CHAPTER

2

Duxbury Thomson Learning

Making Hard Decision Third Edition

Elements of Decision Problems

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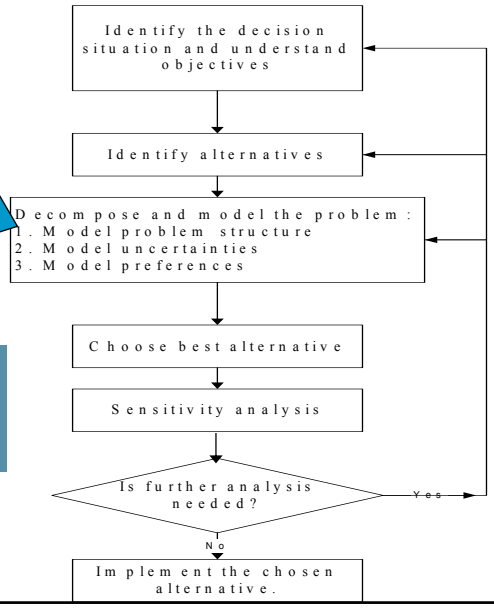
Methodology for Modeling Decision

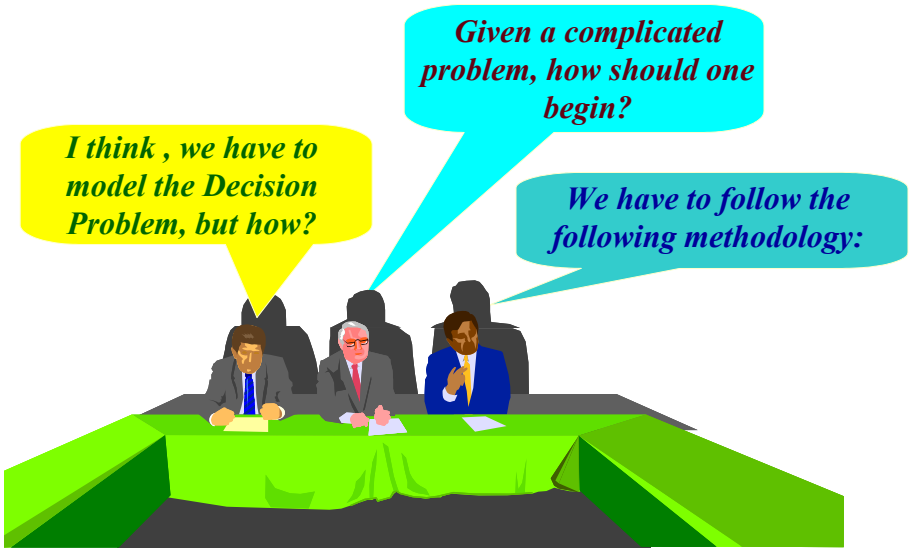
Modeling Decision Problems

The Decision-Analysis Flow Chart

[Source: R.Clemen, 1996]



Methodology for Modeling Decision



Methodology for Modeling Decision

The Methodology of Modeling Decisions is to:

- Understand the problem under study
- Introduce quantitative modeling
- Discuss the elements of a decision.
  - Values and Objectives
  - Decisions to be made
  - Upcoming uncertain events, and
  - Consequences
- Build the decision Model and identify a set of feasible alternatives.
- Evaluate the alternatives and make a choice of a feasible alternative.
- Re-evaluate the alternatives using sensitivity analysis to refine the solution.

Detailed Steps

- *Chapter 2*
- *Chapter 3*
- *Chapter 4*
- *Chapter 5*

Understanding the Problem under Study

- Nature of the Managerial Problem
- Types of Managerial Problems

Nature of the Managerial Problem

- A Managerial Problem may be as :
  - Specific as improving efficiency of a production line (role of Junior Managers) ,or
  - Broad as establishing a long-range corporate strategy involving a combination of financial, marketing and manufacturing operations (role of Senior Managers).

Nature of the Managerial Problem

- Nearly all projects begin with the recognition of a problem that does not have an obvious solution.
- Managers may require quantitative management techniques to assist them in identifying the “best” decision or solution for the problem.

Types of Managerial Problems

- Structured Problems
  - These are the type of routine problems that face Lower Management levels.
  - They are characterized by being narrower and less complex than those at higher managerial level.

Types of Managerial Problems

- Structured Problems (cont'd)
  - These type of problems do not require Top Strategic Managers to solve them.
  - Experts in specific domains are often consulted to provide programmed solutions to such structured problems and/or organized tasks.

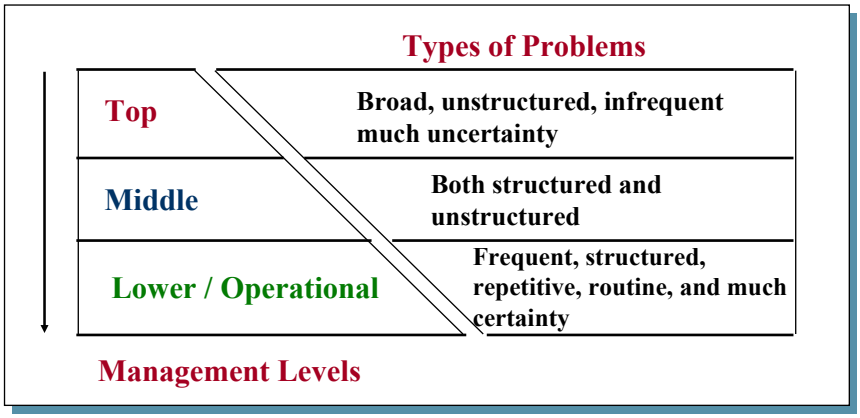
Types of Managerial Problems

- Unstructured Problems
  - These are the type of non-routine problems that face higher Management levels.
  - They are characterized by being broader and more complex than those at lower managerial level.

Types of Managerial Problems

- Unstructured Problems (cont'd)
  - These type of problems require Top Strategic Managers to solve them.
  - Top Managers often use non-programmed solutions to solve such type of unstructured problems and/or non-organized tasks.
  - The relationship between types of managerial problems and the management levels in the organization are as in the next diagram:

Types of Managerial Problems



Types of Problems and Management Levels in the Organization

Introducing Quantitative Modeling

- Quantitative Analysis and Decision Making.
- Structuring and Analyzing the Managerial Problem.
- Importance of Structuring and Analyzing the Managerial Problem.

Quantitative Analysis and Decision Making

- Reasons why a quantitative approach might be used in the decision-making process include the following:
  - The problem is complex, and the manager cannot develop a good solution without the aid of quantitative technique.
  - The problem is very important (e.g. a great deal of money is involved), and the manager desires a thorough analysis before attempting to make a decision.

Quantitative Analysis and Decision Making

- The problem is new and the manager has no previous experience to draw upon a solution.
- The problem is repetitive and the manager saves time and effort by relying upon quantitative procedures to make the routine decision recommendations.

Quantitative Analysis and Decision Making

- A central theme in the quantitative approach to decision making is a the appearance and orientation of a managerial problem.
- The manager responsible for making a decision or selecting a course of action will probably make an analysis of the problem.

Quantitative Analysis and Decision Making

- This problem process includes:
  - A statement of the specific goals or objectives
  - A selection of the apparent "best" decision
  - A solution for the problem.
- This problem analysis process may take two basic forms:

  Qualitative: which is based upon managers' judgment, experience, intuition and feel. It is more an art than a science.

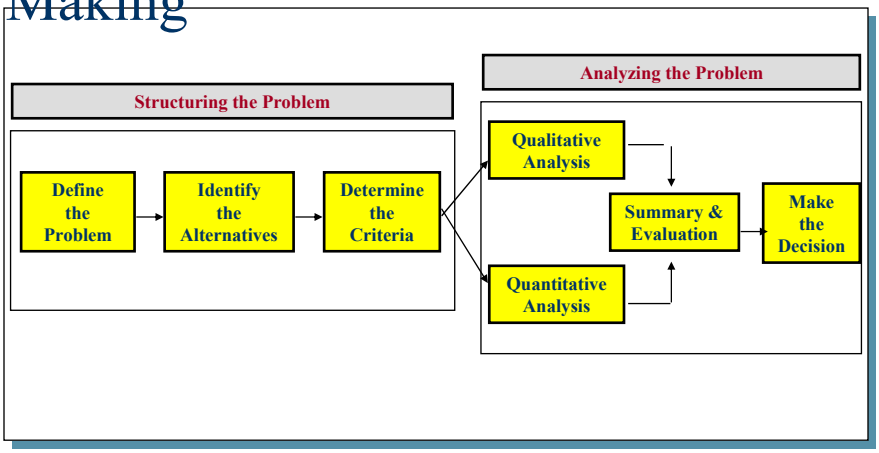
Quantitative Analysis and Decision Making

- **Quantitative:** which is based upon mathematical models that help managers to describe the objectives, constraints and relationships that exist in the problem. It is more a science that an art.
- Both Qualitative and Quantitative analysis of a problem provide important information for the manager or decision maker.

Quantitative Analysis and Decision Making

- In any cases, a manager will draw upon both sources and a thorough comparison and evaluation of the information to make a final decision.
- Problem Solving requires the use of an integrated approach of qualitative and quantitative procedure for making rational decisions. This can be illustrated as in the following flow-chart:

Quantitative Analysis and Decision Making



The Role of Qualitative and Quantitative Analysis

Importance of Structuring and Analyzing the Managerial Problem

All Managers (Juniors and Seniors) in any project or organization have to recognize the:

- ***Importance of adopting quantitative approach in addition to using their intuition, experience and judgment to solving managerial problems and making rational decisions.***

The result will definitely lead to achieving the organizational main Objectives and improving the overall Performance and Effectiveness of their organization.

Detail Steps of Modeling Decisions

Modeling Decisions consists of four steps (Chapters):

- **Step 1: Defining Decision Elements**

  Chapter 2: Elements of Decision Problems

  Step 2: Model Building & Structuring:

- **Chapter 3: Structuring Decisions**

  Step 3: Model Evaluation & Solution:

- **Chapter 4: Making Choices**

  Step 4: Model Re-evaluation:

- **Chapter 5: Sensitivity Analysis**

Focus on Modeling Decisions

- We will consider values and objectives, discussing in depth how multiple objectives can be organized in hierarchies and networks that can provide insight and help to generate creative alternatives.

Focus on Modeling Decisions

- **We will develop both *influence diagrams* and *decision trees* as graphical modeling tools for representing the basic structure of decisions.**
  - An influence diagram is particularly useful for developing the structure of a complex decision problem because it allows many aspects of a problem to be displayed in a compact and intuitive form.

Focus on Modeling Decisions

– A decision-tree representation provides an alternative picture of a decision in which more of the details can be displayed.

Note: Both graphical techniques can be used to represent single-objective decisions. They can be used in multiple-objective situations as well.

- We will discuss measurement, present concepts and techniques which can be used to ensure that we can adequately measure achievement of our objectives, whether those objectives are straightforward or more difficult to quantify.

Focus on Modeling Decisions

- Present the basic tools available to the decision maker for analyzing a decision model.
- Shows how to solve decision trees and influence diagrams.
- The basic concept presented is ***expected value***.

Focus on Modeling Decisions

- Also we will look briefly at the idea of risk analysis and the uses of a stochastic-dominance criterion for making decisions.
- Show how expected value and risk analysis can be used in multiple-objective decisions.

Focus on Modeling Decisions

- We learn how to use sensitivity-analysis tools in concert with *expected monetary value (EMV)* calculations in the iterative decision-structuring and analysis process.
- We bring the discussion of modeling decisions full circle, showing how structuring and analysis are intertwined in the decision-analysis process.

Elements of Decision Analysis

- Values and Objectives
- Making Money: A Special Objective
- Values and Current Decision Context
- Decisions to Make
- Sequential Decisions

Elements of Decision Analysis

- Uncertain Events
- Consequences
- Elements of Decisions …….(Farmer Example)
- The Time Value of Money: A Special Kind of Trade-Off

Highlights of Chapter 2

- This chapter is intended to start the reader thinking about decision problems in decision-analysis terms.
- Thus, we talk about decisions to make, uncertain events, and valuing consequences.

Highlights of Chapter 2

- To make sure that the richness of the terrain is understood, we introduce the concepts of dynamic decision making, a planning horizon, and trade-offs.
- Time value of money (TVM) is mentioned in this chapter because it is a fundamental way that streams of cash flows are valued, and because it provides a nice example of a basic trade-off.

Contents

- Definitions of Key Terms
- Identifying Elements of Decision Situation
- Values and Objectives
- Making Money: A Special Objective
- Values and Current Decision Context
- Decisions to Make
- Sequential Decisions
- Uncertain Events
- Consequences

Definitions of Key Terms

- Key Terms
  - Objectives
  - Values
  - Decision Context
  - Sequential Decisions
  - Decision
  - Alternatives
  - Consequence and Outcomes
  - Uncertain Events (chance event)
  - Time Value of Money

Definitions of Key Terms

- **Objectives:** Specific thing you want to achieve (e.g. a profit minimize pollution, etc.).
- **Values:** are made up of one's objectives, i.e., what matters (e.g., earning money while preserving the environment.

Definitions of Key Terms

- **Decision Context**: refers to the specific identification of the problem (from which we might suspect that when one solves the wrong problem, one has used the wrong decision context).
- **Sequential Decisions:** these are decisions that are based on a series of previous decisions. It creates what is known as a dynamic decision context.

Definitions of Key Terms

- **Decision:** a choice of "best" alternative from a set of feasible alternatives.
- **Alternatives:** a set of options that are identified after the problem is studied.
- **Consequences or Outcomes:** refer to what the decision maker experiences as a result of a combination of alternative (s) chosen and chance outcome (s) or state of nature.

Definitions of Key Terms

- **Uncertain Events (chance events or state of nature):** these are expected future events that will happen for the problem in hand as defined in the problem study step.
- **Time Value of Money (TVM):** refers to a fundamental way that streams of cash flows are valued, and because it provides a nice example of a basic trade-off.

Identifying the Elements of the Decision Situation

- A critical first step is to identify the elements of the situation by defining the following items:

(1) Values and objectives,
(2) Decisions to make
(3) Uncertain events, and
(4) Consequences.

Values and Objectives

- **"Values"** is an overused term that can be somewhat ambiguous; here we use it in a general sense to refer to things that matter to you.
- An "***objective***" is a specific thing that you want to achieve

1. An individual's objectives taken together make up his or her values. They define what is important to that person in making a decision.

Values and Objectives

2. A person's values are the reason for making decisions in the first place!
3. Without objectives, it would not be possible to tell which alternative would be the best choice!

Making Money: A Special Objective

- Money's role as a trading mechanism in our economy puts it in a special role. Although it is typically not one of our basic objectives, it can serve as a proxy objective in many situations.
- We have to balance the volatility of the fund's value against the amount they can expect to earn over the long run.

Making Money: A Special Objective

- Most investment decisions require a trade-off between risk and return.

Making Money: A Special Objective

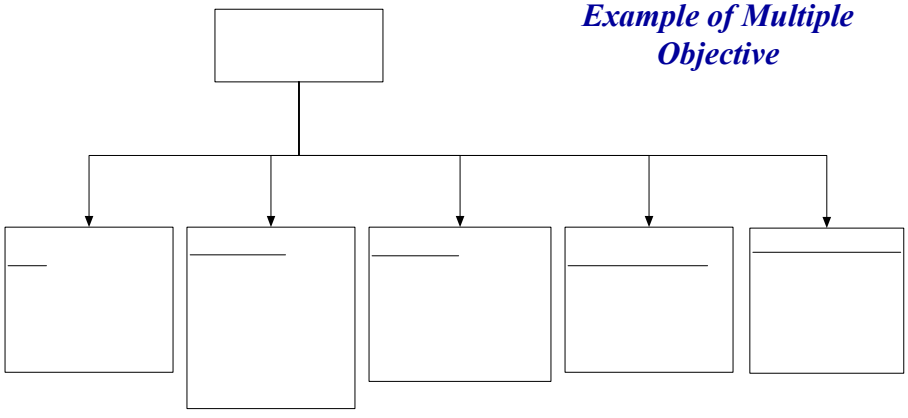
- In many cases, one can price out the value of different objectives.
- We all make related decisions all the time as we decide whether a product or service is worth the price that is asked. In other cases, though, it may not be reasonable to convert everything to dollars.

Making Money: A Special Objective

- **Pricing out the lives and risks to the other patients seems like a cold-hearted way to make this decision. The hospitals will probably be better off thinking in terms of its fundamental objectives and how to accomplish them with or without the wealthy patient's fee.**

Objectives of Boeing's Supercomputer

Example of Multiple Objective



Values and the Current Decision Context

- You can still think hard about your objectives as you consider your decision.
- You can still think hard about your objectives as you consider your decision.

Values and the Current Decision Context

- Values and decision context go hand in hand. It is worthwhile to think about your objectives in advance to be prepared for decisions when they arise or so that you can identify new decision opportunities that you might not have thought about before.

Values and the Current Decision Context

- Every decision situation involves a specific context, and that context determines what objectives need to be considered.
- **Decision context:**

Decision occurs

Example → being in a competitive market

Example → building environmentally safe structures

The setting in which each decision occurs.

Decisions to Make

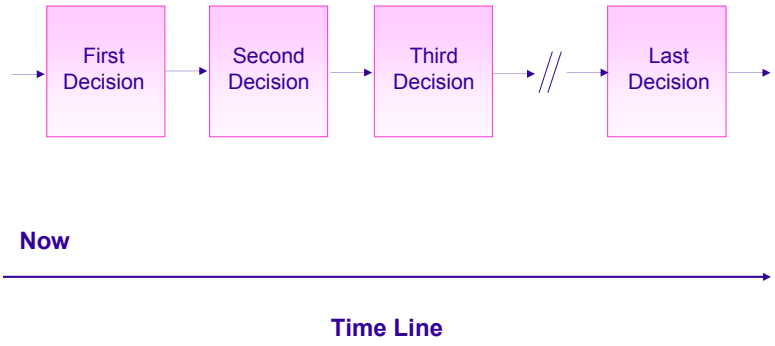
1. **Identifying the immediate decision to make is a critical step in understanding a difficult situation.**
2. **No model of the decision situation can be built without knowing exactly what the decision problem at hand is.**
3. **In identifying the central decision, it is important also to think about possible alternatives.**
4. **Some decisions will have specific alternatives (protect the crop or not), while others may involve choosing a specific value out of arrange of possible values (deciding on an amount to bid).**
5. **Many situations have as the central issue a decision that must be made right away. There would always be at least two alternatives; if there were no alternatives, then it would not be a matter of making a decision!**
6. **Another possibility may be to wait and obtain more information.**
7. **Other possible alternatives are taking out insurance or hedging.**

Sequential Decisions

- Sequential Decisions: A decision maker needs to consider decisions to be made now and later.
  - In many cases, there simply is no single decision to make, but several sequential decisions.
  - When a decision situation is complicated by sequential decisions, a decision maker will want to consider them when making the immediate decisions. A future decision may depend on exactly what happened before. We refer to these kinds of problems as *dynamic decision situations*.

Sequential Decisions

- In identifying elements of a decision situation, we want to know not only what specific decisions are to be made, but the sequence in which they will arise.



Uncertain Events

- Decision problems can be complicated because of uncertainty about what the future holds.
- Many important decisions have to be made without knowing exactly what will happen in the future or exactly what the ultimate outcome will be from a decision made today.

Uncertain Events

- The possible things that can happen in the resolution of an uncertain event are called *outcomes.*
- Many different uncertain events might be considered in a decision situation, but only some are relevant. How can you tell which ones are relevant? The answer is straightforward; the outcome of the event must have some impact on at least one of your objectives.

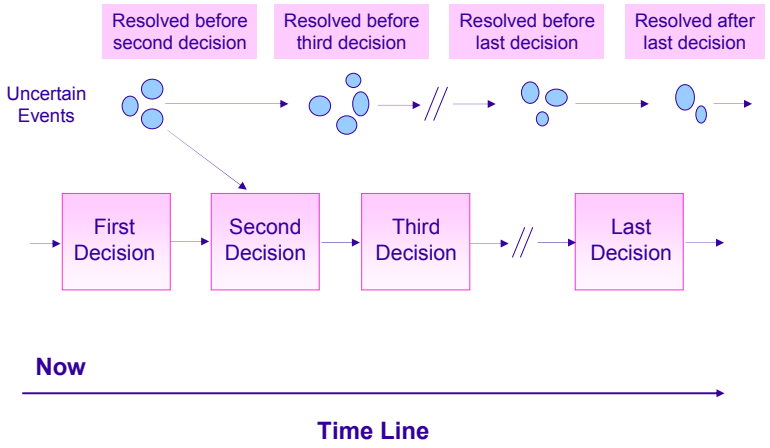
Uncertain Events

- The larger the number of uncertain but relevant events in a given situation, the more complicated the decision.
- Some uncertain events may depend on others.
- There may be interdependencies among the uncertain events that a decision maker must consider.

Uncertain Events

- How do uncertain events relate to sequential decisions? **They must be dovetailed with the time sequence of the decisions to be made; it is important to know at each decision exactly what information is available and what remains unknown.**

Dovetailing Uncertain Events and Sequential Decisions

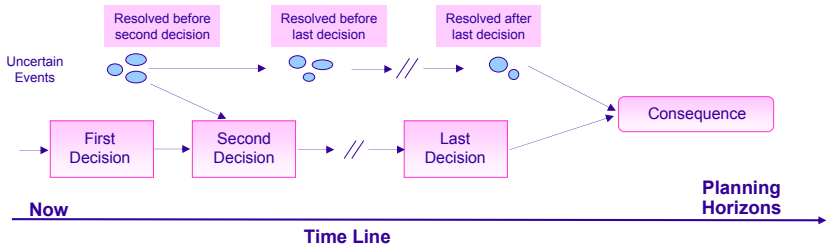


Consequences

- In some cases the final consequences may be a "net value" figure that accounts for both cash outflows and inflows during the time sequence of the decisions.
- If the decision context requires consideration of multiple objectives, the consequence is what happens with respect to each of the objectives.

Consequences

- Looking forward from the current time and current decision, the end of the time line is called the *planning horizon.*



Consequences

- **The Planning Horizon and Value of the Consequences**
- **How far into the future do I look?**
  - "My planning horizon is there. It's not worthwhile for me to think beyond that point in time."
  - For the purpose of **constructing a requisite model**, the idea is to choose a planning horizon such that the events and decisions that would follow after are not essential parts of the immediate decision problem.

Consequences

- Choose a planning horizon that is consistent with your **decision context and the relevant objectives**.
- Once the **dimensions of the consequences** and the planning horizon have been determined, the next step is to figure out how to **value the consequences**.
- It will be possible to **work in terms of monetary values**. That is, the only relevant objective in the decision context is to make money, so all that matters at the end is profit, cost, or total wealth.

Elements of Decision Problems

Example: Farmer whose fruit crop will soon need to be harvested. What should he do?

His object might be to:
- **maximize his profit**
- **maximize production of his crops**
- **minimize use of pesticides**

- **With almost ripe fruit, timing is important:**
  - If weather report is for mild weather ---- no problem with spoilage and rotting
  - If weather report is for freezing, it may be prudent to take protective measurements ---- could be general alternatives

Elements of Decision Problems

Or:

- The farmer could wait for a forecast before actually doing anything.For example, gathering more information first.
  - This strategy may have a cost associated with it (pay farm work for Over Time since you waited later).

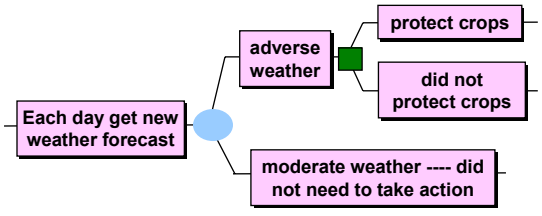
➔ **This information / cost tradeoff is a recurring theme in Decision Analysis.**

Or:

- The farmer could take out insurance to protect his crops
  - Pay a small amount to have harvesting crew for the night-time work.
- Decision are sometimes sequential ---- several decisions to make as seen in this case.

Elements of Decision Problems

In the case of the farmer, maybe the sequential decision is like the following (at nearly the end of the growing season)



Note: In this sort of setting, a future decision may depend on what happened beforehand. These are therefore called

dynamic decision situations

Elements of Decision Problems

- What is the major reason for making decision on farm harvesting difficult?

Elements of Decision Problems

The possible things that can happen in resolving an uncertain event are called **outcomes**

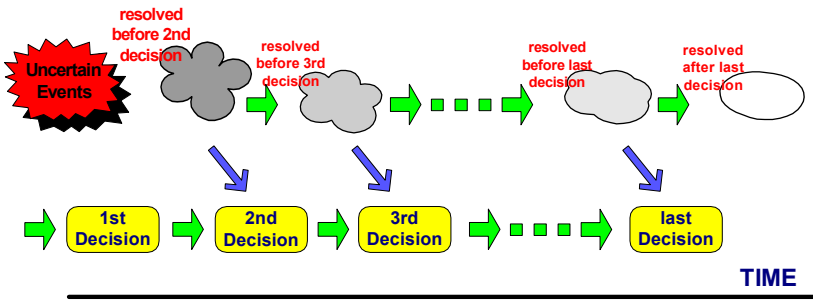
i.e., uncertain event $\Rightarrow$ the weather

Outcomes $\Rightarrow$ { crop damage, no crop damage

This is where probabilities enter Decision Analysis (DA)

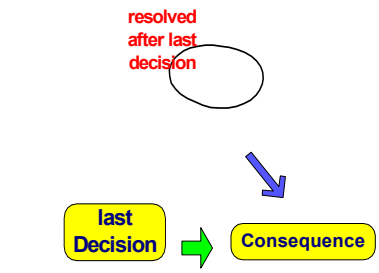
Elements of Decision Problems

- Could be complex in the sense that same uncertain events may depend on others.
- Here is how uncertain events and sequential decisions are related.



Elements of Decision Problems

- After last decision made, we have a consequence



End of time line is the planning horizon (for the farmer, maybe just the season)

TIME

The Time Value of Money (TVM)

- Money has a time value
- One dollar today is worth more than $1 tomorrow.
- Failure to pay the bills results in additional charge termed.

The Interest (i%)

- Interest is usually expressed as a percentage of the amount owed.
- It is due and payable at the close of each period of time involved in the agreed transaction (usually every month).

The Interest (i%)

- Example
  - If $ 1,000.00 is borrowed at 14% interest, then interest on the ***principal*** *of* $ 1,000.00 after one year is **0. 14 x 1, 000, or $140.00.**
  - If the borrower pays back the total amount owed after one year, she will pay **$1,140.00.**
  - If she does not pay back any of the amount owed after one year, then normally the interest owed, but not paid, is considered now to be additional principal, and thus the interest is ***compounded.***
  - After two years she will owe **$1,140.00+0.14 X 1,140.00,or 1,299.60.**

Equivalency

- The banker normally does not care whether you pay him $1,140.00 after one year or $1,299.60 after two years. To him, the three values ($1,000, $1,140, and $1,299.60) are equivalent.
  - $ 1,000 today is equivalent to $1,140 one year from today,
  - $ 1,000 today is equivalent to $1,299.60 two years from today.

Equivalency

- The three values are not equal but equivalent
- Note:
  1. ***The concept of equivalence involves time and a specified rate of interest. The* three preceding values are only equivalent for an interest rate of 14%, and then only at the specified times.**
  2. ***Equivalence* means that one sum or series differs from another only by the accumulated interest at rate *i* for *n* periods of time.**

Symbols

- To generalize the concept of interest the following symbols are used:

  P = a present single amount of money

  F = a future single amount of money, after *n* periods of time

  i = the rate of interest per interest period (usually one year)

  n = the number of periods of time (usually years)

The Time Value of Money

- When NPV is calculated, it reveals the value of the stream of cash flows. A negative NPV for a project indicates that the money would be better invested to earn interest rate *i*.
- Calculating present values establishes trade-offs between dollars at one point in time and dollars at another.

The Time Value of Money

- You would be indifferent between receiving \$1 now or \$1(1 + *i*) at the end of the next time period. More generally, \$1 now is worth \$1$(1 + i)^n$ at the end of n time periods. NPV works by using these trade-off rates to discount all the cash flows back to the present.
- Often we use the interest rate from a savings account, a certificate of deposit, or short-term (money market) securities.

The Time Value of Money

- For a corporation, the appropriate interest rate to use might be the interest rate they would have to pay in order to raise money by issuing bonds. Often the interest rate is called the hurdle rate, indicating that an acceptable investment must earn more than this rate.
- The time value of money can show how a consequence that is a stream of cash flows can be valued through the trade-offs implicit in interest rates.

The Time Value of Money

- Uncertain future events must be dovetailed with the sequence of decisions, showing exactly what is known before each decision is made and what uncertainties still remain.
- The time value of money shows how interest rates imply a special kind of trade-off between cash flows at different points in time.

The Time Value of Money: A Special Kind of Trade-Off

- One of the most common consequences in personal and business decisions is a stream of cash flows. In such a case, there is a special kind of trade-off: spend dollars today to obtain dollars tomorrow.

The Time Value of Money: A Special Kind of Trade-Off

- **Trade-offs between current and future dollars (and between future dollars at different points in time) refer to the fact that the** value of a dollar depends on when it is available to the decision maker**. "Time value of money". The** *present value*.

$$\mathrm{PV}(x, n, i) = \frac{x}{(1+\mathrm{i})^n}$$

The interest rate *i%*
***n* time periods (days, months, years)**

The Time Value of Money: NPV Concept

The formula for calculating NPV for stream of cash flows x_o, …, x_n over *n* periods at interest rate *i* is:

$$\text{NPV} = \frac{x_0}{(1+i)^0} + \frac{x_1}{(1+i)^1} + \ldots + \frac{x_n}{(1+i)^n}$$

$$= \sum_{i=0}^{n} \frac{x_i}{(1+i)^i}$$

The Time Value of Money: NPV Concept

- Example

  A friend is involved in a business deal and offers to let you in on it. For $425 paid to him now, he says, you can have $110.00 next year, $121.00 the following year, $133.10 the third year, and $146.41 at the end of Year 4. Your payments will total $510.51. What is the present value of the stream of payments?

The Time Value of Money: NPV Concept

- Example (cont'd)

$$PV = \frac{110.00}{1.1} + \frac{121 \quad .00}{(1 \; .1)^2} + \frac{133 \quad .10}{(1 \; .1)^3} + \frac{146 \quad .41}{(1 \; .1)^4}$$
$$= \$\,100 \quad + \$\,100 \quad + \$\,100 \quad + \$\,100 \quad = \$\,400$$

Note:
1. You would be paying $425 for a stream of cash flows that has a present value of $400.
2. The ***net present value*** (***NPV***) of the cash flows is the present value of the cash flows ($400) minus the cost of the deal ($425), or -$25.

Some Types of Financial Analysis

- **Discounted present worth analysis**
- **Single payment**
- **Rate of return analysis [ROR]**

Discounted Present Worth Analysis

- Often in engineering economic studies, as well as in general financial analyses, a discounted present worth analysis is made of each alternative under consideration.
- It involves calculating the *equivalent* present worth or present value of all the dollar amounts involved in the alternative to determine its present worth.
- **Definition:**

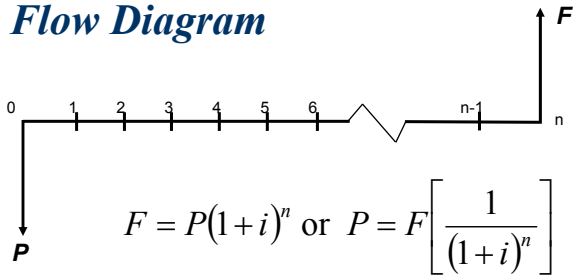
  The present worth is *discounted* at a predetermined rate of interest called the *minimum attractive rate of return* (MARR or *i).**

  The MARR is usually equal to the current rate of interest for borrowed capital plus an additional rate for such factors as risk, uncertainty, and contingencies.

MARR = $i^* = i + i$(risk)

Single Payment

Cash Flow Diagram



$$F = P(1+i)^n \text{ or } P = F\left[\frac{1}{(1+i)^n}\right]$$

Note:

The factor $1/(1+i)^n$ is called the *present worth compound amount factor* (PWCAF)

Example 1: Single Payment

- A contractor wishes to set up a revolving line of credit at the bank to handle her cash flow during the construction of a project. She believes that she needs to borrow $12,000 with which to set up the account, and that she can obtain the money at 1.45% per month.

If she pays back the loan and accumulated interest after 8 months, how much will she have to pay back?

$F = 12{,}000(1 + 0.0145)^8 = 12{,}000(1.122061) = 13{,}464.73 = \underline{\$13{,}465}$

The amount of interest will be:

$13,465 - 12,000 = $1,465.

Example 2: Single Payment

- A construction company wants to set aside enough money today in an interest-bearing account in order to have $ 100,000 five years from now for the purchase of a replacement piece of equipment.

If the company can receive 8% interest on its investment, how much should be set aside now to collect the $100,000 five years from now?

$P = 100{,}000 / (1 + 0.08)^5 = 100{,}000 / (1.46933) = \$68{,}058.32 = \underline{\$68{,}060}$

Or using tables:

$P = 100{,}000\ (P/F, 8, 5) = 100{,}000(0.6805832) = \$68{,}058.32 = \underline{\$68{,}060}$

Example 3

A Construction Company is considering three methods of acquiring company pickups for use by field engineers. The alternatives are:

A. Purchase the pickups for $7,200 each and sell after 4 years for an estimated $1,200 each.

B. Lease the pickups for 4 years for $2,250 per year paid in advance at the beginning of each year. The contractor pays all operating and maintenance costs on the pickups and the leasing company retains ownership.

C. Purchase the pickups on special time payments with $750 down now and $2,700 per year at the end of each year for 3 years. Assume the pickups will be sold after 4 years for $1,200 each.

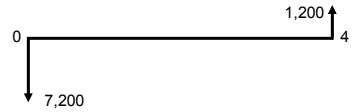
If the contractor's MARR is 15%, which alternative should he choose?

Note: All alternatives involve equal lives.

Example 3 (cont'd)

To solve, calculate the net present worth (NPW) of each alternative at 15% and select the least costly alternative:

A

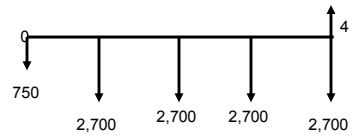


$NPW_A = -7,200 + 1,200(P/F),15,4) = \underline{-\$6,514}$

B

$NPW_B = -2,250 - 2,250(P/A),15,3) = \underline{-\$7,387}$

C



$NPW_C = -750 - 2,700(P/A),15,4)+1,200(P/A),15,4) = \underline{-\$7,772}$

The least costly alternative is A

What to do When Alternatives Involve Different Lives

Approach 1:

Truncate (cut off) the longer-lived alternative(s) to equal the shorter lived alternative and assume a salvage value for the unused portion of the longer lived alternatives. Then make the comparison on the basis of equal lives.

Approach 2:

Assume equal replacement conditions (costs and incomes) for each alternative and compute the discounted present worth on the basis of the least common multiple of lives for all alternatives.

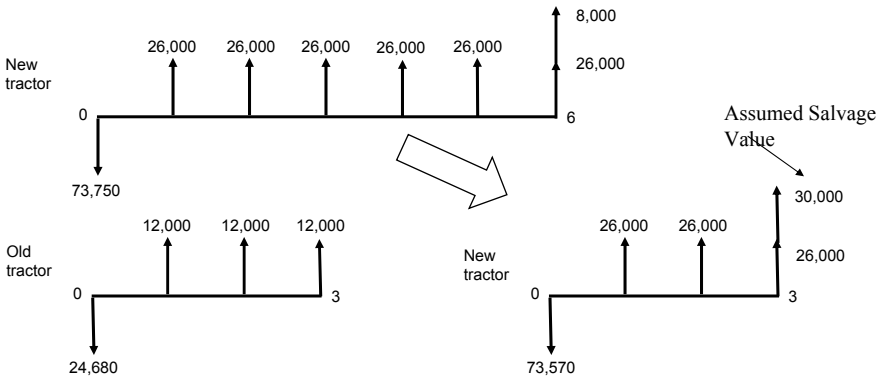
Example 4

A contractor is considering the purchase of either a new track-type tractor for $73,570, which has a 6-year life with an estimated net annual income of $26,000 and a salvage value of $8,000, or a used track-type tractor for $24,680, with an estimated life of 3 years and no salvage value and an estimated net annual income of $12,000.

If the contractor's MARR is 20%, which tractor, if any, should she choose?

Example 4 (cont'd)

Approach 1. ***(comparison on the basis of equal lives)***



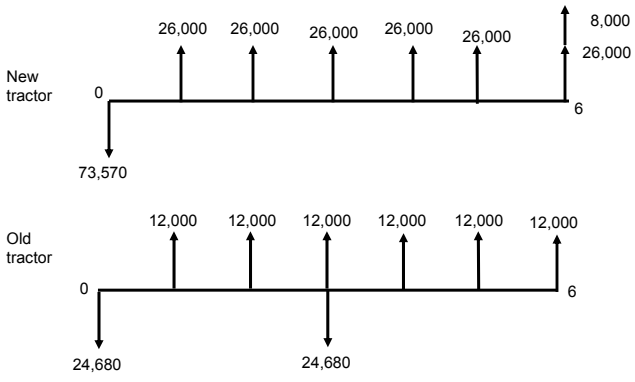
NPW_{new} = -73,570 + 26,000(P/**A**,20,3) + 30,000(P/**F**,20,3) = -73,570 + 26,000(2.10648) + 30,000(0.5787) = -$1,443

NPW_{old} = -24,680 + 12,000(P/A,20,3) = -24,680 + 12,000(2.10648) = + $597

Conclusion: Old tractor is a better Alternative

Example 4 (cont'd)

Approach 2. ***(comparison on the basis of equal replacement conditions)***



NPW_{new} = -73,570 + 26,000(P/**A**,20,6) + 8,000(P/**F**,20,6) = -73570 + 26,000(3.32551) + 8,000(0.33490) = -$15,570

NPW_{old} = -24,680 + 12,000(P/**A**,20,6) - 24,680(P/**F**,20,3) = -24,680 + 12,000 (3.32551) - 24,680(0.57870) = +$944

Conclusion: New tractor is a better Alternative

Rate of Return (ROR) Analysis

- Knowing the anticipated *rate of return* of an investment permits decision maker to have more "perceived" confidence in his decision!

Definition:

> *The **rate of return*** of a proposed investment is that interest rate which makes the discounted present worth of the investment equal to zero.

- To calculate the *rate of return,* simply set up the equation to be equal to zero and solve for *i*.

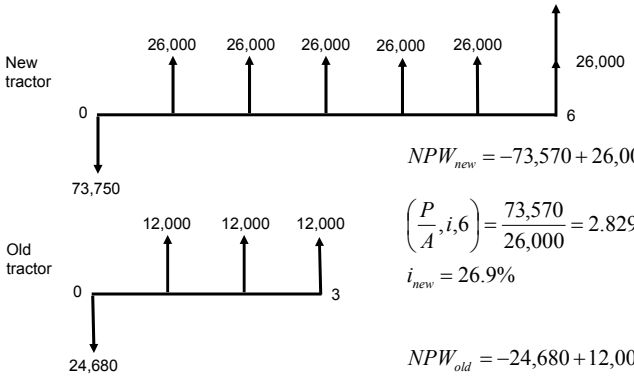
Example 5

- A contractor is considering the purchase of either a new track-type tractor for $73,570, which has a 6-year life with an estimated net annual income of $26,000, or a used track-type tractor for $24,680, with an estimated life of 3 years and no salvage value and an estimated net annual income of $12,000.

If the contractor's MARR is 20%, which tractor, if any, should be chosen?

Example 5 (cont'd)

Approach 1. ***(comparison on the basis of equal lives)***



$$NPW_{new} = -73{,}570 + 26{,}000\left(\frac{P}{A}, i, 6\right) = 0$$

$$\left(\frac{P}{A}, i, 6\right) = \frac{73{,}570}{26{,}000} = 2.82962 = \frac{(1+i)^6 - 1}{i(1+i)^6}$$

$$i_{new} = 26.9\%$$

$$NPW_{old} = -24{,}680 + 12{,}000\left(\frac{P}{A}, i, 3\right) = 0$$

$$\left(\frac{P}{A}, i, 3\right) = \frac{24{,}680}{12{,}000} = 2.05667 = \frac{(1+i)^3 - 1}{i(1+i)^3}$$

$$i_{old} = 21.5\%$$

Example 5 (cont'd)

Iterative Solution

$$NPW_{new} = -73{,}570 + 26{,}000\left(\frac{P}{A}, i, 6\right) = 0$$

$$\left(\frac{P}{A}, i, 6\right) = \frac{73{,}570}{26{,}000} = 2.82962 = \frac{(1+i)^6 - 1}{i(1+i)^6} \Rightarrow \boxed{i_{new} = \frac{(1+i)^6 - 1}{2.82962(1+i)^6}}$$

$$NPW_{new} = -24{,}680 + 12{,}000\left(\frac{P}{A}, i, 3\right) = 0$$

$$\left(\frac{P}{A}, i, 6\right) = \frac{24{,}680}{12{,}000} = 2.05667 = \frac{(1+i)^3 - 1}{i(1+i)^3} \Rightarrow \boxed{i_{old} = \frac{(1+i)^3 - 1}{2.05667(1+i)^3}}$$

Example 5 (cont'd)

Iterative Solution

$$i_{new} = \frac{(1+i)^6 - 1}{2.82962(1+i)^6}$$

i	NPW_{new}
0.200	12893
0.235	5877
0.254	2498
0.262	1027
0.266	416
0.268	168
0.268	67
0.268	27
0.269	11
0.269	4
0.269	2
0.269	1
0.269	0
0.269	0
0.269	0
0.269	0
0.269	0
0.269	0
0.269	0
0.269	0
0.269	0
0.269	0
0.269	0
0.269	0
0.269	0

$$i_{new} = 26.9\%$$

i	$NPWold$
0.150	2719
0.167	1985
0.180	1415
0.190	990
0.198	683
0.203	466
0.207	316
0.210	214
0.212	144
0.213	97
0.214	65
0.214	43
0.215	29
0.215	19
0.215	13
0.215	9
0.215	6
0.215	4
0.215	3
0.215	2
0.215	1
0.215	1
0.215	1
0.215	0
0.215	0

$$i_{old} = \frac{(1+i)^3 - 1}{2.05667(1+i)^3}$$

$$i_{new} = 21.5\%$$

Example 5 (cont'd)

If MARR is 20%

Then, the new tractor is selected.