

Solution to Homework Set #8
ENCE 627 – Decision Analysis for Engineering - Fall 2003

Assigned T, 11/18 Due T, 12/2

Problem 1

Textbook (CR): 5.10

*** SOLUTION ***

The model is a linked tree where the uncertainty node for the amount of fun is linked to cell \$F\$6 in the spreadsheet model (“Fun Level for Forest Job”), and the uncertainty node for the amount of work is linked to cell \$G\$7 in the spreadsheet model (“Salary Level for In-town Job”). The outcome nodes for the Forest Job are linked to cell \$F\$8 and the outcome nodes for the In-Town Job to cell \$G\$8. The user can then vary the weights to see that Sam will still prefer the forest job. The sensitivity analysis gives the following results:

	Expected Overall Score	
k_s	Forest Job	In-Town Job
0.50	71.25	57.50
0.75	76.125	56.25

Thus, regardless of the precise value of k_s , the optimal choice is the forest job. In fact, a much stronger statement can be made; it turns out that for no value of k_s between zero and one is the in-town job preferred. Smaller values of k_s favor the in-town job, but even setting $k_s = 0$ leaves the expected overall scores equal to 60 and 61.5 for the in-town and forest jobs, respectively.

Another way to show the same result is to realize that the expected overall scores are linear in the weights and in the expected scores for the individual attributes. Because the forest job has higher expected scores on both attributes, there cannot exist a set of weights that makes the in-town job have the higher overall expected score.

Problem 2

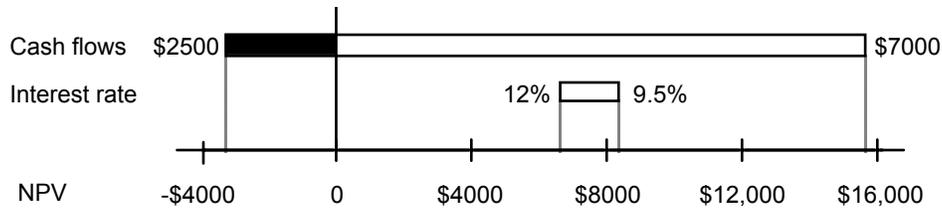
Textbook (CR): 5.11

*** SOLUTION ***

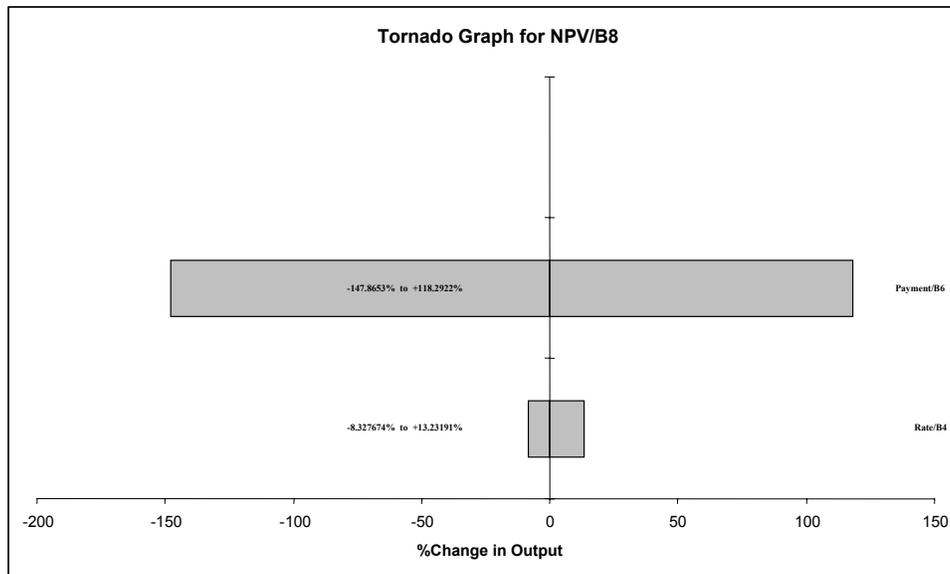
Using the base values of \$5000 for the cash flows and 11% for the interest rate,

$$\begin{aligned} \text{NPV} &= -14000 + \frac{5000}{1.11} + \frac{5000}{1.11^2} + \dots + \frac{5000}{1.11^6} \\ &= \$7153. \end{aligned}$$

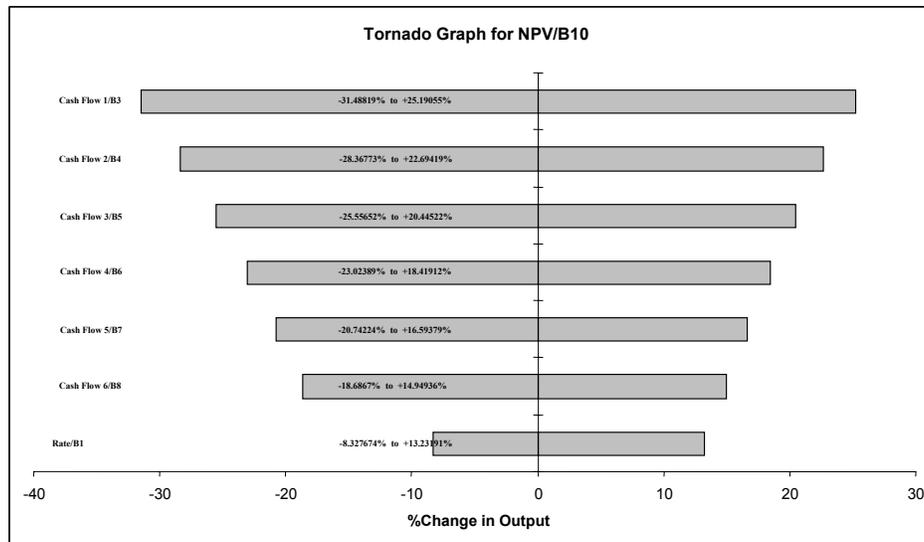
When the cash flows are varied from \$2500 to \$7000, and the interest rate is varied from 9.5% to 12%, the following tornado diagram results:



This graph assumes that while the cash flows may vary between \$2500 and \$7000, whatever the amount, it occurs each year. The range of possible interest rates appears not to pose a problem; NPV remains positive within a relatively narrow range. On the other hand, NPV is very sensitive to the range of cash flows. It would be appropriate to set the interest rate at 11% for the analysis but to model the uncertainty about the cash flows with some care. The tornado diagram generated by TopRank shows the difference as a percentage change of NPV, not by the absolute amount. You cannot get the absolute amounts from the graph, but if you click on the “Actuals” button in the results window, TopRank will provide these values. This model is shown in the Excel file “Problem 5.11.xls”.



The problem can also be modeled where each year can vary independently between \$2500 and \$7000. This model is shown in the Excel file “Problem 5.11 alt.xls”. In the student version, you cannot create both models in the same spreadsheet or have both files open at the same time. This will give you an error “Model Extends Beyond Allowed Region of Worksheet,” and will crash the program and require restarting the computer.



Because of the discounting factor, the early years are more sensitive than the later years.

When using TopRank, the program automatically tries to create a name for each input and output cell by scanning the spreadsheet around the cell where the input/output is located and combining any row and column headings. These are often nonsensical, and can be changed by clicking in the name box in the inputs by outputs dialog box and entering the desired name. After the new names are entered, a what-if analysis must be run for the new names to take effect.

To use the graphs that are generated in reports, use the menu command “Graph in Excel.” This command places the selected graph data points in Excel and thus a native format graph is created on the worksheet from the data points.

Problem 3

Textbook (CR): 6.7

*** SOLUTION ***

Answers will vary, but look for ways to accomplish the various objectives. For example, a balanced strategy would include ways to increase the use of safety features and ways to minimize accidents. How about an education program promoting the use of safety features? Or the development of a set of laws requiring adults to take refresher driving courses (education) if they accumulate too many traffic tickets?

Problem 4

Textbook (CR): 11.1

*** SOLUTION ***

Constructing a simulation model can help a decision maker specify clearly what is uncertain in the environment and the nature of the uncertainty. This is part of the structuring phase of decision analysis, and insight can be gained in the process of iterative modeling and analysis with the goal of reaching a requisite decision model. The analysis itself (running the simulation) can produce simulation-generated probability

distributions for those uncertain quantities in which the decision maker is interested (e.g., NPV, profit, payoff, cost).

Problem 5

The rate of steady water flow per time unit at a constant depth in a prismatic open channel can be estimated using the following Manning formula:

$$Q = \frac{C_m}{n} AR^{2/3} S^{1/2}$$

where C_m is a constant of value = 1 in SI system, n = Manning roughness factor, R = hydraulic radius, and S = slope of the bottom of the channel. Consider a trapezoidal cross section made with gravel. Assume C_m is a constant. However, n , A , R , and S are statistically independent random variables with mean values of 0.029, 8.0 m², 1.1 m, and 0.003, respectively, and the corresponding COV 's are 0.30, 0.1, 0.1, and 0.1, respectively. No distributional information is available.

- (a) Determine the first-order mean and standard deviation of Q .
- (b) Determine the second-order mean of Q using the following equation:

$$E(Y) \approx g(\mu_{X_1}, \mu_{X_2}, \dots, \mu_{X_n}) + \frac{1}{2} \sum_{i=1}^n \left(\frac{\partial^2 g}{\partial X_i^2} \right) \text{Var}(X_i)$$

- (c) Compare the results of Parts (a) and (b).

***** SOLUTION *****

- (a) First-order mean:

$$E(Q) \approx \frac{1}{0.029} (8)(1.1)^{2/3} (0.003)^{1/2} = 16.1 \frac{\text{m}^3}{\text{s}}$$

First-order variance:

$$\begin{aligned} \text{Var}(Q) &= \left(\frac{1}{0.029^2} (8)(1.1)^{2/3} (0.003)^{1/2} \right)^2 (0.029 \times 0.3)^2 + \left(\frac{1}{0.029} (8)(1.1)^{2/3} (0.003)^{1/2} \right)^2 (8 \times 0.1)^2 \\ &\quad + \left(\frac{1}{0.029} (8) \left(\frac{2}{3} \right) (1.1)^{-1/3} (0.003)^{1/2} \right)^2 (1.1 \times 0.1)^2 + \left(\frac{1}{0.029} (8)(1.1)^{2/3} \left(\frac{1}{2} \right) (0.003)^{1/2} \right)^2 (0.003 \times 0.1)^2 \\ &= 23.33 + 2.59 + 1.15 + 0.65 = 27.72 \end{aligned}$$

Therefore, standard deviation $\sigma_Q = 5.265 \frac{\text{m}^3}{\text{s}}$

- (b) Second-order mean:

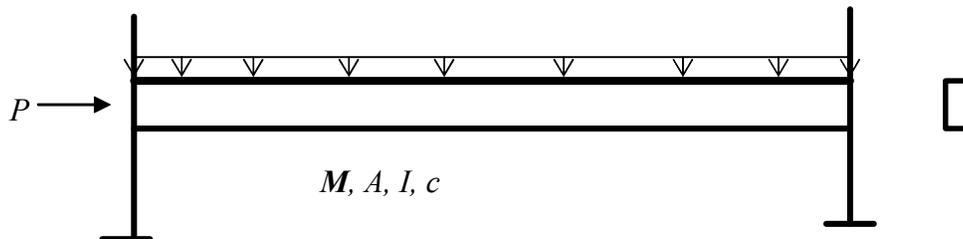
$$\begin{aligned}
E(Q) &\approx 16.1 + \frac{1}{2} \left[\left(\frac{(2)(1)}{0.029^2} (8)(1.1)^{2/3} (0.003)^{1/2} \right) (0.029 \times 0.3)^2 \right] \\
&\quad + (8 \times 0.1)^2 \times 0 + \left(\frac{1}{0.029} (8) \left(\frac{2}{3} \right) \left(-\frac{1}{3} \right) (1.1)^{-4/3} (0.003)^{1/2} \right) (1.1 \times 0.1)^2 \\
&\quad + \left(\frac{1}{0.029} (8) (1.1)^{2/3} \left(\frac{1}{2} \right) \left(\frac{-1}{2} \right) (0.003)^{-3/2} \right) (0.003 \times 0.1)^2 \\
&= 16.1 + \frac{1}{2} (2.898 + 0 - 0.036 - 0.04) = 17.51 \frac{\text{m}^3}{\text{s}}
\end{aligned}$$

Comparison of means: $\left| \frac{16.1 - 17.51}{17.51} \right| \times 100 = 8.05\% \text{ error}$

Problem 6

Investigate the compressive stress at the extreme fiber of a steel beam shown below using Monte Carlo simulation on a spreadsheet. The compressive stress at the extreme fiber is given by

$$\sigma = \left(\frac{P}{A} + \frac{Mc}{I} \right) \leq F_y$$



where σ = computed compressive stress, c = distance from the neutral axis to the extreme fiber, P = applied axial load, M = applied moment due to external loads, A = cross sectional area of the beam, I = centroidal moment of inertia of the cross section, and F_y = yield strength of steel. The variables c , P , M , A , I , and F_y are called basic random variables. The probabilistic characteristics of these variables are provided in the table shown below (Table 1).

1. Simulate the stress σ for 1,000, 5,000, and 10,000 cycles.
2. Determine the descriptive measures (i.e., mean value and standard deviation) of the compressive stress σ based on the simulated values for each cycle.
3. Compare the three values of the mean and the three values of the standard deviation. Which values for the mean and standard deviation are the most accurate? Why?
4. Based on the 10,000 cycles of simulation, plot a histogram for σ .
5. Based on the 10,000 cycles of simulation, compute the stress failure probability, that is $P(\sigma < F_y)$.

Table 1. Probabilistic Characteristic of Basic Random Variables for the Stress σ

Random Variable	Mean	Coefficient of Variation (<i>COV</i>)	Distribution Type
C (in)	10	0.05	Normal
P (kip)	300	0.20	Lognormal
M (kip-in)	3,000	0.30	Lognormal
A (in ²)	200	0.05	Normal
I (in ⁴)	1,500	0.08	Normal
F_y (ksi)	36	0.15	Lognormal

*** SOLUTION ***

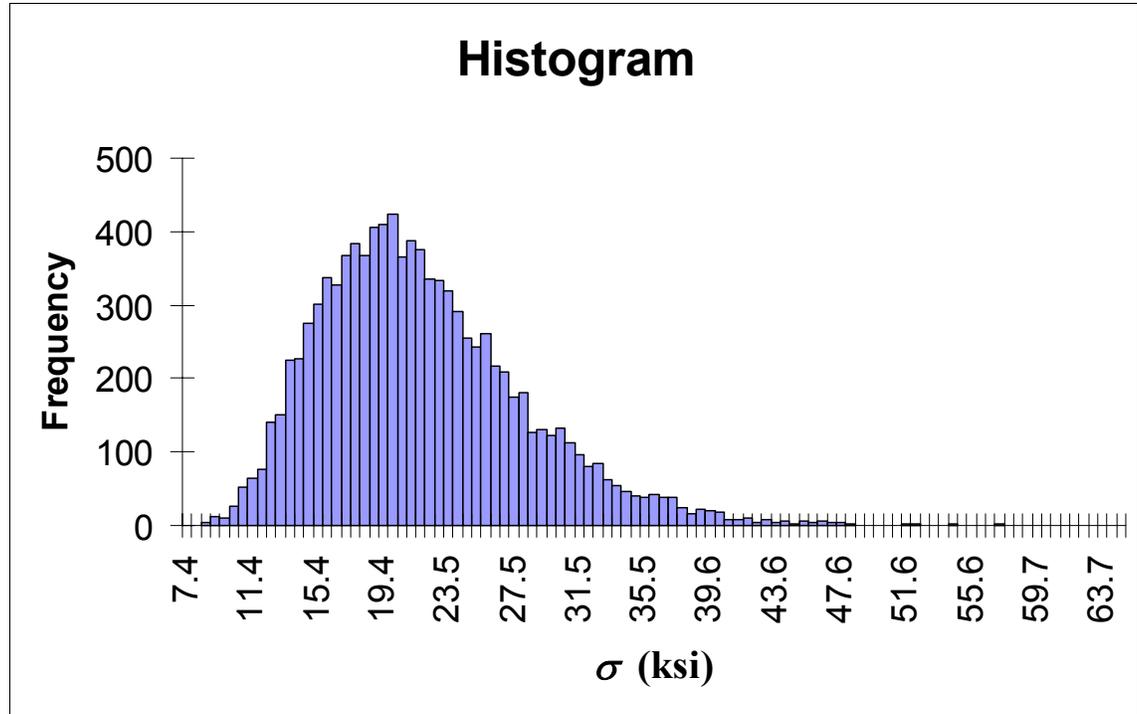
1. Samples of simulated values for σ .

Cycle	σ
1	12.66361
2	19.98396
3	31.76617
4	29.37954
5	16.08795
6	16.69813
.	.
.	.
9,997	30.19026
9,998	29.11275
9,999	36.85558
10,000	14.05618

2. Descriptive Measures for σ .

# Cycles	Mean	Standard Deviation
1,000	21.56764	6.519653444
5,000	21.58783	6.390889703
10,000	21.59032	6.418730263

3. The most accurate values are those values based on 10,000 cycles. As the number of simulation cycles increases, the accuracy increases.
4. Histogram for σ based on 10,000 cycles:



5. Failure Probability:

The failure probability $P(\sigma < F_y)$ is approximately 0.0484.

Problem 7

Textbook (CR): 11.8 (solve using the program @RISK)

***** SOLUTION *****

11.8. The process for simulating the call option is as follows:

1. Generate a normal random number (Stock price) with mean \$45.50 and standard deviation \$5.00.
2. Calculate the option value with the following IF statement:

$$\text{IF}(\text{Stock price} < \$45, 0, 100 * (\text{Stock price} - \$45.00))$$

This statement checks the price of the stock. If the price is less than \$45, then the option is worthless (0). If the price is greater than \$45, then the value is 100 times the difference between the stock price and the exercise price.

3. Repeat steps 1 and 2 many times.
4. With 1000 iterations, we obtained $E(\text{Option value}) = \221.89 , standard deviation = \$329.65.

To run the @RISK model, make sure that you have the @RISK add-in open, and then click the "Start Simulation" button. The results of 1 trial simulation of 100 iterations appear in the "Results" sheet in this workbook. Cell \$B\$5 (stock price) was defined as an Input for the simulation with a Normal Distribution(45.5, 5), and Cell \$B\$6 (option value) was defined as the Output cell for the simulation.

Problem 11.8- Excel Solution
 Making Hard Decisions with Decision Tools by Robert T. Clemen & Terence Reilly

Stock Price \$45.50
 Option Value \$50.00

To run the @Risk simulation, make sure that you have the @Risk add-in open, and then click the "Start Simulation" button. The results of 1 trial simulation of 100 iterations appear in the "Results" sheet in this workbook.

Name	Option Value	Stock Price
Description	Output	RiskNormal(45.5, 5)
Cell	B6	B5
Minimum	0	28.18242
Maximum	1651.126	61.51126
Mean	225.368	45.49932
Std Deviation	308.5604	4.998631
Variance	95209.53	24.98631
Skewness	1.478505	-5.15E-03
Kurtosis	4.73402	2.970535
Errors Calculated	0	0
Mode	0	44.6827
5% Perc	0	37.24318
10% Perc	0	39.06855
15% Perc	0	40.30712
20% Perc	0	41.27778
25% Perc	0	42.12525
30% Perc	0	42.86743
35% Perc	0	43.57178
40% Perc	0	44.22203
45% Perc	0	44.87004
50% Perc	49.55032	45.4955
55% Perc	111.8176	46.11818
60% Perc	177.0539	46.75449
65% Perc	241.8808	47.41881
70% Perc	312.4673	48.11086
75% Perc	387.0522	48.87052
80% Perc	469.1323	49.69132
85% Perc	568.3192	50.68187
90% Perc	689.5953	51.89595
95% Perc	874.3171	53.69205
Filter Minimum		
Filter Maximum		
Type (1 or 2)		
# Values Filtered	0	0
Scenario #1	>75%	
Scenario #2	<25%	
Scenario #3	>90%	