview or @RISK

- Example: Marketing Soft Pretzels (cont'd)
- Using RISKview or @RISK, the following result can be obtained:

1. Graph of Density Mass Function
2. Graph of Cumulative Distribution Function
3. $P(10,000 \leq x \leq 40,000)=0.738$
4. $\mathrm{P}(x>40,000)=0.121$

- Example: Marketing Soft Pretzels (cont'd)


Example: Marketing Soft Pretzels (cont'd)



Constructing Distribution Using
RISKview or @RISK

- Example: Marketing Soft Pretzels (cont'd)


