CHAPTER 4b

Making Choices

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ENCE 627 – Decision Analysis for Engineering

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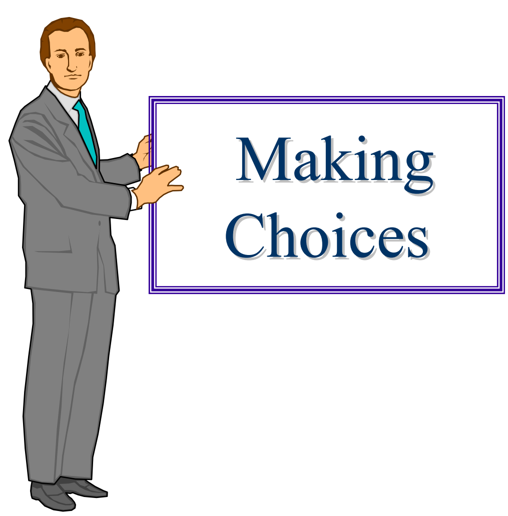
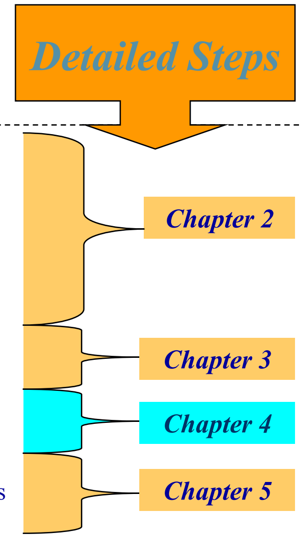


Methodology of Modeling Decisions

The Methodology of Modeling Decisions is to:

- Understand the problem under study
- Introduce quantitative modeling
- Discuss the elements of a decision.
  - Values and Objectives
  - Decisions to be made
  - Upcoming uncertain events, and
  - Consequences
- Build the decision Model and identify a set of feasible alternatives.
- **Evaluate the alternatives and make a choice of a feasible alternative.**
- Re-evaluate the alternatives using sensitivity analysis to refine the solution.



Risk Profiles and Specific Strategies

- **We can intuitively grasp the relative riskiness of alternatives by studying their *risk profiles*.**

Definition:

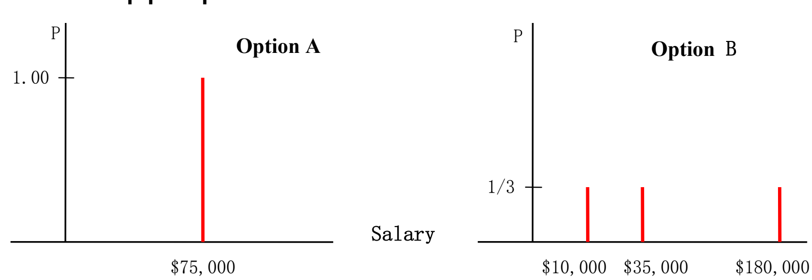
- **A risk profile is a graph that shows the chances associated with possible consequences. Each risk profile is associated with a *strategy*, a particular immediate alternative, as well as specific alternatives in future decisions.**
- **In constructing a risk profile, we collapse a decision tree by multiplying out the probabilities on sequential chance branches. At a decision node, only one branch is taken.**
- **You can think about the process as one in which nodes are gradually removed from the tree in much the same sense as we did with the folding-back procedure, expect that in this case we keep track of the possible outcomes and their probabilities.**

Risk Profiles and Specific Strategies

- **By looking at the risk profiles, the decision maker can tell a lot about the riskiness of the alternatives.**
- **In some cases a decision maker can choose among alternatives on the basis of their risk profiles.**
- **Although risk profiles can in principle be used as an alternative to EMV to check every possible strategy, for complex decisions it can be tedious to study many risk profiles.**
- **Thus, a compromise is to look at strategies only for the first one or two decisions, on the assumption that future decisions would be made using a decision rule such as maximizing expected value, which is itself a kind of strategy.**

Examples of Risk Profile

- Sometimes the concept of compressing the decision down to one number, an EMV may not be appropriate



$EMV_A = \$75,000$

$EMV_B = 1/3(\$10,000) + 1/3(\$35,000) + 1/3(\$180,000) = \$75,000$

Which one would you prefer and why?

Examples of Risk Profile (cont'd)

- What was shown was the risk profile for each option
- It really depends on your preference for risk (i.e. variance of each option)

Note:

- VarA < VarB, since

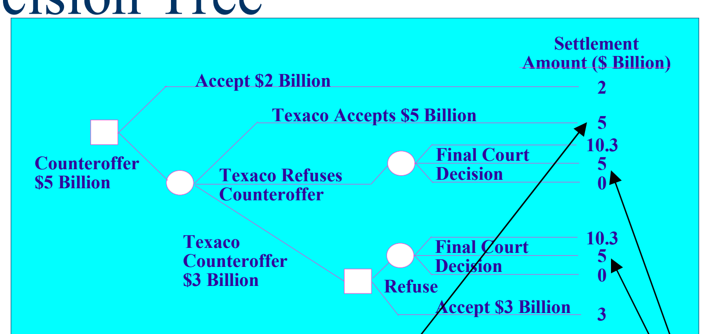
  $Var_A = 0$,

  $Var_B = \text{Sum } p(X-(E(\bar{X}))^2$

- Let's go back to the Texaco-Pennzoil problem using risk profile

Texaco-Pennzoil Problem

Decision Tree



Hugh Liedtke's decision in the Texaco-Pennzoil affair.

The Options:

Liedtke could end up with :

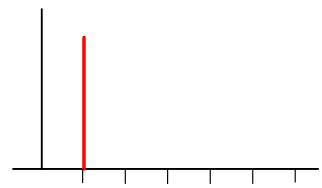
1- $10.3 Billion [court settlement]

2- $5 Billion (counter offer (upper branch) or court settlement (lower branches)]

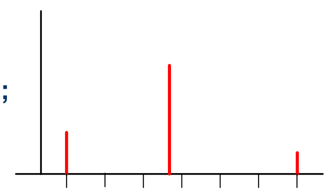
3- $0 Billion [court settlement]

Texaco-Pennzoil Problem Risk Profiles

- **Profile #1: Risk Profile for Accept $2 Billion alternative**



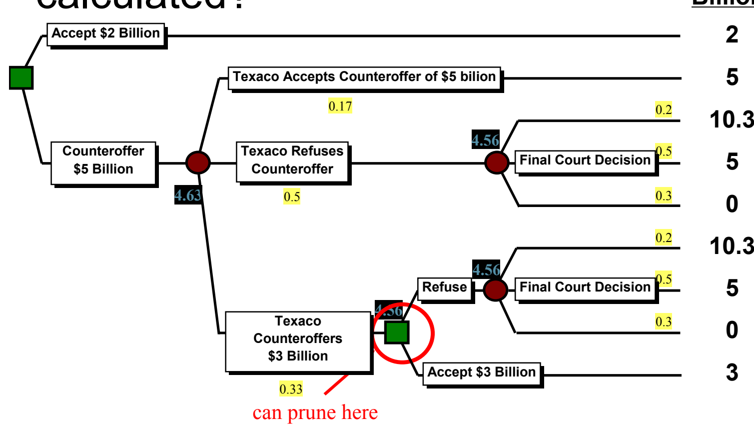
- **Profile #2: Risk Profile for the Strategy "Counteroffer $5 Billion; Refuse Texaco Counteroffer"**



Calculations Steps of Risk Profiles

- How are these values calculated?

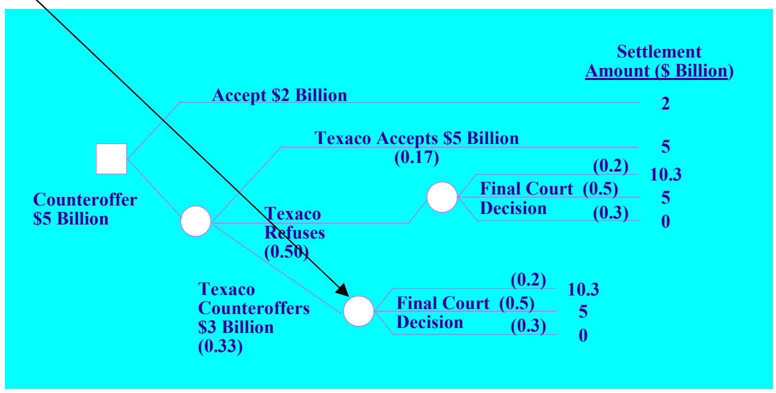
Settlement Amount ($ Billion)



Calculations Steps of Risk Profile #2

1- First step in collapsing the decision tree to make risk profile:

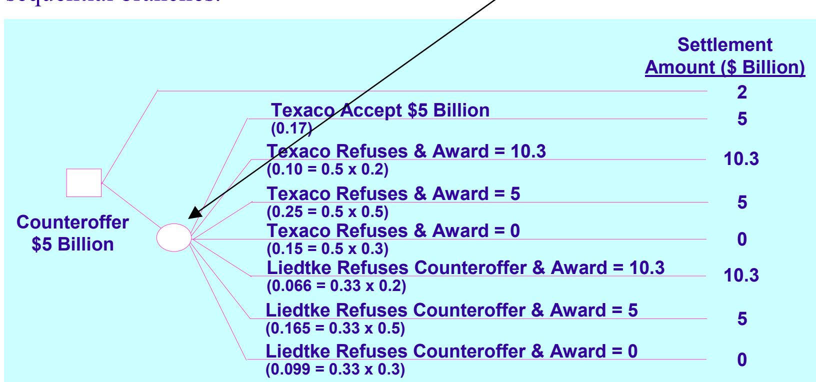
For "Counteroffer $5 Billion; Refuse Texaco Counteroffer" strategy, the decision node has been removed to leave only the outcomes associated with "Refuse" branch.



Calculations Steps of Risk Profiles #2 (cont'd)

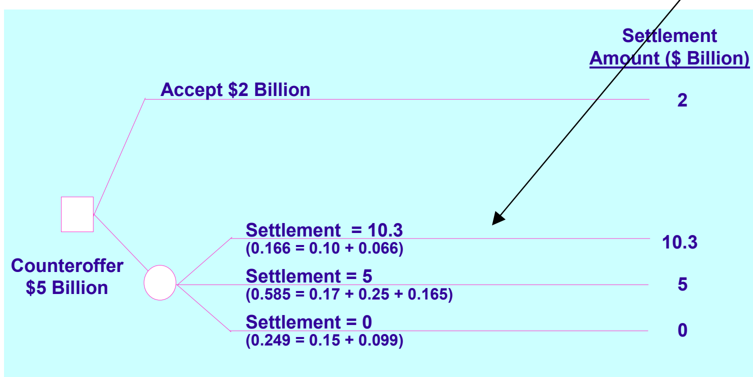
2- Second step in collapsing the decision tree to make a risk profile.

The third chance nodes have been collapsed into one chance node. The probabilities on the branches are the product of the probabilities from sequential branches.



Calculations Steps of Risk Profiles #2 (cont'd)

3- Third step in collapsing the decision tree to make a risk profile:
The seven branches from the chance node have been combined into three branches.



Interpretation of Risk Profile #2 Probability Calculations

- For example how did we get $0?
- Only 2 end nodes with $0, can add up the probabilities for each branch, why?
- Probability of a branch is the product of the pieces, why? (because the have to add to 1.0)

$0 Billion

Branch #1: 0.50 × 0.3 = 0.150
(0.50: Texaco Refuses Counteroffer; 0.3: Final Court Settlement)

Branch #2: 0.33 × 0.3 = 0.099
(0.33: Texaco Counteroffers $3 Billion; 0.3: Final Court Settlement)

0.249 (24.9%)

Interpretation of Risk Profile #2

Probability Calculations (cont'd)

- For example how did we get $5?
- Only 3 end nodes with $5, can add up the probabilities for each branch, why? (Because they have the same settlement value of $5)

$5 Billion		
	Branch #1	0.17
	Branch #2	0.50 × 0.5 = 0.25
	Branch #3	0.33 × 0.5 = 0.165
		0.585 (58.5%)

Interpretation of Risk Profile #2

Probability Calculations (cont'd)

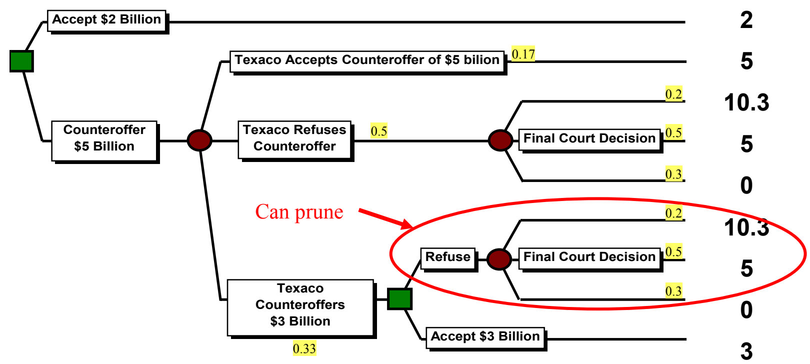
- For example how did we get $10.3?
- Only 2 end nodes with $10.3, can add up the probabilities for each branch, why? (Because they have the same settlement value of $10.3)

$10.3 Billion		
	Branch #1	0.50 × 0.2 = 0.100
	Branch #2	0.33 × 0.2 = 0.066
		0.166 (16.6%)

Risk Profile #3

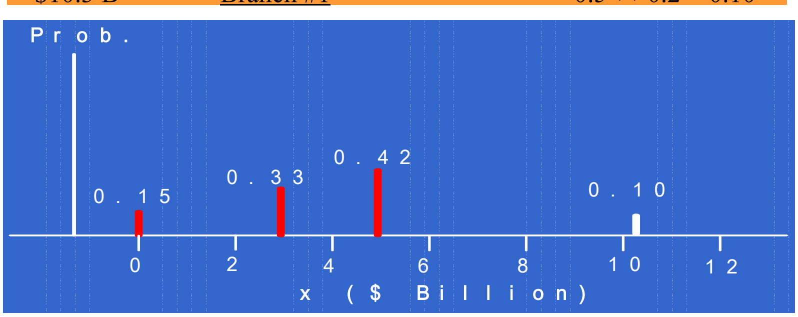
- **Profile #3**: *(did not appear as optimal in an EMV analysis)*
- **Strategy: "Counteroffer $5 Billion; Accept $3 Billion"**

Settlement Amount ($ Billion)



Calculations Steps of Risk Profile #3

$0 B	Branch #1	$0.50 \times 0.3 = 0.15$
$3 B	Branch #1	0.33
$5 B	Branch #1	0.17
	Branch #2	$0.50 \times 0.5 = 0.25$
		0.42
$10.3 B	Branch #1	$0.5 \times 0.2 = 0.10$



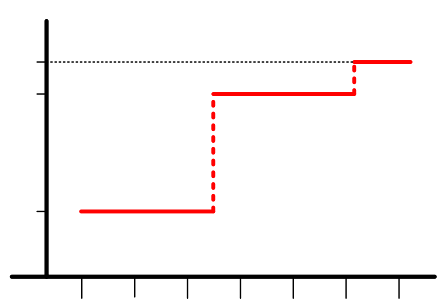
Cumulative Risk Profiles

- Between profiles #1, 2 & 3, which would you prefer and why?
- We can also consider (cdf's):
- **Creating a <u>cumulative risk profile</u> is just a matter of adding up, or accumulating, the chances of the individual payoffs.**
- **For any specific value along the horizontal axis, we can read off the chance that the payoff will be <u>less than or equal to that specific value</u>:** P ($X \leqslant x$)

Cumulative Risk Profiles (cont'd)

- <u>**For Profile #2:** ***Cumulative risk profile for "Counteroffer \$5 Billion; Refuse Texaco Counteroffer."***</u> we'd have:

	<u>Prob.</u>	<u>Cum.</u>
\$0 B	0.249	0.249
\$5 B	0.585	0.834
\$10.3 B	0.166	1.00



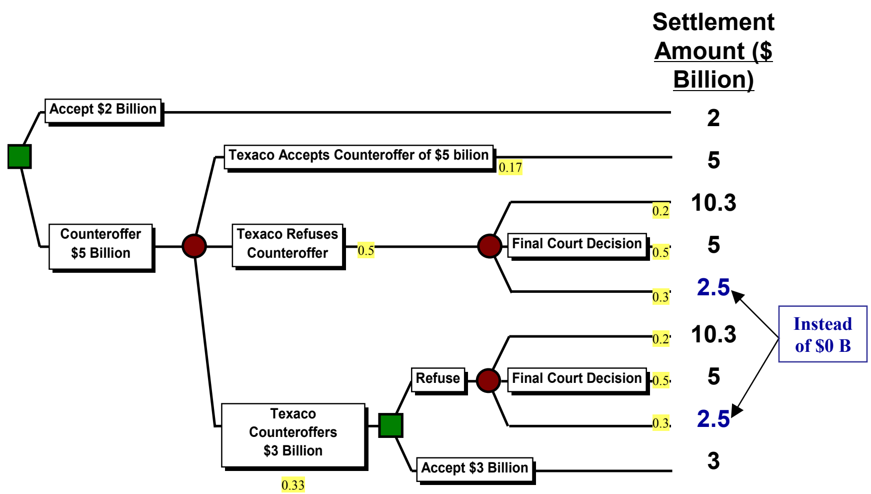
Dominance: An Alternative to EMV

- In many cases EMV inadequately captures the nature of the risks that must be compared.
- With risk profiles, however, we can make a more comprehensive comparison of the risks.
- Question:
- How can we choose one risk profile over another?
- Answer:
- By using the idea of ***dominance***, we can identify those profiles (and their associated strategies) that can be ignored.
- Such strategies are said to be ***dominated***, because we can show logically, according to some rules relating to cumulative risk profiles, that there are better risks (strategies) available.
- **Dominance: another concept in comparing strategies can be either:**
  - **Deterministic;**
  - **Stochastic (probabilistic): uses cumulative risk profiles**

Deterministic Dominance in Texaco-Pennzoil Problem

- Suppose Liedtke's decision model changed to have $2.5 not $0B as the min count settlement amount
- Why would this be plausible?
- What should he do in this case?
- Refuse $2 Billion offer since the $2.5 billion court settlement beats it
  - That is, deterministic dominance of refusing counteroffer of $2 Billion to accepting it
- We can use the cumulative risk profiles (CRPs) to show deterministic dominance as well

Deterministic Dominance



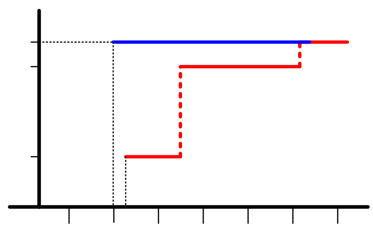
Deterministic Dominance

- **Deterministic dominance, signifying that the dominating alternative pays off at least as much as the one that is dominated**

- **We can show deterministic dominance in terms of cumulative risk profiles.**

- **Deterministic dominance can be detected in the risk profiles by comparing the value where one cumulative risk profile [CRFs] reaches 100% with the value where another risk profile begins.**

Cumulative Risk Profiles (CRPs)



Strategy 1: Accept $2B

Strategy 2: Counteroffer $5B, refuse Texaco counteroffer (min. court decision=$2.5 billion)

- Strategy 2 deterministically dominates strategy 1 since:
- Strategy one has 100% chance that value ⩽ 2B but
- Strategy 2 has some probability of being ⩾ 2.5B
- Basically strategy 2's CRP "starts" at a value ($2.5B) higher than where strategy 1's CRP "ends" ($2B)
- This is not always the case.

Stochastic Dominance (Probabilistic Dominance)

- **In some situations the two alternatives that offer the same possible consequences, but the dominating alternative is more likely to bring a better consequence.**
- **Deterministic dominance is a special case of stochastic dominance.**
- **Sometimes stochastic dominance may emerge as a mixture of the two; both <u>slightly better payoffs and slightly better probabilities</u> may lead to one alternative dominating another.**
- **Stochastic dominance is represented in the cumulative risk profiles by the fact that the two profiles do not cross and that there is some space between them.**
- **That is, if two cumulative risk profiles are such that no part of Profile A <u>lies to the left</u> of B, and at least some part of it <u>lies to the right</u> of B, then the strategy corresponding to Profile A stochastically dominates the strategy for Profile B.**

Pr

1.00
0.834

0.249

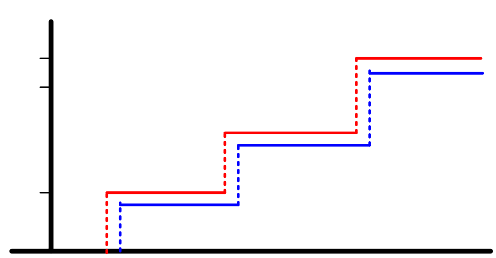
Stochastic Dominance (Probabilistic Dominance)

- **<u>If one alternative dominates another, then the dominating alternative must have the higher expected value</u>. This a <u>property of dominant alternatives</u> that can be proven mathematically. To get a feeling for why it is true, think about the cumulative risk profiles, and imagine the EMV for a dominated Alternative B. Alternative A dominates B, then its cumulative risk profile must lie at least partly <u>to the right</u> of the profile for B. Because of this, the EMV for A must also lie to the right of, and hence be greater than, the EMV for B.**

- **Relationship between Stochastic Dominance and EMV**
  - **If Option A stochastically dominates Option B**
    - **⇒ EMVA > EMVB**

- **Screening alternatives formally on the basis of dominance is an important decision-analysis tool. If an alternative can be eliminated early in the selection process on that basis, considerable cost can be saved in large-scale problems.**

Stochastic Dominance (Probabilistic Dominance)

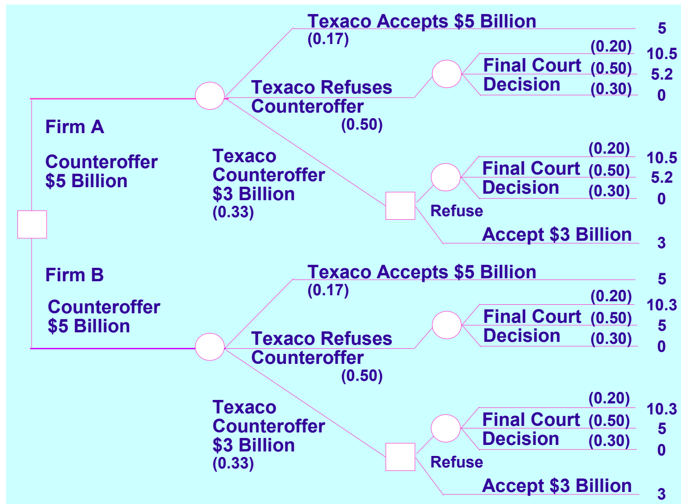


Option A stochastically dominates Option B since A's CRP is to the right Of B's for some part and it never crosses B's CRP.

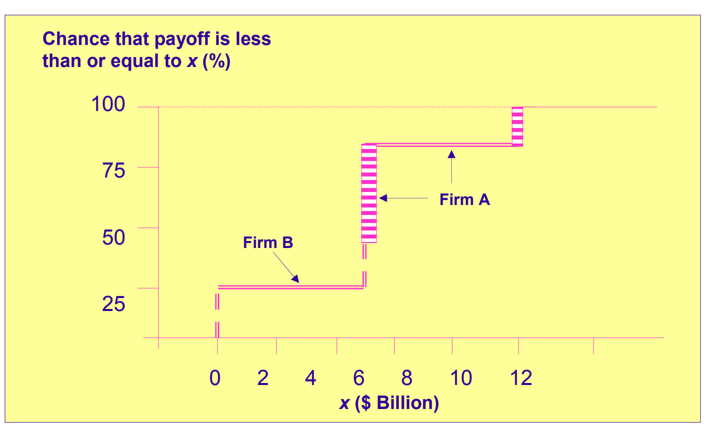
Stochastic Dominance in Texaco-Pennzoil Problem

A decision tree comparing two law firms.

Firm A charges less than Firm B if Pennzoil is awarded damages in court.



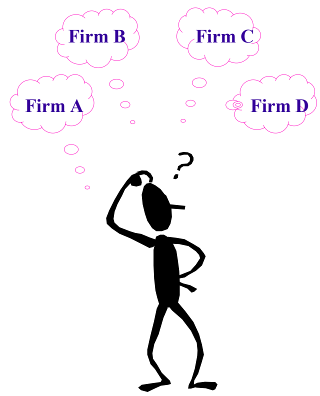
Cumulative Risk Profiles (CRPs)



Cumulative risk profiles for two law firms. Firm A stochastically dominates Firm B.

Making Decisions with Multiple Objectives

- **Extending <u>concepts of expected value and risk profiles</u> to multiple-objective situations.**



Summer-Job Example of Multiple Objectives

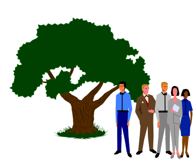
- **<u>Example: Summer Job Multiple Objective Decision</u>**
- **Sam Chu was in a quandary. With two job offers in hand, the choice he should make was far from obvious.**
- ***<u>The first alternative</u>* was a job as an assistant at a local small business; the job would pay minimum wage ($5.25 per hour), require 25 to 35 hours per week, and the hours would be primarily during the week, leaving the weekends free. The job would last for three months, but the exact amount of work, and hence the amount Sam could earn, was uncertain. On the other hand, the free weekends could be spent with friends.**

Summer-Job Example of Multiple Objectives (cont'd)

- *The second alternative* was to work as a member of a trail-maintenance crew for a conservative organization. This job would require 10 weeks of hard work, 40 hours per week at $6.50 per hour, in a national forest in neighboring state. The job would involve extensive camping and backpacking. Members of the maintenance crew would come from a large geographic area and spend the entire 10 weeks together, including weekends. Although Sam had no doubt about the earnings this job would provide, the real uncertainty was what the staff and other members of crew would be like. Would new friendships develops? The nature of the crew and the leaders could make for 10 weeks of a wonderful time, 10 weeks of misery, or anything in between.

Summer-Job Example of Multiple Objectives (cont'd)

- **Sam has two objectives in this context:**
- **1. Earning money**



- **2. Having fun this summer.**

Both are reasonable, and the two jobs clearly differ in these two dimensions; they offer different possibilities for the amount of money earned and the quality of summer fun.

- **Note:**

1. **The amount of money to be earned has a natural scale (dollars), and like most of us Sam prefers more money to less.**
2. **The objective of having fun has no natural scale, though.**

Attribute Fun Scale

5 (Best) - A large, congenial group. Many new friendships made. Work is enjoyable, and time passes quickly.

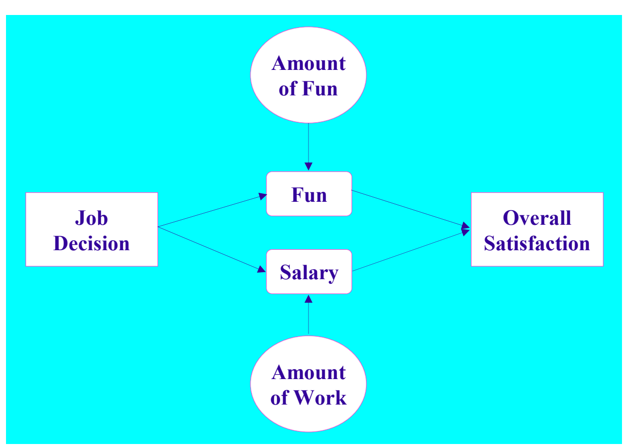
4 A small but congenial group of friends. The work is interesting, and time off work is spent with a few friends in enjoyable pursuits.

3 No new friends are made. Leisure hours are spent with a few friends doing typical activities. Pay is viewed as fair for the work done.

2 Work is difficult. Coworkers complain about the low pay and poor conditions. On some weekends it is possible to spend time with a few friends, but other weekends are boring.

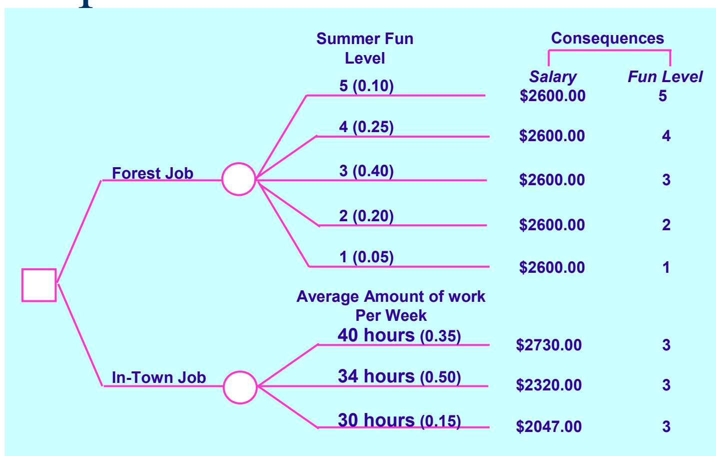
1 (Worst) - Work is extremely difficult, and working conditions are poor. Time off work is generally boring because outside activities are limited or no friends are available.

Influence diagram for Summer-Job Example



Influence diagram for summer-job example.

Decision Tree for Summer-Job Example

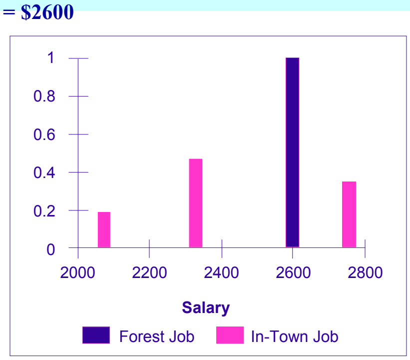


Decision tree for summer- job example

Analysis: One Objective at a Time

- **One way to approach the analysis of a multiple-objective decision is to <u>calculate the expected value or create the risk profile for each individual objective</u>.**

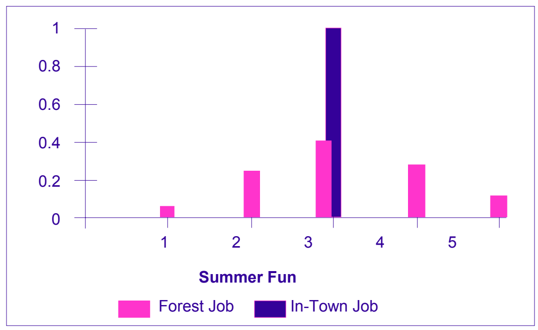
E(Salary-in Town Job) = 0.35($2730.00) + 0.50($2320.50) + 0.15($2047.50)
= $2422.88
E(Salary-Forest Job) = $2600



Risk profiles for salary in the summer-job

Subjective Ratings for Constructed Attribute Scales

- **The summer-fun objective constructed attribute scale, risk profiles can be created and compared, but expected-value calculations are not meaningful because no meaningful numerical measurements are attached to the specific levels in the scale.**



Risk profiles for summer fun in the summer-job example.

Subjective Ratings for Constructed Attribute Scales

- **Before we can do any meaningful analysis, Sam must rate the different levels in the scale, indicating how much each fun level is worth (to Sam) relative to the other levels.**

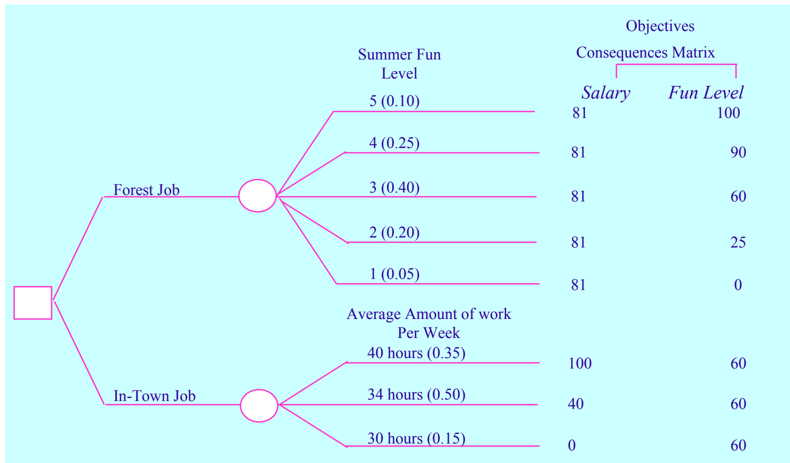
To make the necessary ratings:

1. We begin by setting the endpoints of the scale. Let the best possible level (Level 5 in the summer-job example) have a value of 100 and the worst possible level (Level 1) a value of 0.
2. Indicate how the intermediate levels rate on this scale from 0 to 100 points.

Note:

1. There is no inherent reason for the values of the levels to be evenly spaced; in fact, it might be surprising to find perfectly even spacing.
2. This same procedure can be used to create meaningful measurements for any constructed scale. The best level is assigned 100 points, the worst 0 points, and the decision maker must then assign rating points between 0 and 100 to the intermediate levels.
3. A scale like this assigns more points to the preferred consequences, and the rating points for intermediate levels should reflect the decision maker's relative preferences for those levels.

Decision Tree With Ratings for Consequences



Decision tree with ratings for consequences

Calculations of Expected Rated Value

- **Now we can calculate and compare expected values for the amount of fun in the two jobs:**

***E*(Fun Points – Forest Job) = 0.10(100) + 0.25(90) + 0.40(60) + 0.20(25) + 0.50(0) = 61.5**

***E*(Fun Points- in Town Job) = 0.35(60) + 0.50(60) + 0.15(60) = 60**

- **With individual expected values and risk profiles, alternatives can be compared.**

- **In doing so, we can hope for a clear winner, an alternative that dominates all other alternatives on all attributes.**

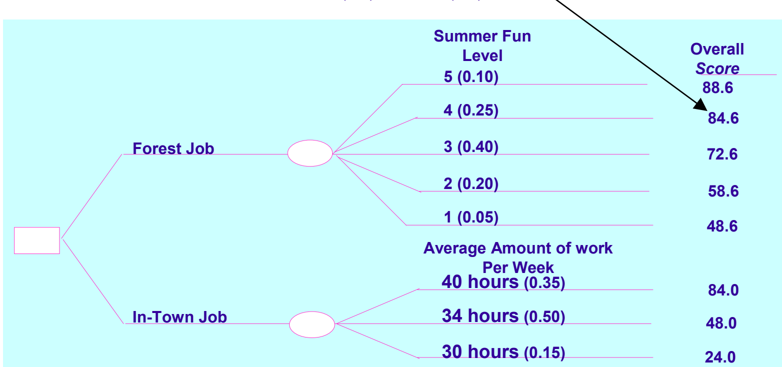
Assessing Trade-Off Weights

- In order to make the comparison between salary and fun, it is helpful to measure these two on similar scales, and the most convenient arrangement is to put salary on the same 0 to 100 scale that we used for summer fun.
- Use the weights to calculate a weighted average of the two ratings for any given consequence in order to get an overall score.
- Sam *must* take into consideration the ranges of the two attributes. The two weights should reflect the relative value of going from best to worst on each scale. Paying attention to the ranges of the attributes in assigning weights is crucial.
- Example:
- Suppose that *ks* = 0.70 and *kf* = 0.30, reflecting a judgment that salary is a little more than twice a important as fun.
- For example, the overall score (U) for the forest job with fun at <u>Level 3</u> would be:

U(Salary)= 81, U(Fun)= 60; Therefore: U (overall score) = 0.70(81) + 0.30(60) = 74.7

Decision Tree With Overall Scores for Summer-Job Example

- Weights used are k_s= 0.60 and k_f = 0.40.
- For example, consider the forest job that has an outcome of Level 4 on the fun scale. The rating for salary is 81, and the rating for fun is 90.
- Thus, the overall score is: 0.60(81) + 0.40(90) = 84.6



Analysis: Expected Values and Risk Profiles for Two Objectives

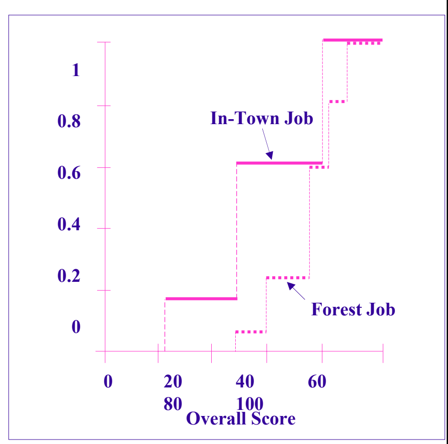
- Fold back the tree to calculate expected values. Using the overall scores the expected values are:

E(Score for Forest Job) = 0.10(88.6) + 0.25(84.6)
+ 0.40(72.6) + 0.20(58.6) + 0.05(48.6) = 73.2
E(Score for In-Town Job) = 0.35(84) + 0.50(48) + 0.15(24) = 57

- **The risk profiles would represent the uncertainty associated with the overall weighted score Sam will get from either job.**
- **To the extent that this weighted score is meaningful to Sam as a measure of overall satisfaction, the risk profiles will represent the uncertainty associated with Sam's overall satisfaction.**

Risk Profiles For the Two Alternatives

Frequency risk profile for summer jobs.



Cumulative risk profiles for summer jobs.

The Forest Job Stochastically Dominates the In-Town Job

Decision Analysis Using Precision Tree

Class Exercise to Show Analysis of a Decision Model

Run-Demo

Analyzing Decision Tree for Oil Drilling Problem