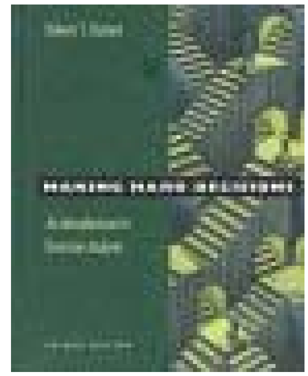
UNIVERSITY OF MARYLAND

Department of Civil & Environmental Engineering
ENCE 627 – Decision Analysis for Engineering

<u>Honor Pledge Code</u>

"I pledge on my honor that I have not given or received any unauthorized assistance on this examination."

Signature: ___________________________________



Grading:

Problem 1:	______	/ 20
Problem 2:	______	/ 20
Problem 3:	______	/ 20
Problem 4:	______	/ 20
Problem 5:	______	/ 20
Total:	______	/ 100

Solution to Midterm Exam
(Open Book & Open Class Notes)

Tuesday, October 28, 2003
3:30 PM – 6:00 PM, EGR 3102

Instructor: Dr. I. Assakkaf

"Show your work & state all your assumptions"

Student Name:____________________SAMPLE________________

SSN:__123-45-6789_

Grade: ___**100**___ ☺

Problem 1 (20 points)

If P(A) = 0.68, P($B \mid A$) = 0.30, and P($B \mid \overline{A}$) = 0.02,

(a) Compute the following:

$\mathrm{P}(\overline{A})$, $\mathrm{P}(A \text{ and } B)$, and $\mathrm{P}(\overline{A} \text{ and } B)$

(b) Use the given data and the result of Part (a) to complete the construction of the following probability table for the random variables A and B (i.e., fill in the missing entries):

	B	$\overline{B}$	
A	**0.204**	**0.476**	0.680
$\overline{A}$	**0.006**	**0.314**	**0.320**
	0.210	**0.790**	1.000

(c) Now use the probability table to find P(A or B), that is the probability where either A occurs or B occurs (or both).

***** SOLUTION *****

(a)

$\mathrm{P}(\overline{A}) = 1 - \mathrm{P}(A) = 1 - 0.68 = 0.320$

P(A and B) = P (B and A) = P(B | A) P(A) = 0.30 (0.68) = **0.204**

P($\overline{A}$ and B) = P(B and $\overline{A}$) = P(B | $\overline{A}$) P($\overline{A}$) = 0.02 (0.32) = **0.006**

(b) Probability Table:

P(A and $\overline{B}$) = 0.680 – 0.204 = 0.476

P($\overline{A}$ and $\overline{B}$) = 0.320 – 0.006 = 0.314

P(B) = 0.204 + 0.006 = 0.210

P($\overline{B}$) = 0.476 + 0.314 = 0.790

(c) P(A or B) = 0.204 + 0.476 + 0.006 = **0.686**

Problem 2 (20 points)
Should you drop your decision-analysis (DA) course? Suppose you faced the following problem: if you drop the course, the anticipated salary in your best job offer will depend on your current GPA:

$$\text{Anticipated Salary} \mid \text{Drop} = (\$4000 \times \text{Current GPA}) + \$16{,}000$$

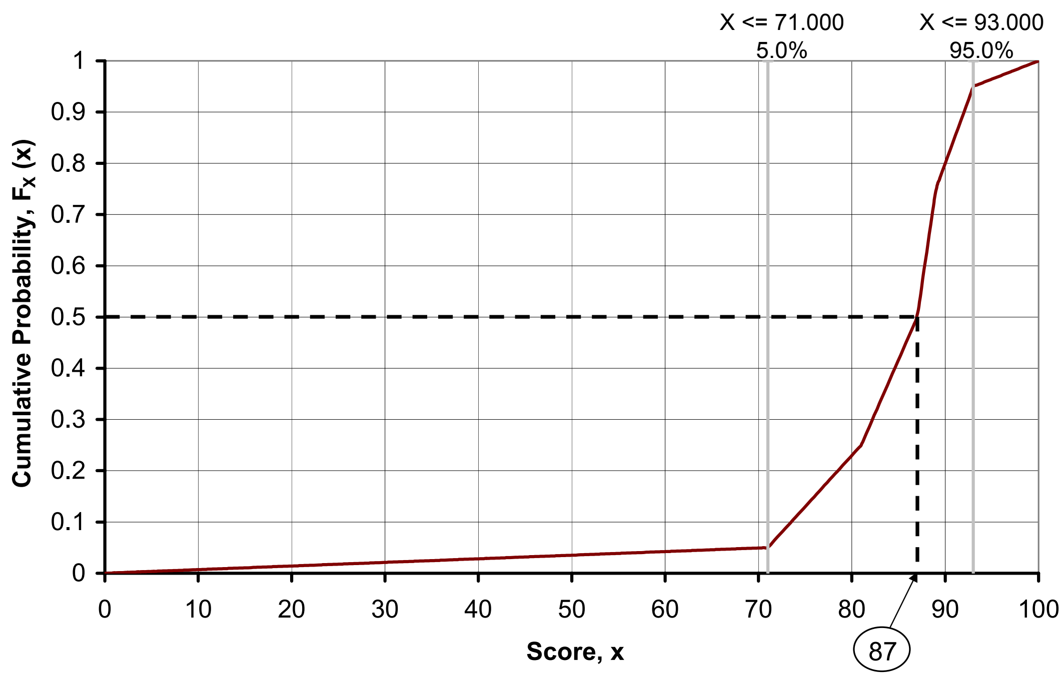
If you take the course, the anticipated salary in your best job offer will depend on both your current GPA and your overall score (on a scale from 0 to 100) in the course:

$$\text{Anticipated Salary} \mid \text{Do Not Drop} = 0.6\,(\$4000 \times \text{Current GPA}) + 0.4\,(\$170 \times \text{Course Score}) + \$16{,}000$$

The problem is that you do not know how well you will do in the course. Suppose that your current GPA is 2.9 and you have been able to assess a continuous probability distribution for your score as shown in the figure below based on a criterion that implies 90 – 100 is an A, 80-89 a B, 70-79 a C, 60-69 a D, and 0-59 an F, determine

(a) Your expected DA score using the Pearson-Tukey method;
(b) The expected salary if you drop the course;
(c) The expected salary if you do not drop the course; and
(d) Whether or not to drop the course (Justify your answer)

***** SOLUTION *****



(a) The Pearson-Tukey method approximates the expected DA score as:

From the graph, find the score that corresponds to P = 0.5 will be the score of x ≈87, therefore,

$E_{P\text{-}T}(\text{DA Score}) \approx 0.185\,(71) + 0.63\,(87) + 0.185\,(93) = \mathbf{85.15}$

(b) Expected salary if drop the course:
E(Salary | Drop Course) = \$4000 (2.9) + \$16,000 = **\$27,600**

(c) Expected salary if not drop the course:
E(Salary | Don't drop) = 0.6 (\$4000 × GPA) + 0.4 (\$170 ×EP-T(DA Score)) + \$16,000
= 0.6 (\$4000 × 2.9) + 0.4 (\$170 × 85.15) + \$16,000= **\$28, 750**

(d) Since {E(Salary | Don't drop) = \$28,750} > {E(Salary | Drop Course) = \$28,750}
Thus, the optimal choice is **<u>not to drop the course</u>**.

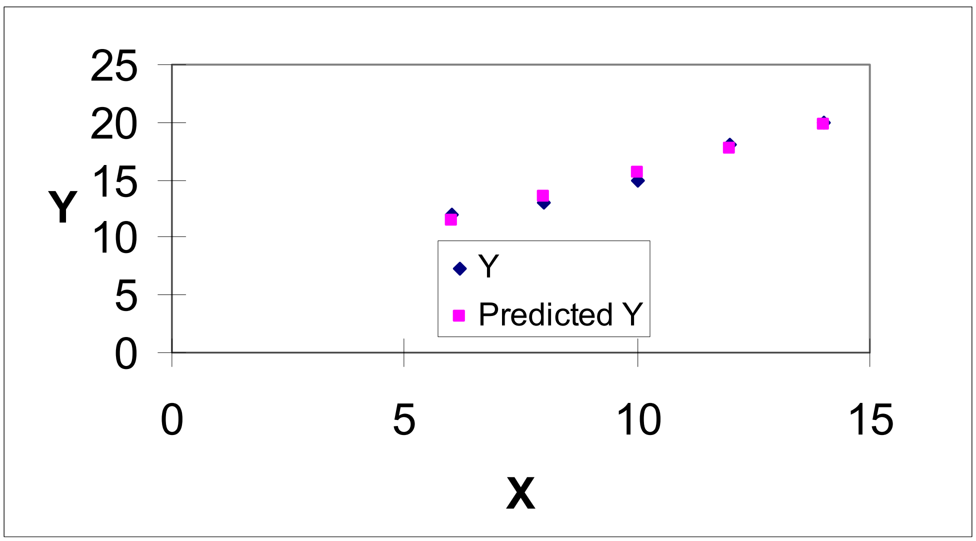
Problem 3 (20 points)
Use linear regression to fit the model $\hat{y} = b_0 + b_1 x$ in the following data:

x	6	8	10	12	14
y	12	13	15	18	20

(a) Plot the data-based CDF of Y;
(b) Determine the coefficients b_0 and b_1 using least squares fit;
(c) Compute the coefficient of variation R; and
(d) Determine the standard error S_e.

***** SOLUTION *****

(a) The plot:



(b) $b_0 = \mathbf{5.100}$ and $b_1 = \mathbf{1.050}$

(c) $R = \mathbf{0.988}$

(d) $S_e = \mathbf{0.60553}$

Problem 4 (20 points)

(A) A company owns two different computers, which are in *separate buildings* and operated entirely separately. Based on past history, Computer A is expected to break down 5.0 times a year, with a variance of 6, and costing \$200 per breakdown. Computer B is expected to break down 3.6 times per year, with a variance of 7, and costing \$165 per breakdown. What is the company's expected cost for computer breakdowns and the standard deviation of the breakdown cost? What assumption must you make to find the standard deviation? Is this a reasonable assumption?

(B) A friend asks you for a loan of \$1000 and offers to pay you back at the rate of \$90 per month for 12 months. Using an annual rate of 10%, find the net present value *NPV* (to you) of loaning your friend the money.

***** SOLUTION *****

(A) Let X_A = random number of breakdowns for Computer A, and X_B = random number of breakdowns for Computer B.

$$\text{Cost} = \$200\,(X_A) + \$165\,(X_B)$$

$$\text{E(Cost)} = \$200\,\text{E}(X_A) + \$165\,\text{E}(X_B) = \$200\,(5) + \$165\,(3.6) = \mathbf{\$1594}$$

If X_A and X_B are independent, then

$$\begin{aligned}\text{Var(Cost)} &= 200^2\,\text{Var}(X_A) + 165^2\,\text{Var}(X_B) = 200^2\,(6) + 165^2\,(7)\\ &= 430{,}575 \text{ “dollars squared”}\end{aligned}$$

$$\sigma\text{Cost} = \sqrt{430{,}575 \text{ “dollars squared”}} = \mathbf{\$656.18}$$

The assumption made for the variance computation (or equally the standard deviation) is that the computers break down independently of one another. Given that they are in separate buildings and operated separately, this seems like a reasonable assumption.

(B) If the annual rate = 10%, then the monthly (periodic) rate i = 10% / 12 = 0.83%.

$$\text{NPV}(0.83\%) = -1000 + \frac{90}{1.0083} + \frac{90}{1.0083^2} + \ldots + \frac{90}{1.0083^{12}}$$

$$= \mathbf{\$23.71}$$

Or

$$\text{NPV}(0.83\%) = -P + A\left(\frac{(1+i)^n - 1}{i(1+i)^n}\right) = -\$1000 + \$90\left(\frac{(1+0.0083)^{12} - 1}{0.0083(1+0.0083)^{12}}\right) = \mathbf{\$23.92}$$

Problem 5 (20 points)

(A) The compressive strength of concrete specimen follows normal distribution with a mean value (μ) of 2.4 ksi and a coefficient of variation (COV) of 0.2. If the applied stress is 2.5 ksi, find the probability of failure, that is P(x <2.5).

(B) A continuous random variable X has the following uniform density function:

$$f_X(x) = \begin{cases} 0.5 & \text{for } 3 \le x \le 5 \\ 0 & \text{otherwise} \end{cases}$$

Sketch a graph of this function, verify that the area under its curve equal 1, and find both its expected value and variance.

(C) The probability of a flood in any 1 year is 0.1 in a 10-year period, what is the probability of no floods occur?

***** SOLUTION *****

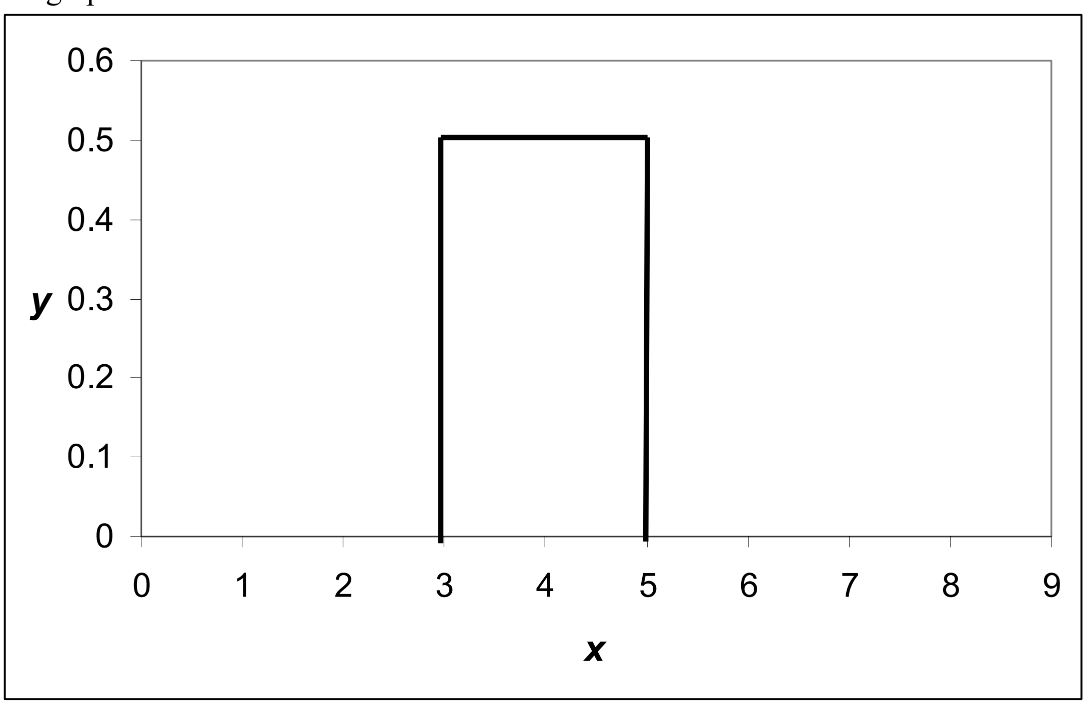
(A) From the Appendix, Φ(0.20) = 0.57926 and Φ(0.21) = 0.583166. Therefore, using linear interpolation:

z	$\Phi(z)$
0.20	0.57926
0.208	$\Phi(z)$
0.21	0.583166

$$\Rightarrow \frac{\Phi(z) - 0.57926}{0.583166 - 0.57926} = \frac{0.208 - 0.2}{0.21 - 0.2} \Rightarrow \Phi(z) = 0.5824$$

P(failure) = P(x <2.5) = Φ(0.208) = $\boxed{\mathbf{0.5824}}$

(B) Sketch graph:



Referring to the graph, Area = $0.5 \times (5-3) =$ **1.0** (Proof)

Or

$$\int_a^b f_X(x)dx = \int_3^5 0.5dx = 0.5x\Big|_3^5 = 0.5(5-3) = \mathbf{1.0} \text{ (Proof)}$$

For uniform distribution:

$$\mathrm{E}(x) = \frac{a+b}{2} = \frac{3+5}{2} = \mathbf{4}$$

$$\mathrm{Var}(x) = \frac{(b-a)^2}{12} = \frac{(5-3)^2}{12} = \mathbf{0.333}$$

(C) Poisson distribution with the following parameters:

$\lambda = 0.1$ $t = 10$

Therefore, $\lambda t = 0.1(10) = 1$

$$\mathrm{P}(X = x) = \frac{(\lambda t)^x e^{-\lambda t}}{x!}$$

$$\mathrm{P}(X = 0) = \frac{(\lambda t)^x e^{-\lambda t}}{x!} = \frac{(1)^0 e^{-1}}{1!} = \mathbf{0.3679}$$