CHAPTER

4a

Duxbury Thomson Learning

Making Hard Decision Third Edition

Making Choices

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ENCE 627 – Decision Analysis for Engineering

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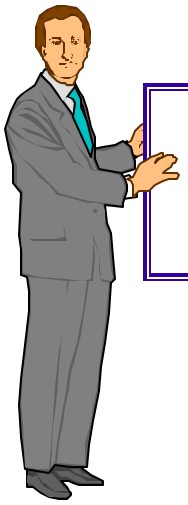


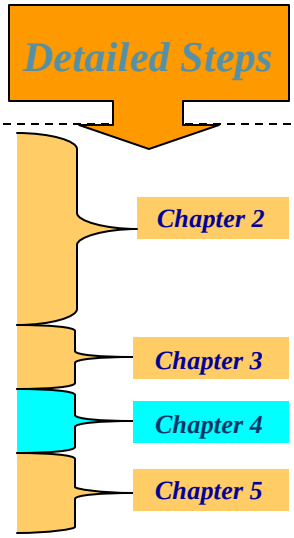
Making Choices

Methodology of Modeling Decisions

The Methodology of Modeling Decisions is to:

- Understand the problem under study
- Introduce quantitative modeling

- Discuss the elements of a decision.
  - Values and Objectives
  - Decisions to be made
  - Upcoming uncertain events, and
  - Consequences
- Build the decision Model and identify a set of feasible alternatives.
- **Evaluate the alternatives and make a choice of a feasible alternative.**
- Re-evaluate the alternatives using sensitivity analysis to refine the solution.



Contents

Contents

- Analysis: One Objective at a Time
- Subjective Ratings for Constructed Attribute Scales
- Assessing Trade-Off Weights
- Analysis: Expected Values and Risk Profiles for Two Objectives
- Decision Analysis Using Precision Tree

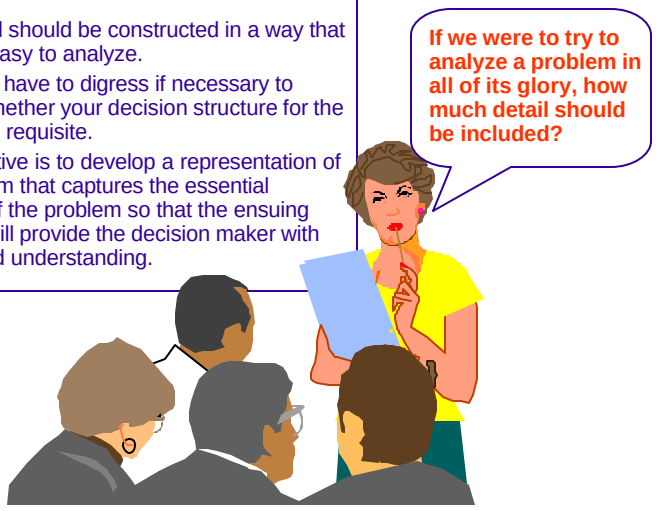
Learning About Making Choices

- Use the details in a structured problem to find a preferred alternative in uncertain and risky environments.
- Introduce risk profiles and dominance considerations, ways to make decisions without doing many calculations.
- Show the analysis of decision models that involve only one objective or attribute.

Learning About Making Choices

- Discuss calculation of expected values and the use of risk profiles for single-attribute decisions.
- Introduce Decisions with multiple attributes and present some simply analytical approaches.
- Discuss Application of software for doing decision-analysis calculations on personal computers using the Precision Tree package.

1. All of the relevant information should be included.
2. The model should be constructed in a way that makes it easy to analyze.
3. You might have to digress if necessary to explore whether your decision structure for the problem is requisite.
4. The objective is to develop a representation of the problem that captures the essential features of the problem so that the ensuing analysis will provide the decision maker with insight and understanding.



Decision Making Under Uncertainty

- There are two type of Decision-making procedures:
  1. Decision-making without Probabilities
  2. Decision-making with Probabilities

Decision Making Under Uncertainty

- Decision-making without Probabilities
  - This includes two types:
    - A. Matrix Decision Trees or Pay-off tables
      1. Maximax criterion: optimistic approach
      2. Maximin criterion: pessimistic approach
      3. Minimax criterion: minimum regret method
    - B. Decision Trees without probabilities

Decision Making Under Uncertainty

- Decision-making with Probabilities
  - This includes:
    - A. Expected value approach
    - B. Decision Trees with probabilities

Decision-Making without Probabilities

Pay-off Table

In decision-theory, we refer to the outcome that results from a specific decision alternative and the occurrence of a particular state of nature as a payoff.

A table showing payoffs for all combination of decision alternatives and states of nature is a payoff table as follows:

	State of Nature	
Decision alternative	High Market Acceptance	Low Market Acceptance
	S1	S2
d1 = Small Complex	8	7
d2 = Medium Complex	14	5
d3 = Large Complex	20	-9

Maximax Criterion: Optimistic Approach

[*A*] Optimistic: Maximax Criterion: i.e. Max of Max

Decision alternative	High Market S1	Low Market S2	Maximum	Decision
d1 = Small Complex	8	7	8	max
d2 = Medium Complex	14	5	14	of max =
d3 = Large Complex	20	-9	20	d3

Maximin Criterion: Pessimistic Approach

[B] Pessimistic: Maximin Criterion: i.e Max of Min

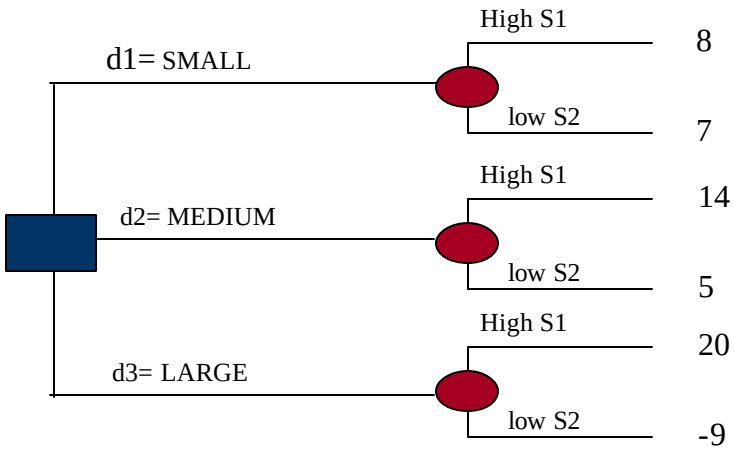
Decision alternative	High Market S1	Low Market S2	Minimum	Decision
d1 = Small Complex	8	7	7	max
d2 = Medium Complex	14	5	5	= of min
d3 = Large Complex	20	-9	-9	d1

Minimax Criterion: Regret Approach

[C] Minimax Regret Approach: Minimax Criterion i.e Min of Max

Decision alternative	High Market S1	Low Market S2	Maximum Regret	Decision
d1 = Small Complex	20 - 8 = 12	7 -7 = 0	12	min
d2 = Medium Complex	20 - 14 = 6	7 - 5 = 2	6	of max=
d3 = Large Complex	20 - 20 = 0	7 - (-9) = 16	16	d2

Decision Tree without Probabilities



Criteria for Comparing Results

- Civil Lawsuit

	Trial Outcome	Trial Outcome
Claimant's Decision	Claimant Wins	Claimant Loses
D1 = Settlement	**$450,000**	$450,000
D2 = Jury	**$4,900,000**	-$50,000

- **Maximax Criterion >> Select the alternative that gives the largest of the best outcomes for the alternatives.**
- **For example,**
  D1 = Settle >> Best outcome = $450,000
  D2 = Jury >> Best outcome = $4,950,000
  Decision = D2 = Jury (optimistic, gambler) = $4,900,000

Criteria for Comparing Results

- Civil Lawsuit (cont'd)

	Trial Outcome	Trial Outcome
Claimant's Decision	Claimant Wins	Claimant Loses
D1= Settlement	**$450,000**	**$450,000**
D2 = Jury	$4,950,000	**-$50,000**

- **Maximin Criterion >> Select the alternative that gives the best of the worst outcomes for the alternatives.**
- **For example,**
  D1= Settle >> Worst outcome = $450,000
  D2= Jury >> Worst outcome = -$50,000
  Decision = D1= Settle = $450,000

Criteria for Comparing Results

- Civil Lawsuit (cont'd)

	Trial Outcome	Trial Outcome
Claimant's Decision	Claimant Wins	Claimant Loses
D1 = Settlement	$4,950,000 - $450,000 =$4,500,000	$4,50,000 – $450,000 = 0
D2 = Jury	$4,950,000 - $4,950,000 = 0	450,000 - (-$50,000) = **500,000**

- **Minimax Regret Criterion >> Select the alternative that gives the minimum of the maximum regret outcomes for the alternatives.**
- **For example,**
  D1 = Settle >> Maximum regret outcome = $4,500,000
  D2 = Jury >> Maximum regret outcome = $500,000
  Decision = D2= Jury = $500,000 = minimum of maximum

Decision Making Under Risk

- The risk of an event can be defined as a combination of both its occurrence probability and its occurrence consequence.
- The combination can be in the form of their product

Decision Making Under Risk

- Alternatively, it can be considered to be an ordered pair of occurrence probability and its occurrence consequence, i.e., (probability, consequences).
- For several events of interest, risk plots can be produced using these ordered pairs for the events and a coordinate system of occurrence probability and consequence.

Elements of the Risk-Decision Model

- A decision model is a systematic framework for decision making in risk analysis.
- In order to construct a decision model, the following elements of the decision model need to be defined:
  - objectives of decision analysis,
  - decision variables,
  - decision outcomes, and
  - associated probabilities and consequences

Objectives of Risk-Decision Analysis

- In order to construct a decision model, the following elements of the decision model need to be defined:
  - Engineering decision problems can be classified into single- and multiple-objective problems.
  - Example objectives are minimizing the total expected cost, maximizing safety, maximizing the total expected utility value, and maximizing the total expected profit.
  - Decision analysis requires the definition of these objectives.

Decision Variables

- The decision variables are the feasible options or alternatives available to the decision maker at any stage of the decision-making process.
- Ranges of values that can be taken by the decision variables should be defined.

Decision Variables

- Example decision variables can include, for example, what and when to inspect components or equipment, which inspection methods to use, assessing the significance of detected damage, and repair/replace decisions.

Decision Variables

- Assigning a value to a decision variable means making a decision at that point of a decision-making process. These points within the decision-making process are called decision nodes( ).

Decision Outcomes

- The decision outcomes are the events that can happen as a result of a decision.
- They are random in nature, and their occurrence cannot be fully controlled by the decision maker.

Decision Outcomes

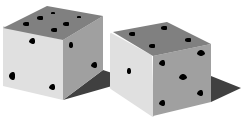
- Example decision outcomes can include, for example, the outcomes of an inspection (detection or non-detection of a damage), and the outcomes of a repair (satisfactory or non-satisfactory repair).

Decision Outcomes

- Therefore, the decision outcomes with the associated occurrence probabilities need to be defined. The decision outcomes can occur after making a decision at points within the decision-making process called chance nodes (●).

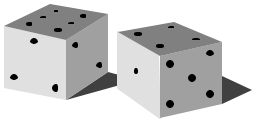
Associated Probabilities and Consequences

- The decision variables take values that can have associated costs. These costs can be considered as the direct consequences of making these decisions.



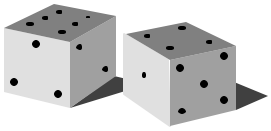
Associated Probabilities and Consequences

- The decision outcomes have both consequences and occurrence probabilities. The probabilities are needed due to the random (chance) nature of these outcomes.

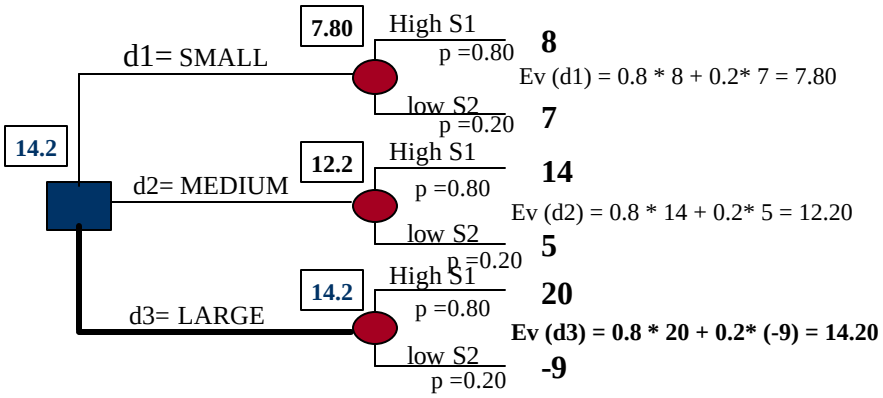


Associated Probabilities and Consequences

- The consequences can include, for example, the cost of failure due to damage that was not detected by an inspection method.

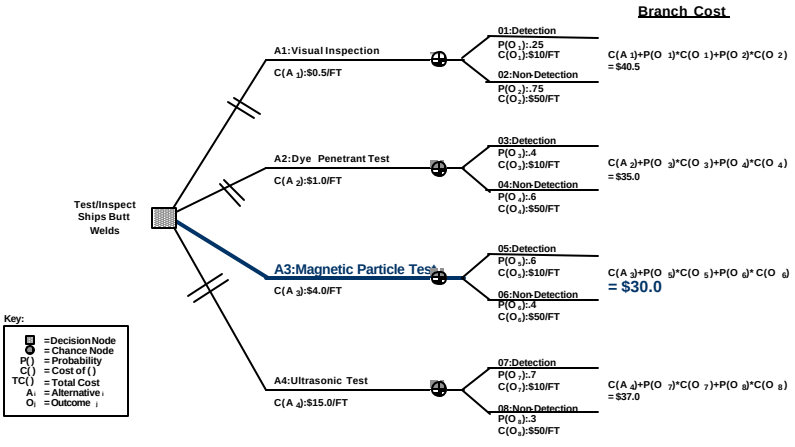


Decision Tree With Probabilities



Example of Risky Decision Analysis

Decision Analysis in Inspection Planning



Decision Trees and Expected Monetary Value (EMV)

- **One way to choose among risky alternatives is to pick the alternative with the highest *expected value* (EV).**
- **When the decision's consequences involve only money, we can calculate the *expected monetary value*** (EMV). **Finding EMVs when using decision trees is called *"folding back the tree"* or *"rolling back"*.**
- **We start at the endpoints of the branches on the far right-hand side and move to the left:**
  - (1) Calculating expected values when we encounter a ***chance node***, or
  - (2) Choosing the branch with the highest value or expected value when we encounter a ***decision node***.

Expected Monetary Value (EMV) The Double-Risk Dilemma

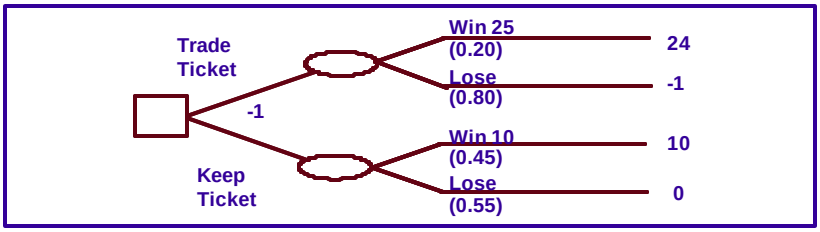
Example: The Double-Risk Dilemma

- **A double-risk dilemma is a matter of choosing between two risky alternatives.**
- **You have a ticket that will let you participate in a game of chance (a lottery) that will pay off $10 with a 45% chance, and nothing with a 55% chance. Your friend has a ticket to a different lottery that has 20% chance of paying $25 and an 80% chance of paying nothing. Your friend has offered to let you have his ticket if you will give him your ticket plus one dollar. Should you agree to the trade and play to win $25, or should you keep your ticket and have a better chance of winning $10?**

Expected Monetary Value (EMV)

The Double-Risk Dilemma

Example: The Double-Risk Dilemma (cont'd)



A double-risk dilemma

Expected Monetary Value (EMV)

The Double-Risk Dilemma

Solve the decision tree using EMV:

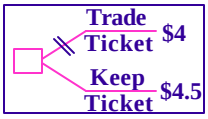
1. **Calculate the expected value of keeping the ticket and playing for $10. This expected value is simply the weighted average of the possible outcomes of the lottery, the weights being the chances with which the outcomes occur.**

EMV (Keep Ticket) = 0.45(10)+0.55(0) = $4.5

- **One interpretation of this EMV is that playing this lottery many times would yield an average of approximately $4.50 per game. Calculating EMV for trading tickets gives**

EMV (Trade Ticket) = 0.20(24)+0.80(-1) = $4

2. **Replace the chance nodes in the decision tree with their expected values.**

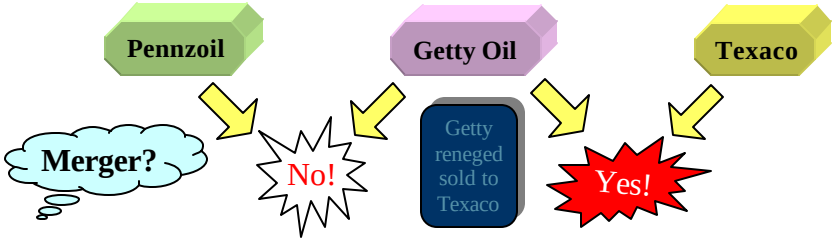


Replacing chance nodes with EMVs.

3. **Choosing between trading and keeping the ticket amounts to choosing the branch with the highest expected value. The double slash through the "Trade Ticket" branch indicates that this branch would not be chosen.**

Example: Texaco Versus Pennzoil Decision Problem

- **The Problem:**
  - **In Early 1984…**



  - Pennzoil sued Texaco.
  - In late 1985, Pennzoil won, it was awarded $11.1 billion.
  - Texas appealed to court and the following scenario prevailed:

Example: Texaco Versus Pennzoil Decision Problem

- Texaco court dropped settlement amount by $2 billion (to **$10.3 billion** after interest and penalties).
- Texaco said it would go for bankruptcy if it had to pay up if Pennzoil filed lien against Texaco's assets.
- April 1987, Texaco **offered to pay Pennzoil $2 billion** to settle the entire case.
- Head of Pennzoil, Hugh Liedtke, (pronounced "Lid-key") was told should settle for **$3-$5 billion**

Example: Texaco Versus Pennzoil

Decision Alternatives

- What are the decision alternatives of this problem from the perspective of the Pennzoil Chairman, Hugh Liedtke?
- Risky Decision Alternatives based on the assumption that there is one fundamental objective– ***make the most dollars possible :***

Example: Texaco Versus Pennzoil

Decision Alternatives

- Alternatives Scenarios:
  - 1- Pennzoil Accepts a sure $2 billion or
  - 2- Refuse and make a counteroffer to settle
  - – If Liedtke counteroffer with $5 billion
  - – Accordingly:
    - Texaco might agree to pay $5 Billion, or
    - Texaco refuses the counteroffer wait for final court decision with different outcomes, or

Example: Texaco Versus Pennzoil

Decision Alternatives

- Texaco counteroffers with $3 billion only
  - Accordingly Lid Key of Pennzoil would accept the $3 billion offer, or
  - Refuses and wait for final court decision with different outcomes

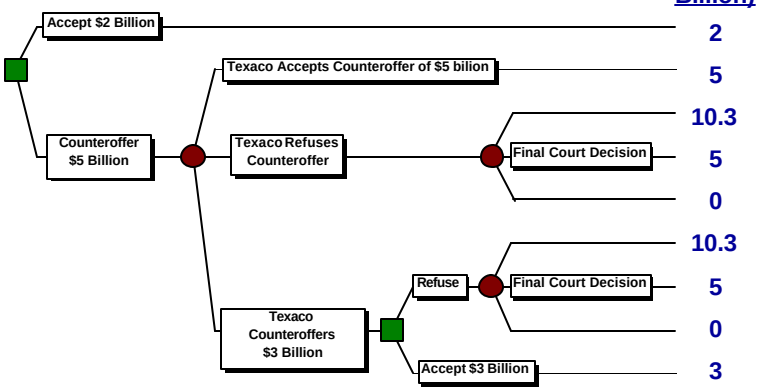
■ What should you advise the Chairman to do in this case and why and How can you construct a decision tree to help him with this problem??

Corresponding Simplified Decision

Tree without Probabilities

In a decision tree for the problem, what are the:

- **Decision nodes (shown in green square)**
  - **Chance nodes (shown in red circles)**
  - **Other data (payoff values shown in blue)**

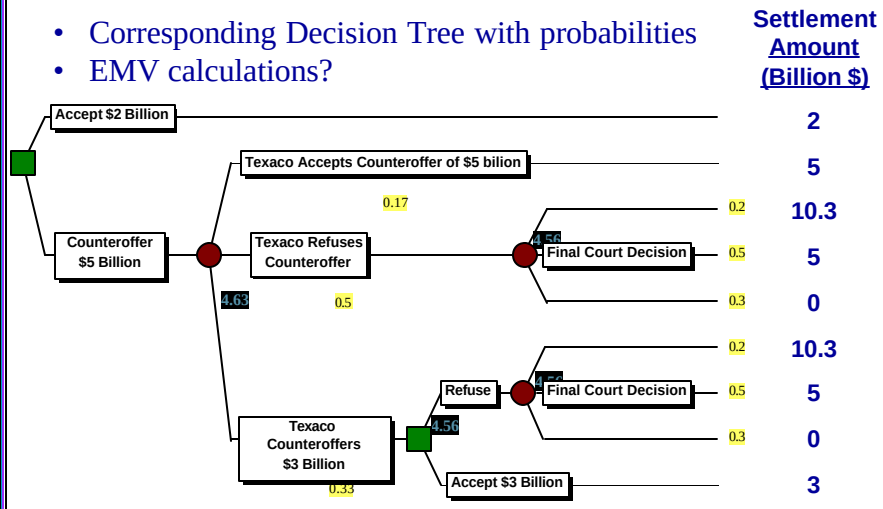


Comments on The Decision Tree

- This tree is adequate for a first cut.
- However, needs to add probabilities:
  - Not something that has been observed (like crop yields)
  - More of a subjective rating
  - Could do a sensitivity analysis on them
- What should Pennzoil's Chairman Liedtke do?
  - Really need to know the probabilities for the chance nodes to make an informed decision
- Read analysis in page 114 and how probabilities can be derived.

Corresponding Simplified Decision Tree with Probabilities

- Corresponding Decision Tree with probabilities
- EMV calculations?



Solving The Decision Tree Using EMV "Folding Back the Tree"

- Use the concept of Expected Monetary Value (EMV) to resolve this risky decision.
- **"Folding Back the Tree"**
  - We will "fold back the tree" work from the right-hand side of the tree first, moving to the left.

Solving The Decision Tree Using EMV "Folding Back the Tree"

- Calculate the expected value of the final court decision. The expected value of the court decision is the weighted average of the possible outcomes:

Solving The Decision Tree Using EMV "Folding Back the Tree"

- The solution of The problem is as follows:

EMV (Court Decision) = [P(Award = 10.3) x 10.3] + [P(Award = 5) x 5] +[P(Award = 0) x 0]
= [0.2 x 10.3] + [0.5 x 5] + [0.3 x 0] = 4.56

1. Replace both uncertainty nodes representing the court decision with this expected value of $4.56.

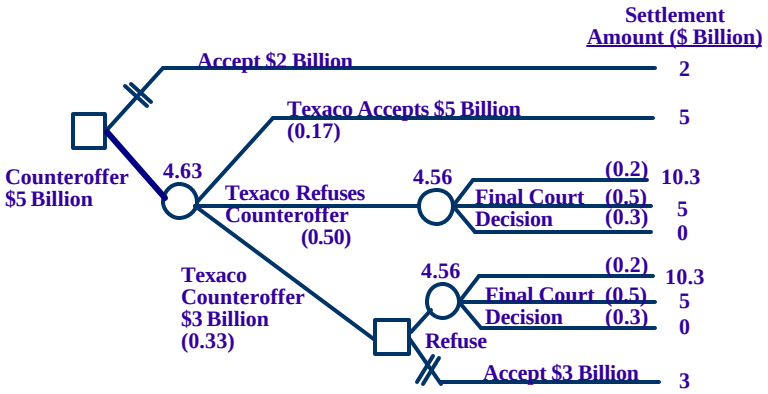
   The expected value of $4.56 is greater than the certain value of $3 billion, and hence the slash through the "Accept $3 Billion" branch.

   Folding back the decision tree, replaced is the decision node with the preferred alternative.
2. Next calculate the expected value of the alternative "Counteroffer $5 Billion)," This expected value is:

EMV (Counteroffer $5 Billion)= [P(Texaco Accepts) x 5] +[P(Texaco Refuses) x 4.56]
= [P(Texaco Counteroffers) x 4.56]
= [0.17 x 5] + [0.50 x 4.56] + [0.33 x 4.56]= 4.63

Solving The Decision Tree Using EMV "Folding Back the Tree"

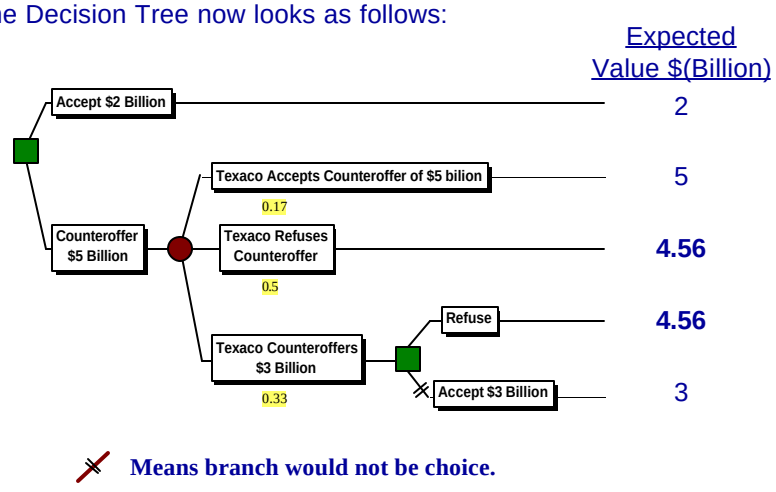
- The Decision Tree now looks as follows:



Hugh Liedtke's solved decision tree.

Further Reduction of the Decision Tree

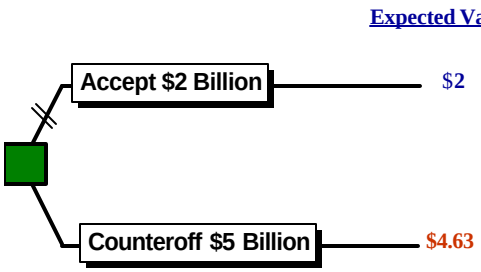
- The Decision Tree now looks as follows:



Means branch would not be choice.

Final Reduction of the Decision Tree

- **Now replace the next node with the preferred alternative**
- **What should Liedtke do based on an expected monetary value concept?**
- **What does this concept ignore?**



Interpretation of Decision Results

- Liedtke should go for the counteroffer of $5 billion because on the average, Pennzoil will have better chance of having $4.63 billions (better than having $2 billions only).
- Also, we learned:
  - If Texaco turns down the $5 billion offer and makes a $3 billion of counteroffer, Pennzoil should refuse since it would gain $4.56 billion on the average.

Solving Influence Diagrams: Overview

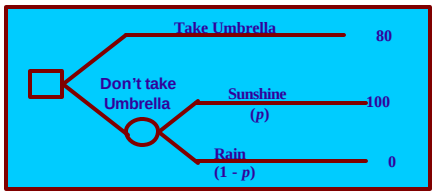
- Solving Influence Diagrams: Overview
- More complicated than solving decision trees.
- First, a comparison with decision trees on a small problem.

Solving Influence Diagrams: Overview

An influence diagram "thinks" about a decision in terms of a symmetric expansion of the decision tree from one node to the next.

Example: The Umbrella Problem

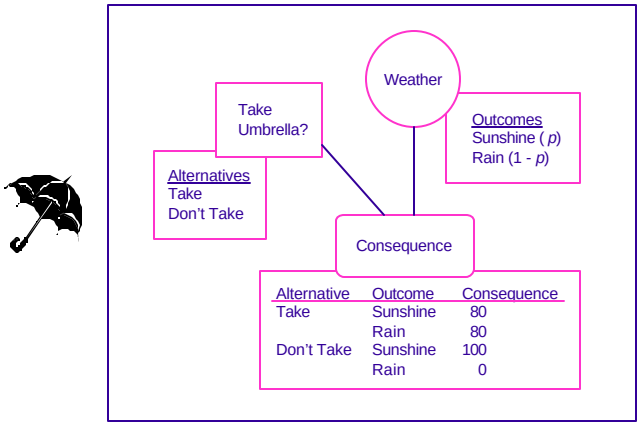
Suppose we have the basic decision tree which represents the "umbrella problem".



Umbrella problem.

■**The issue is whether or not to take your umbrella. If you do not take the umbrella, and it rains, your good clothes (and probably your day) are ruined, and the consequence is zero (units of satisfaction). However, if you do not take the umbrella and the sun shines, this is the best of all possible consequences with a value of 100. If you decide to take your umbrella, your clothes will not get spoiled. However, it is a bit of a nuisance to carry the umbrella around all day. Your consequence is 80, between the other two values.**

How the Influence Diagram "Thinks" About the Umbrella Problem

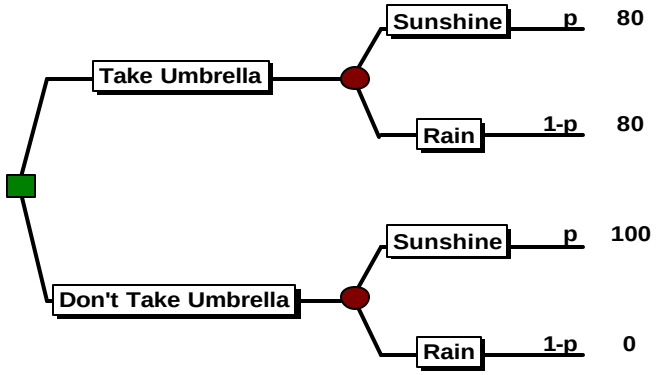


Influence diagram of the umbrella problem.

Note:
It does not matter whether the sun shines or not if you take the umbrella.

How the Influence Diagram "Thinks" About the Umbrella Problem

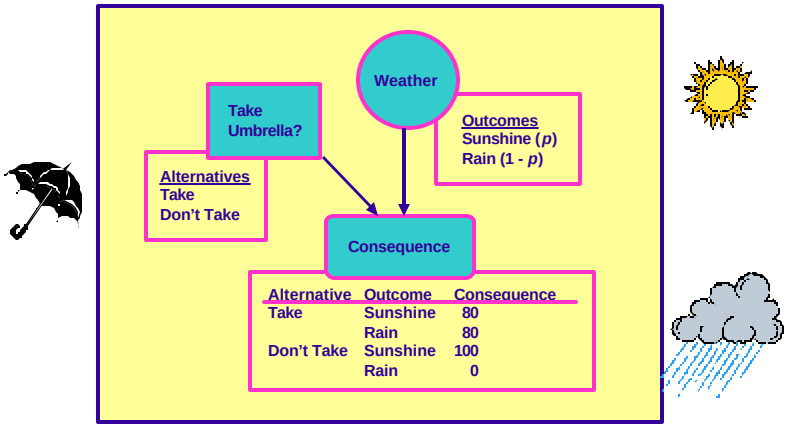
■ If we were to reconstruct exactly how the influence diagram "thinks" about the umbrella problem in terms of a decision tree, the representation would be that shown below:



Decision Trees Vs. Influence Diagrams

- The uncertainty chance node on the "Take Umbrella" branch is an unnecessary node. The payoff is the same regardless of the weather.
- In a decision-tree model, we can take advantage of this fact by not even drawing the unnecessary node.
- Influence diagrams, however, use the symmetric decision tree, even though this may require unnecessary nodes (and hence unnecessary calculations).

How the Influence Diagram "Presents" the Umbrella Problem



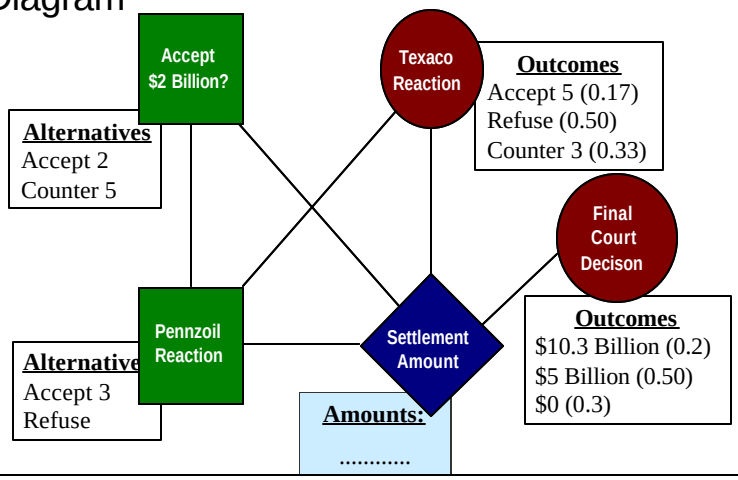
Influence diagram of the umbrella problem.

Note:

It does not matter whether the sun shines or not if you take the umbrella.

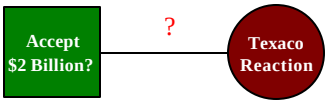
Influence Diagram "Node Presentation" of the Texaco-Pennzoil Problem

- Texaco-Pennzoil Example as an Influence Diagram



Solving Influence Diagrams: The Details

- **Some comments about this influence diagram**
- Initial decision for Pennzoil: Accept Texaco's offer of $2 Billion
- Shouldn't there be an arrow from the decision node?



(Texaco's reaction depends on Pennzoil's acceptance or no acceptance?)

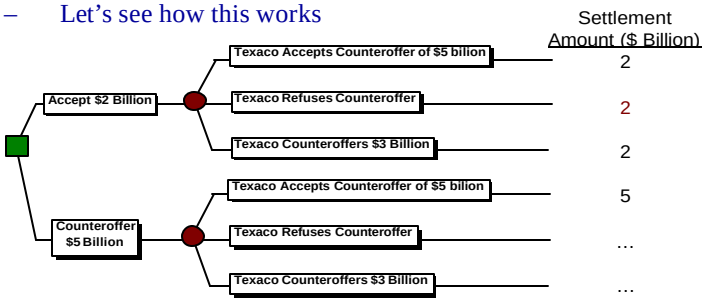
Solving Influence Diagrams: The Details

- **Answer:**
  - **Yes. But an arrow is unnecessary and would only make it more complicated.**
  - **An arrow would indicate that the decision outcomes of accepting / rejecting the $2 billion offer would affect the chances with Texaco's reaction of a counteroffer. (It really does not depend on the decision.)**
- **Similarly, no arrows between "Final Court Decision" and the other 3 nodes.**

Solving Influence Diagrams: The Details

Now:

- How do things get resolved in an influence diagram?
- In the consequence node's table, e.g.
  - Liedtke's settlement for every possible combination of decisions and outcomes
  - Let's see how this works



Solving Influence Diagram: Table Presentation

Liedtke's settlement for every possible combination of decision and outcomes:

Accept $2 Bilion?	Texaco Reaction ($ Billion)	Pennzoil Reaction ($ Billion)	Final Court Decision ($ Billion)	Settlement Amount ($ Billion)
Accept 2	Accept 5	Accept 3	10.3	2.0
			5	2.0
			0	2.0
		Refuse	10.3	2.0
			5	2.0
			0	2.0
	Offer 3	Accept 3	10.3	2.0
			5	2.0
			0	2.0
		Refuse	10.3	2.0
			5	2.0
			0	2.0
	Refuse	Accept 3	10.3	2.0
			5	2.0
			0	2.0
		Refuse	10.3	2.0
			5	2.0
			0	2.0

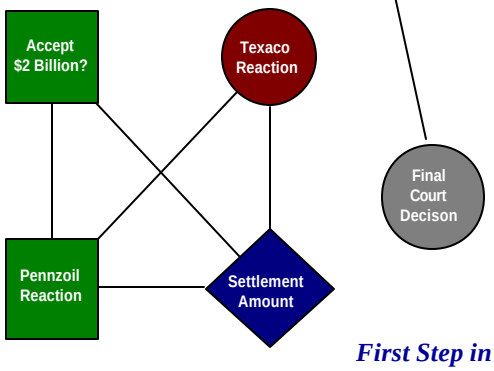
Solving Influence Diagram: Table Presentation

Accept $2 Bilion?	Texaco Reaction ($ Billion)	Pennzoil Reaction ($ Billion)	Final Court Decision ($ Billion)	Settlement Amount ($ Billion)
Offer 5	Accept 5	Accept 3	10.3	5.0
			5	5.0
			0	5.0
		Refuse	10.3	5.0
			5	5.0
			0	5.0
	Offer 3	Accept 3	10.3	3.0
			5	3.0
			0	3.0
		Refuse	10.3	10.3
			5	5.0
			0	0.0
	Refuse	Accept 3	10.3	10.3
			5	5.0
			0	0.0
		Refuse	10.3	10.3
			5	5.0
			0	0.0

Solving Influence Diagram: Node Presentation

- **Want to reduce nodes one at a time [Read pages 127-128].**

Step 1: Reduce "Final Court Decision". Calculate expected values to reduce this column (node).



First Step in Solving Influence Diagram

Solving Influence Diagram: Table Presentation

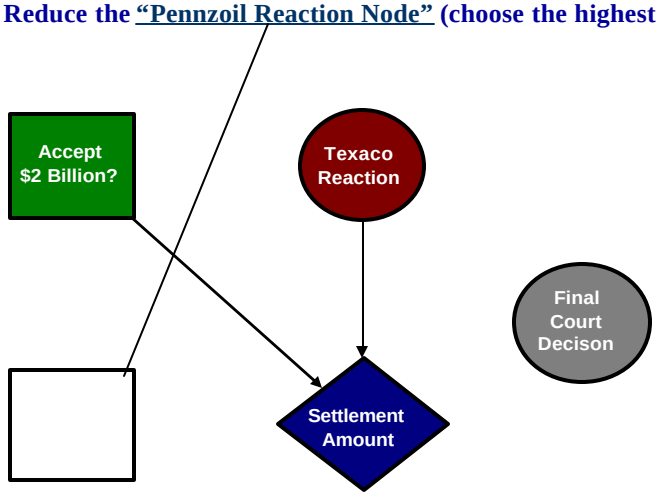
Accept $2 Bilion?	Texaco Reaction	Pennzoil Reaction	Settlement Amount Expected Value ($ Billion)
Accept 2	Accept 5	Accept 3	2.0
		Refuse	2.0
	Offer 3	Accept 3	2.0
		Refuse	2.0
	Refuse	Accept 3	2.0
		Refuse	2.0
Offer 5	Accept 5	Accept 3	5.0
		Refuse	5.0
	Offer 3	Accept 3	3.0
		Refuse	4.56
	Refuse	Accept 3	4.56
		Refuse	4.56

How?

Table for Liedtke's decision after reducing "Final Court Decision" nodes

Solving Influence Diagram: Node Presentation

Step 2: Reduce the "Pennzoil Reaction Node" (choose the highest EMV)



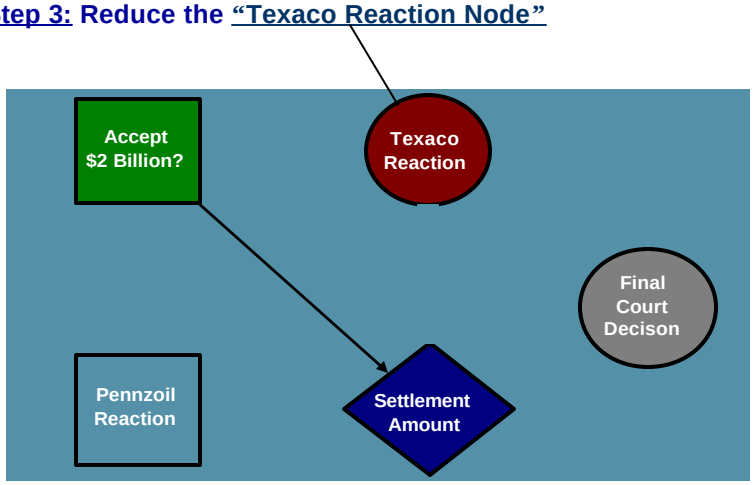
Solving Influence Diagram: Table Presentation

Accept $2 Bilion?	Texaco Reaction	Expected Value ($ Billion)
Accept 2	Accept 5 (0.17)	2
	Offer 3 (0.5)	2
	Refuse (0.33)	2
Offer 5	Accept 5 (0.17)	5
	Offer 3 (0.5)	4.56
	Refuse (0.33)	4.56

Table for Liedtke's decision after reducing "Final Court Decision" and "Pennzoil Reaction" nodes

Solving Influence Diagram: Node Presentation

Step 3: Reduce the "Texaco Reaction Node"



Solving Influence Diagrams: Final Table Presentation

Accept $2 Bilion?	Expected Value ($ Billion)
Offer 5	4.63

Step 4: Choose Best Alternative and Make the Decision

Accept $2 Bilion?	Expected Value ($ Billion)
Accept 2	2
Offer 5	4.63