CHAPTER

12

Duxbury Thomson Learning

Making Hard Decision

Third Edition

VALUE OF INFORMATION

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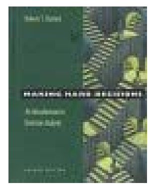

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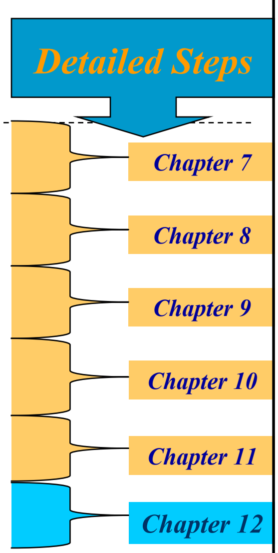
Methodology for Modeling Decision

The Methodology of Modeling Uncertainty is described in five chapters that mainly concentrating on how to model uncertainty using probabilities and information as follows:

- Probability Basics: reviews fundamental probability concepts.
- Subjective probability: translates beliefs & feelings about uncertainty in probability for use in decision modeling.
- Theoretical Probability Models: helps with representing uncertainty in decision modeling
- Using Data: uses historical data for developing probability distributions
- Monte Carlo Simulation: to give the decision-maker a fair idea about the probabilities associated with various outcomes.
- **Value of Information: explores the value of information within the decision-analysis framework.**



Contents

Gathering Information

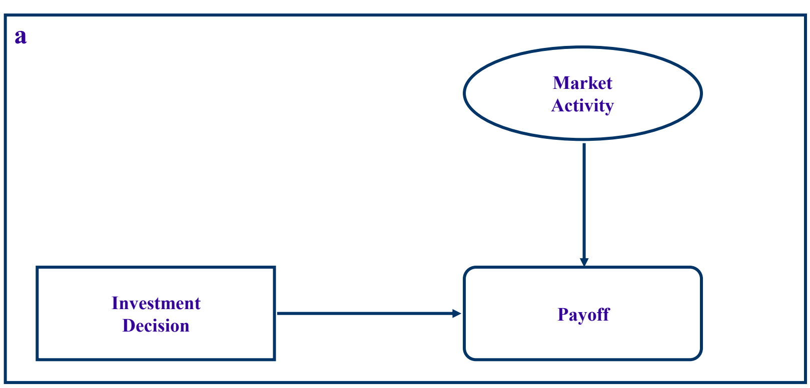
- Decision makers who face uncertain prospects often gather information with the intention of reducing uncertainty.
- Information gathering includes:
  1. consulting experts,
  2. conducting surveys,
  3. performing mathematical or statistical analyses
  4. doing research, or
  5. reading books, journals, and newspapers
- The intuitive reason for gathering information is straightforward; to the extent that we can reduce uncertainty about future outcomes, we can make choices that give us a better chance at a good outcome.
- We will work a few examples that should help you understand the principles behind information valuation.
- We will demonstrate the techniques used to calculate information value.

Investing in the Stock Market Example

Example:

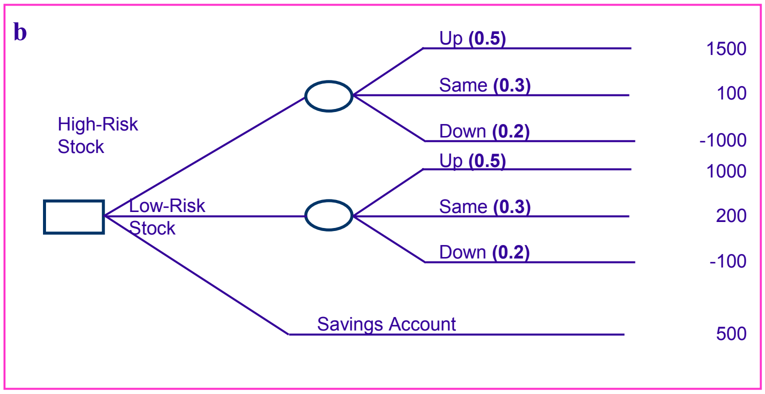
- An investor has some funds available to invest in one of the three choices:
  - a high-risk stock,
  - a low-risk stock, or
  - a savings account that pays a sure $500.
- If he invests in the stocks, he must pay a brokerage fee of $200.
- His payoff for the two stocks depends on what happens to the market. If the market goes up, he will earn $1,700 from the high-risk stock and $1,200 from the low-risk stock. If the market stays at the same level, his payoffs for the high- and low-risk stocks will be $300 and $400, respectively. Finally, if the stock market goes down, he will lose $800 with the high-risk stock but still gain $100 with the low-risk stock.

Influence Diagram for Investment Problem



(a) Influence-diagram

Decision Tree for Investment Problem



(b) Decision-tree representations of the investor's problem.

Value of Information: Some Basic Ideas

- What does it mean for an expert to provide perfect information?
- How does probability relate to the idea of information?
- What is an appropriate basis on which to evaluate information in a decision situation?

Probability and Perfect Information

- We can use conditional probabilities to model perfect information.

■Example:

- **Imagine an expert who always correctly identifies a situation in which the market will increase:**
- **P (Expert Says "Market Up" | Market Really Does Go Up) = 1**
- **Because the probabilities must add to 1, we also must have**
- **P (Expert Says "Market Will Stay the Same or Fall" | Market Really will Go Up) = 0**
- **P (Expert Says "Market Will Go Up" | Market Really Will Stay the Same or Fall) = 0**
- **We can use Bayes' theorem to "flip" the probabilities as there is no uncertainty after we have heard the expert.**

Perfect Information and Bayes Theorem

- Some Notations:
  - Market Up = The market really goes up
  - Market Down = The market really stays flat or goes down
  - Exp says "Up" = The expert says the market will go up
  - Exp says "Down" = The expert says the market will stay flat or go down

Perfect Information and Bayes Theorem

- Using Bayes Theorem and calculating the conditional probabilities:

$$
\begin{aligned}
&P(\text{Market Up} \mid \text{Exp Says "Up"}) \\
&= \frac{P(\text{Exp Says "Up"} \mid \text{Market Up})P(\text{Market Up})}{[P(\text{Exp Says "Up"} \mid \text{Market Up})\,P(\text{Market Up}) + P(\text{Exp Says "Up"} \mid \text{Market Down})\,P(\text{Market Down})]} \\
&= \frac{1\,P(\text{Market Up})}{1\,P(\text{Market Up}) + 0\,P(\text{Market Down})} \\
&= 1
\end{aligned}
$$

The Expected Value of Information [EVPI]

- **How can we place a value on information in a decision problem?**
- **For example , how could we decide whether to hire the expert described in the last section? Does it depend on what the expert says?**
- **Information appears to have no value when the investor would have taken the same action regardless of the expert's information.**
- **On the other hand, the expert might say that the market will fall or remain the same, in which case the investor would be better off with the savings account.**
- **The information has value when it leads to a different action, one with a higher expected value than what would have been experienced without the expert's information.**

The Expected Value of Information

- We can think about information value after the fact, but it is much more useful to consider it before the fact—that is, before we actually get the information or before we hire the expert.
- What effects do we anticipate the information will have on our decision?
- By considering the expected value, we can decide whether an expert is worth consulting, whether a test is worth performing, or which of several information sources would be the best to consult.

The Expected Value of Information

- The worst possible case would be that, regardless of the information we hear, we still would make the same choice that we would have made in the first place. In this case, the information has zero expected value!
- But but there are certain cases—things an expert might say or outcomes of an experiment—on the basis of which we would change our minds and make a different choice, then the expected value of the information must be positive; in those cases, the information leads to a greater expected value.
- The expected value of information can be zero or positive, but never negative.

The Expected Value of Information

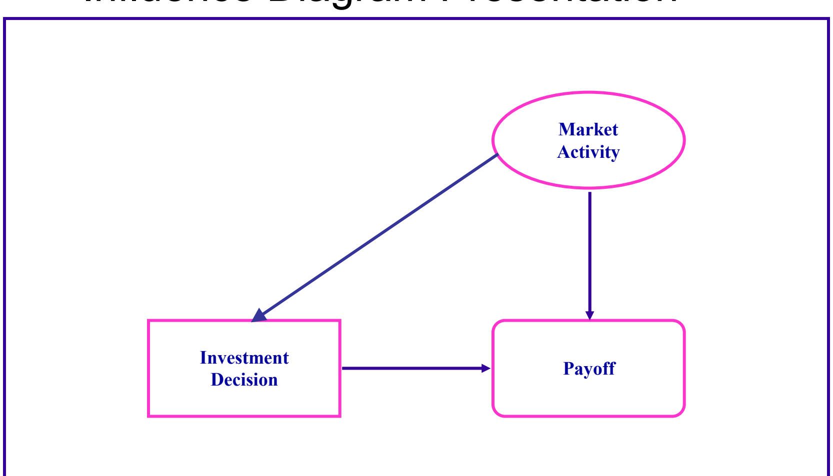
- The expected value of perfect information provides an upper bound for the expected value of information in general.
- The expected value of any information source must be somewhere between zero and the expected value of perfect information.
- For this reason, different people in different situations may place different values on the same information.

Example:

- General Motors may find that economic forecasts from an expensive forecaster may be a bargain in helping the company refine its production plans.
- The same economic forecasts may be an extravagant waste of money for a restaurateur in a tourist town.

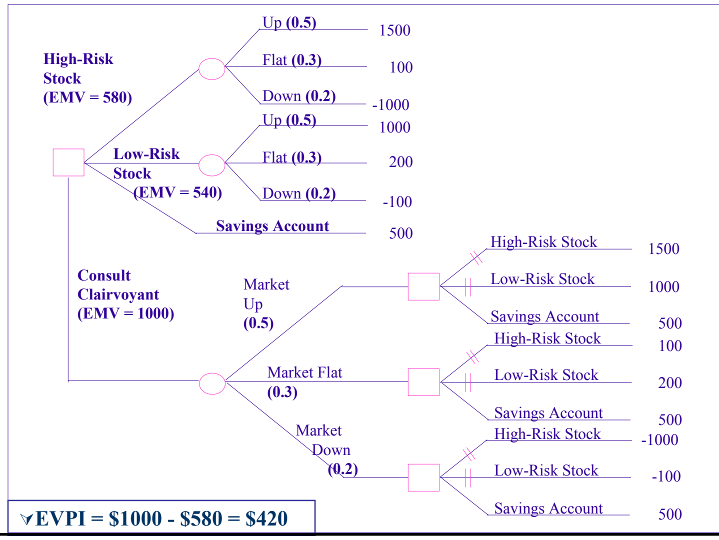
Perfect information in the Investor's Problem

- Influence Diagram Presentation



Perfect information in the Investor's Problem (Decision Tree)

Investment decision tree with the perfect-information alternative



Expected Value of Perfect Information

- **How can we find the EVPI in the investment problem?**
- **Solve each influence diagram.**
- **Find the EMV of each situation.**
- **Subtract the EMV ($580) from the EMV ($1000).**
- **EVPI = $1000 - $580 = $420**
- **We can interpret this quantity as the maximum amount that the investor should be willing to pay the clairvoyant for perfect information.**

The Expected Value of Imperfect Information

- The analysis of imperfect information parallels that of perfect information.
- We still consider the expected value of the information before obtaining it, and we will call it the *expected value of imperfect information* [EVII].
- This can be seen as notion of collecting some information from a sample.
- Some times it is called *expected value of sample of information* [EVSI].
- An example is provided next.

The Expected Value of Imperfect Information

- Example 1
  - In the investment example, suppose that the investor hires an economist who specializes in forecasting stock market trends.
  - Because he can make mistakes, however, he is not a clairvoyant, and his information is imperfect.

The Expected Value of Imperfect Information

- Example 1 (cont'd)
  - For example, suppose his track record shows that if the market actually will rise, he says "up" 80% of the time, "flat" 10%, and "down" 10%. We construct a table (Table 12.1) to characterize his performance in probabilistic terms. The probabilities therein are conditional; for example, P (Economist Says "Flat" | Flat) = 0.70.
  - The table shows that he is better when times are good (market up) and worse when times are bad (market down); he is somewhat more likely to make mistakes when times are bad.

The Expected Value of Imperfect Information

- Example 1 (cont'd)
  - How should the investor use the economist's information?
  - Figure 12.4 shows an influence diagram that includes an uncertainty node representing the economist's forecast.
  - The structure of this influence diagram should be familiar from Chapter 3; the economist's information is an example of imperfect information.

The Expected Value of Imperfect Information

- Example 1 (cont'd)
  - The arrow from "Market Activity" to "Economic Forecast" means that the probability distribution for the particular forecast is conditioned on what the market will do.
  - This is reflected in the distributions in Table 12.1. In fact, the distributions contained in the "Economic Forecast" node are simply the conditional probabilities from that table.

The Expected Value of Imperfect Information

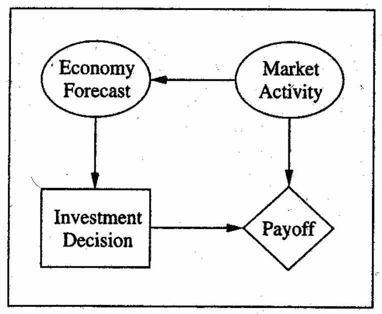
Table 12.1

Conditional probabilities characterizing economist's forecasting ability.

	True Market State		
Economist's Prediction	Up	Flat	Down
"Up"	0.80	0.15	0.20
"Flat"	0.10	0.70	0.20
"Down"	0.10	0.15	0.60
	1.00	1.00	1.00

Influence Diagram of Example 1

Figure 12.4
Imperfect information in the investor's problem.



The Expected Value of Imperfect Information

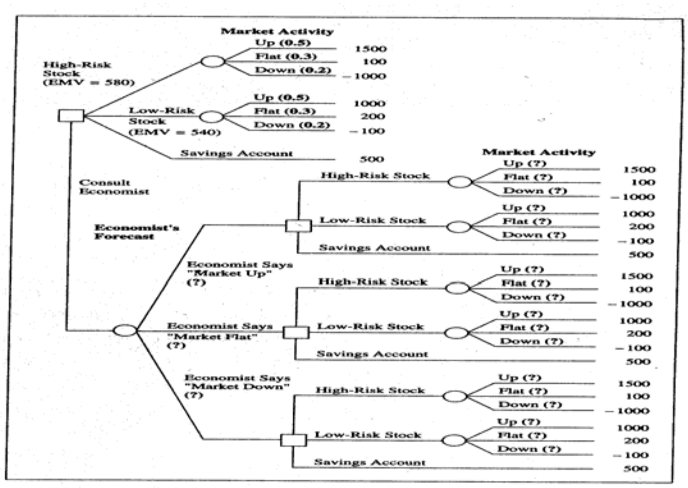
- Example 1 (cont'd)
  - Solving the influence diagram in Figure 12.4 gives the EMV associated with obtaining the economist's imperfect information before action is taken. The EMV turns out to be \$822.
  - As we did in the case of perfect information, we calculate EVII as the difference between the EMVs from Figures 12.4 and 12.1a, or the situation with no information.
  - Thus, EVII equals \$822- \$580 = \$242

The Expected Value of Imperfect Information

- Example 1 (cont'd)
  - The influence-diagram approach is easy to discuss because we actually do not see the detail calculations.
  - On the other hand, the decision-tree approach shows the calculation of EVII in its full glory.
  - Figure 12.5 shows the decision-tree representation of the situation, with a branch that represents the alternative of consulting the economist. Look at the way in which the nodes are ordered in the "Consult Economist" alternative. The first event is the economist's forecast.

The Expected Value of Imperfect Information

Figure 12.5
Incomplete decision tree for the investment example, including the alternative for consulting the economist.



The Expected Value of Imperfect Information

- Example 1 (cont'd)
  - Thus, we need probabilities P(Economist Says "Up"), P(Economist Says "Flat"), and P(Economist Says "Down").
  - Then the investor decides what to do with his money.
  - Finally, the market goes up, down, or sideways.

The Expected Value of Imperfect Information

- Example 1 (cont'd)
  - Because the "Market Activity" node follows the "Economists Forecast" node in the decision tree, we must have conditional probabilities for the market such as P(Market Up | Economist Says "Up") or P(Market Flat | Economist Says "Down"). What we have, however, is the opposite. We have probabilities such as P(Market Up) and conditional probabilities such as P(Economist Says "Up" | Market Up).

The Expected Value of Imperfect Information

- Example 1 (cont'd)
  – We must use Bayes' theorem to find the pos- tenor probabilities for the actual market outcome. For example, what is P(Market Up | Economist Says "Up")? It stands to reason that after we hear him say "up," we should think it more likely that the market actually will go up than we might have thought before.

The Expected Value of Imperfect Information

- Example 1 (cont'd)
  – We used Bayes' theorem to "flip" probabilities in Chapter 7.
  – There are several ways to think about this situation.
  – First, applying Bayes' theorem is tantamount to reversing the arrow between the nodes "Market Activity" and "Economic Forecast" in Figure 12.4.

The Expected Value of Imperfect Information

- Example 1 (cont'd)
  – In fact, reversing this arrow is the first thing that must be done when solving the influence diagram (Figure 12.6). Or we can think in terms of flipping a probability tree as we did in Chapter 7.
  – Figure 12.7a represents the situation we have, and Figure 12.7b represents what we need.

The Expected Value of Imperfect Information

Figure 12.6
First step in solving the influence diagram. We reverse the arrow between "Economic Forecast" and "Market Activity."

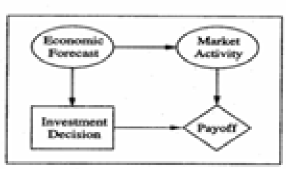
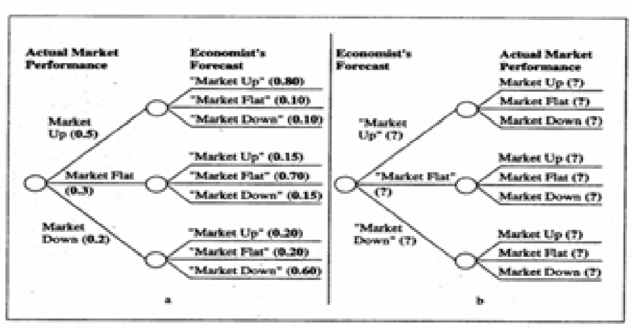


Figure 12.7
Flipping the probability tree to find posterior probabilities required for value-of-information analysis. In (a) we see what we have; in (b) we see what we need.



The Expected Value of Imperfect Information

- P(Market Up | Economist Says "Up")
- = P(Up | "Up")

$$= \frac{P(\text{"Up"} \mid Up)\,P(Up)}{P(\text{"Up"} \mid Up)\,P(Up) + P(\text{"Up"} \mid Flat)\,P(Flat) + P(\text{"Up"} \mid Down)\,P(Down)}$$

- P(Up), P(Flat), and P(Down) are the investor's prior probabilities, while P(Economist Says "Up"| Up), and so on, are the conditional probabilities shown in Table 12.1.

- From the principle of total probability, the denominator is
- P (Economist Says "Up").

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The Expected Value of Imperfect Information

Table 12.2
Posterior probabilities for market trends depending on economist's information.

	Posterior Probability for:		
Economist's Prediction	Market Up	Market Flat	Market Down
"Up"	0.8247	0.0928	0.0825
"Flat"	0.1667	0.7000	0.1333
"Down"	0.2325	0.2093	0.5581

The Expected Value of Imperfect Information

■Substituting in values for the conditional probabilities and priors,

$$P(\text{Market Up Economist Says "Up"}) = \frac{0.8(0.5)}{0.8(0.5)+0.15(0.3)+0.2(0.2)} = \frac{0.400}{0.485} = 0.8247$$

■P(Economist Says "Up") is given by the denominator and is equal to 0.485.

■ Of course, we need to use Bayes' theorem to calculate nine different posterior probabilities to fill in the gaps in the decision tree in Figure 12.5. Table 12.2 shows the results of these calculations; these probabilities are included on the appropriate branches in the completed decision tree (Figure 12.8).

■We also noted that we needed the marginal probabilities P("Up"), P("Flat"), and P("Down").

■ These probabilities are P("Up") = 0.485, P("Flat") = 0.300, and P("Down") = 0.215; they also are included in Figure 12.8 to represent our uncertainty about what the economist will say.

The Expected Value of Imperfect Information

- Example 1 (cont'd)
  - As usual, the marginal probabilities can be found in the process of calculating the posterior probabilities because they simply come from the denominator in Bayes' theorem.
  - From the completed decision tree in Figure 12.8 we can tell that the EMV for consulting the economist is \$822, while the EMV for acting without consulting him is (as before) only \$580.

The Expected Value of Imperfect Information

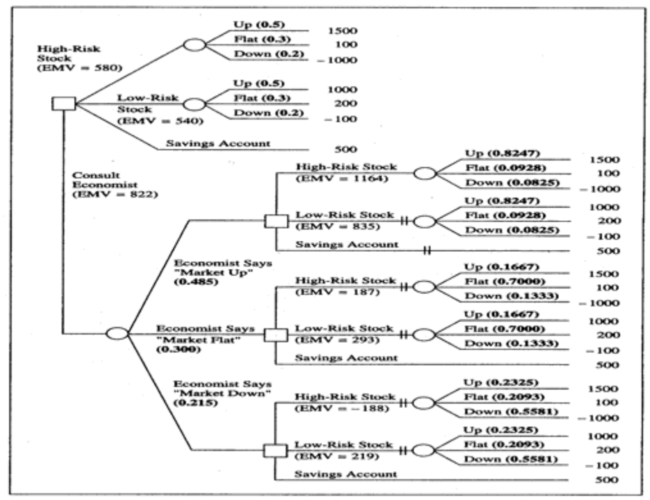
- Example 1 (cont'd)
  - The EVII is the difference between the two EMVs. Thus, EVSI is $242 in this example, just as it was when we solved the problem using influence diagrams.
  - Given this particular decision situation, the investor would never want to pay more than $242 for the economic forecast

The Expected Value of Imperfect Information

- **As with perfect information, $242 is the value of the information only in an expected-value sense.**
- **If the economist says that the market will go up, then we would invest in the high-risk stock, just as we would if we did not consult him.**
- **Thus, if he does tell us that the market will go up, the information turns out to do us no good. But if he tells us that the market will be flat or go down, we would put our money in the savings account and avoid the relatively low expected value associated with the high-risk stock.**
- **In those two cases, we would "save" 500 -187 = 313 and 500 -(-188) = 688, respectively, with the savings in terms of expected value.**
- **Thus, EVSI also can be calculated as the "expected incremental savings," which is 0(0.485) + 313(0.300) + 688(0.215) = 242.**

The Expected Value of Imperfect Information

Figure 12.8 *Completed decision tree for the investment example.*



Value of Information and Experts

- In Chapter 8 we discussed the role of experts in decision analysis. One of the issues that analysts face in using experts is how to value them and how to decide how many and which experts to Consult.
- Because experts tend to read the same journals, go to the same conferences, use the same techniques in their studies, and even communicate with each other, it comes as no surprise that the information they provide can be highly redundant.

Value of Information and Experts

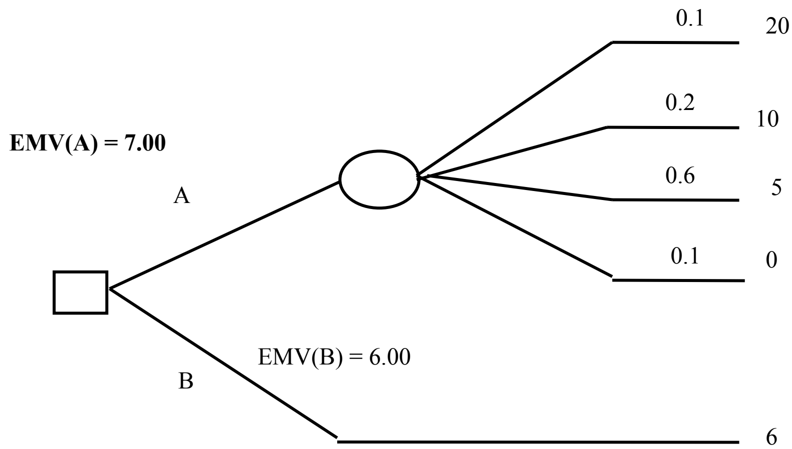
- The real challenge in expert use is to recruit experts who look at the same problem from very different perspectives. Recruiting experts from different fields, for example, can be worthwhile if the information provided is less redundant.

Value of Information and Experts

- It can even be the case that a highly diverse set of less knowledgeable (and less expensive) experts can be much more valuable than the same number of experts who are more knowledgeable (and cost more) but give redundant information!

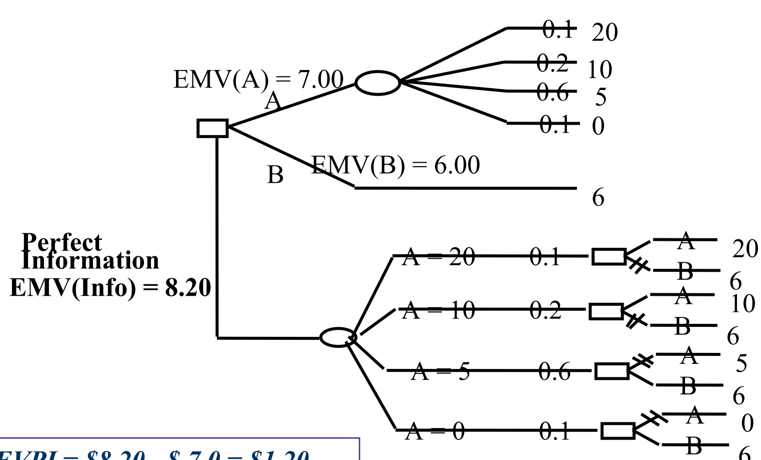
Additional Example

- Example 12.2
  - Calculate the EVPI of this decision tree:



Additional Example

- Example 12.2 (cont'd)



EVPI = $8.20 - $ 7.0 = $1.20

Efficiency between EVPI and EVII

- If we consider that EVPI is 100% then
- the efficiency of sampling or imperfect information can be calculated by the following formula:

$$\text{Efficiency} = \frac{\text{EVII}}{\text{EVPI}}$$

- Ex: Efficiency = 15.280 / 17.40 = **88%**
- That is to say that EVII is as 88 % accurate as EVPI.

Calculating EVPI and EVII with Precision Tree

- Performing EVPI calculations involves nothing more than reordering nodes in a decision tree.
- This is easy to do in any decision-tree program, and it can be done for any combination of timing and event information simply by restructuring the tree.

Calculating EVPI and EVII with Precision Tree

- DPL has no specific EVPI command but does provide a menu command called "Reorder Node," which facilitates restructuring the tree for EVPI calculations.
- One final comment about DPL: Once the decision tree has been created, EVPI calculations must be done by restructuring the tree; adding arcs in the influence-diagram view will not do the trick, because DPL uses the decision tree to specify sequencing of the nodes.
- The Precision Tree can help calculating EVPI and EVII

Calculating EVPI and EVII with Precision Tree

- Read a detailed example in text pages 512 – 517