Homework Set #3

ENCE 627 – Decision Analysis for Engineering - Fall 2003

Assigned T, 9/30 Due T, 10/7

Problem 1

There are two cars in the garage. The operating condition of each car can be described as excellent (E), good (G), or bad (B). Identify all possible combinations of operating conditions of these two cars (i.e., identify all sample points). Suppose there are five cars, and each car can have the three conditions just described. Calculate the total number of sample space.

***** SOLUTION *****

The total number of sample points in the sample space = $3^2 = 9$.
E = excellent, G = good, and B = bad.

$$S = \begin{Bmatrix} \text{EE}, & \text{EG}, & \text{EB}, \\ \text{GE}, & \text{GG}, & \text{GB}, \\ \text{BE}, & \text{BG}, & \text{BB} \end{Bmatrix}$$

For 5 cars, each with 3 possible operating conditions, the total number of sample points is $3^5 = 243$.

Problem 2

Piles are used to support the foundation of a structure. The failure probability of a pile during proof testing is 0.1. Determine the following:

(a) the probability of three failed piles to out of ten tested piles;
(b) The probability of no failures in 20 tested piles;
(c) The probability of 10 tested piles to obtain the first failure; and
(d) The probability of 3 consecutive pile failures.

***** SOLUTION *****

Use the binomial distribution with the following parameters:

$p = 0.1$

(a) $\binom{10}{3} p^3 (1-p)^{10-3} = \dfrac{10!}{3!7!}(0.1)^3(0.9)^7 \approx 0.0574$

(b) $\binom{20}{0} p^0 (1-p)^{20-0} = (0.9)^{19} \approx 0.1216$

(c) Geometric: $(1-p)^9 p^1 = (1-0.1)^9 (0.1)^1 \approx 0.0387$

(d) $p^3 = (0.1)^3 = 0.001$

Problem 3

The probability that a flood of a specified magnitude occurs in any 1 year is 0.05. What is the probability that in the next 10 years:

(a) Exactly two such floods will occur;
(b) No more than one such flood will occur;
(c) No such floods will occur;
(d) At least four floods will occur; and
(e) At least three but no more than six such floods will occur.

***** SOLUTION *****

Use Poisson distribution with the following parameter:

$$\lambda = 0.05, \qquad t = 10, \qquad \lambda t = 0.5$$

Therefore,

$$P(X = x) = \frac{(\lambda t)^x e^{-\lambda t}}{x!} = \frac{1^x e^{-1}}{x!}$$

(a) $$P(X = 2) = \frac{0.5^2 e^{-0.5}}{2!} = 0.075816$$

(b) $$P(X \leq 1) = P(X = 0) + P(X = 1)$$
$$= \frac{0.5^0 e^{-0.5}}{0!} + \frac{0.5^1 e^{-0.5}}{1!} = 0.60653 + 0.303265 = 0.909795$$

(c) $$P(X = 0) = \frac{0.5^0 e^{-0.5}}{0!} = 0.60653$$

(d) $$P(X \geq 4) = 1\text{- } P(X \leq 3) = \sum_{i=0}^{3} \frac{0.5^i e^{-0.5}}{i!}$$

(e) $$P(3 \leq X \leq 6) = \sum_{i=3}^{6} \frac{0.5^i e^{-0.5}}{i!}$$

Problem 4

Textbook (CR): 9.13

***** SOLUTION *****

Because of independence among arrivals, the probability distribution for arrivals over the next 15 minutes is independent of how many arrived previously. Thus, for both questions,

$P_P(X = 1 \text{ in 15 minutes} \mid m = 6 \text{ per hour})$
$= P_P(X = 1 \text{ in 15 minutes} \mid m = 1.5 \text{ per 15 minutes})$
$= 0.335$ from Appendix C.

Using RISKview, select a function as the distribution source, the Poisson as the distribution type, and 1.5 for the λ parameter. In the statistics grid, set the Left X value to 0.5 and the Right X value to 1.5. The Difference P value then shows the desired probability: 33.47%.

Problem 5

The probability of failure of a structure due to earthquake is estimated as 10^{-5} per year. Assuming that the design life of the structure is 60 years and the probability of failure in each year remains constant and independent during its life, what is the probability of failure of the structure in the 10^{th} year?

***** SOLUTION *****

Consider Geometric distribution: (first failure):

$$\mathrm{P}(X=10)=p(1-p)^{x-1}=10^{-5}\left(1-10^{-5}\right)^{10-1}=9.999\times10^{-6}$$

Problem 6

Textbook (CR): 9.25 (only Parts a, b, and c)

***** SOLUTION *****

(a) No, because the second investment is substantially riskier as indicated by the higher standard deviation.

(b) It makes some sense. We need a single peak (mode) and a reasonably symmetric distribution for the normal distribution to provide a good fit. Returns can be positive or negative, and we might expect deviations around a central most-likely value to be evenly balanced between positive and negative deviations.

(c) $P(R_1 < 0\%) = P_N(R_1 \leq 0 \mid \mu = 10, \sigma = 3)$

$= P(Z \leq \frac{0 - 10}{3}) = P(Z < -3.33) = 0.0004.$

To use RISKview, select a function as the distribution source, the Normal as the distribution type, 0.1 for the μ parameter, and 0.3 for the σ parameter. Set the Left X value to 0, then the Left P value is 0.04%.

$P(R_2 < 0\%) = P_N(R_2 \leq 0 \mid \mu = 20, \sigma = 12)$

$= P(Z \leq \frac{0 - 20}{12}) = P(Z < -1.67) = 0.0475.$

To use RISKview, select a function as the distribution source, the Normal as the distribution type, 0.2 for the μ parameter, and 0.12 for the σ parameter. Set the Left X value to 0, then the Left P value is 4.78%.

$P(R_1 > 20\%) = P_N(R_1 > 20 \mid \mu = 10, \sigma = 3)$

$= P(Z > \frac{20 - 10}{3}) = P(Z > 3.33) = 0.0004.$

To use RISKview, select a function as the distribution source, the Normal as the distribution type, 0.1 for the μ parameter, and 0.3 for the σ parameter. Set the Left X value to 0.2 and the Right X value to 0.5, then the Difference P value is 0.04%.

$$P(R_2 < 10\%) = P_N\,(R_2 < 10 \mid \mu = 20, \sigma = 12)$$
$$= P\left(Z \le \frac{10 - 20}{12}\right) = P(Z < -0.83) = 0.2033.$$

<u>Problem 7</u>

Textbook (CR): 9.31

***** SOLUTION *****

Let L denote the uncertain length of an envelope.

(a) $P_N\,(L > 5.975 \mid \mu = 5.9, \sigma = 0.0365) = P\left(Z > \frac{5.975 - 5.9}{0.0365}\right) = P(Z > 2.055) = 0.02$

To use RISKview, select a function as the distribution source, the Normal as the distribution type, 5.9 for the μ parameter, and 0.0365 for the σ parameter. Set the Left X value to 5.975 and the Right X value to 8. The Difference P value is the desired probability: 1.99

(b) We will use a binomial model, in which p = P(Envelope fits) = 0.98. We need $P_B(R \le 18 \mid n = 20, p = 0.98) = P_B(R \ge 2 \mid n = 20, p = 0.02)$

$$= 1 - P_B(R \le 1 \mid n = 20, p = 0.02) = 1 - 0.94 = 0.06$$

<u>Problem 8</u>

The Reynolds number, R_e, for a pipe flow was found to follow lognormal distribution with a mean value and standard deviation of 2650 and 700, respectively. The flow can be classified as laminar flow if $R_e < 2700$ and as turbulent flow if $R_e > 4000$. Find the probability that the flow is in transitional state.

***** SOLUTION *****

$$\mu_{R_e} = 2650,\ \sigma_{R_e} = 700,$$
$$\sigma_Y^2 = \ln\left[1 + \left(\frac{\sigma_{R_e}}{\mu_{R_e}}\right)^2\right] = \ln\left[1 + \left(\frac{700}{2650}\right)^2\right] = 0.06745$$
$$\sigma_Y = 0.2597$$
$$\mu_Y = \ln(\mu_{R_e}) - \frac{1}{2}\sigma_Y^2 = \ln(2650) - \frac{1}{2}\sigma_Y^2 = 7.8486$$
$$P(2700 < R_e < 4000) = \Phi\left(\frac{\ln 4000 - 7.8486}{0.2597}\right) - \Phi\left(\frac{\ln 2700 - 7.8486}{0.2597}\right)$$
$$\simeq \Phi(1.72) - \Phi(0.20) = 0.957284 - 0.57926 = 0.378024$$

Problem 9

The extreme wave height that is used in the design of an offshore facility for a design life of 30 years is a random variable with a mean of 25 ft and standard deviation of 5 ft. Determine the probability that the extreme wave height will exceed 30 ft in 30 years. Assume that the extreme wave height follows the largest extreme value distribution of type.

***** SOLUTION *****

Using a mean of 25 ft and standard deviation of 5 ft, the parameters of extreme (largest) type I are:

$$\alpha_n = \sqrt{\frac{\pi^2}{6(5)}} = 0.25638 \quad \text{and} \quad \mu_n = 25 - \frac{0.577216}{0.25638} = 22.7486$$

Therefore, the following probability can be computed:

$P(X > 30) = 1 - F_X(30) = 1 - \exp(-\exp(-0.25638\ (30 - 22.7486)))$

$= 1 - 0.855721 = 0.1443$