Homework #7 Solution
ENCE 302 - FALL 2001
Due M, 11/5

Problem 1:

Textbook: 3-43

Using Table A-2 of the textbook:

(a) $\mathrm{P}(t > 2.45, k = 6) = 0.025 - \dfrac{2.45 - 2.447}{3.143 - 2.447}(0.025 - 0.01) = 0.024935$

(b) $\mathrm{P}(t < 2.72, k = 11) = 0.990026$

(c) $\mathrm{P}(t > -1.75, k = 16) = \mathrm{P}(t > 1.75, k = 16) = 0.049733$

Problem 2:

Textbook: 3-45

(a) $\mathrm{P}(C > 5.024, k = 1) = 0.05 - \dfrac{5.024 - 3.842}{5.405 - 3.842} = (0.05 - 0.02) = 0.027313$

(b) $\mathrm{P}(C < 0.831, k = 5) = 1 - 0.975 = 0.025$

(c) $\mathrm{P}(C > 2.555, k = 10) = 0.9900$

$\mathrm{P}(C < 6.251 \text{ or } C > 10.864, k = 18) = (1 - 0.9950) + 0.9000 = 0.905$

Problem 3:

Textbook: 3-48

$$\alpha = \sqrt{\frac{\pi^2}{6(5)}} = 0.25638 \quad \text{and} \quad \sigma = 25 - \frac{0.577216}{0.25638} = 22.7486$$

Therefore,

$$\mathrm{P}(X > 30) = 1 - F_X(30) = 1 - \exp(-\exp(-0.25638(30 - 22.7486)))$$
$$= 1 - 0.855721 = 0.1443$$

Problem 4:

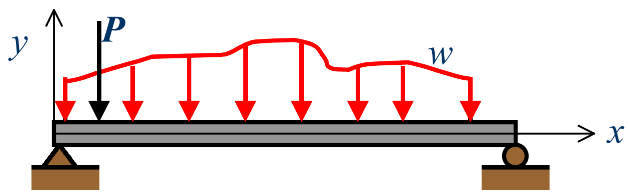
Consider the beam in the figure that has the T-cross section shown. The maximum shearing stress on the cross section is given by

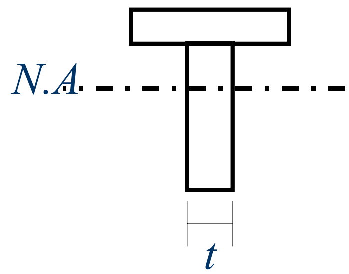
$$\tau = \frac{VQ}{It}$$

where V = maximum shear force along the beam, Q = the first moment of area with respect to neutral axis, I = moment of inertia of the cross section, and t = thickness of the web of the cross section. If the probabilistic characteristics of the basic random variables are as follows:

Random Variable	Mean	*COV*	Distribution Type
V	900 lb	0.15	Lognormal
Q	64 in^3	0.10	Normal
I	533 in^4	0.12	Normal
t	2 in	-	Deterministic

1) Simulate the maximum shearing stress τ_{max} for 100 values using a spreadsheet.
2) Find the mean, variance, standard deviation, and coefficient of variation of τ_{max} using the simulated values.





***** SOLUTION *****

Sample Calculations:

In reference to Table 2, consider the second row, where u_1, u_2, and u_3 are random numbers for V, Q, and I. Note t is given as deterministic, and therefore, should be treated as constant and not a random variable.

1) V is lognormal:

$$\sigma_V = \mu_V COV_V = 900(0.15) = 135 \text{ lb}$$

$$\sigma_Y = \sqrt{\ln\left[1 + \left(\frac{\sigma_V}{\mu_V}\right)^2\right]} = \sqrt{\ln\left[1 + \left(\frac{135}{900}\right)^2\right]} = 0.1492$$

$$\mu_Y = \ln(\mu_V) - \frac{1}{2}\sigma_Y^2 = \ln(900) - \frac{1}{2}(0.1492)^2 = 6.7913$$

$$u_1 = \Phi\left(\frac{\ln V - \mu_Y}{\sigma_Y}\right) \qquad \text{or} \qquad \ln V = \mu_Y + \sigma_Y \Phi^{-1}(u_1)$$

or

$$V = e^{\left[\mu_Y + \sigma_Y \Phi^{-1}(u_1)\right]} = e^{\left[6.7913 + 0.1492 \times \Phi^{-1}(0.75918)\right]}$$

$$= e^{\left[6.7913 + 0.1492 \times 0.70366\right]} = 988.596 \text{ lb}$$

2) Q is normal:

$$\sigma_Q = \mu_Q COV_Q = 64(0.10) = 6.4 \text{ in}^3$$

$$u_2 = F_Q(Q) = \Phi(z) = \Phi\left(\frac{Q - \mu_Q}{\sigma_Q}\right)$$

or $$z = \frac{x - \mu_Q}{\sigma_Q}$$

Therefore,

$$Q = \mu_Q + \sigma_Q z = \mu_Q + \sigma_Q \Phi^{-1}(u_2)$$
$$= 64 + 6.4 \times \Phi^{-1}(0.09696) = 64 + 6.4 \times -\Phi^{-1}(1 - 0.09696)$$
$$= 64 - 6.4 \times \Phi^{-1}(0.90304) = 64 - 6.4(1.29907) \approx 55.685 \text{ in}^3$$

3) I is normal:

$$\sigma_I = \mu_I COV_I = 533(0.12) = 63.96 \text{ in}^4$$

$$u_3 = F_I(I) = \Phi(z) = \Phi\left(\frac{I - \mu_I}{\sigma_I}\right)$$

or $$z = \frac{x - \mu_I}{\sigma_I}$$

Therefore,

$$I = \mu_I + \sigma_I z = \mu_I + \sigma_I \Phi^{-1}(u_3)$$
$$= 533 + 63.96 \times \Phi^{-1}(0.4402) = 533 + 63.96 \times -\Phi^{-1}(1 - 0.4402)$$
$$= 533 - 63.96 \times \Phi^{-1}(0.5598) = 533 - 63.96(0.15046) \approx 523.338 \text{ in}^4$$

4) Therefore, for row 2 of the table,

$$\tau = \frac{VQ}{It} = \frac{988.596(55.685)}{523.338(2)} = 52.595 \text{ lb/in}^2$$

Statistical Data for τ based on the simulated values:

Table 2 gives sample results of the spreadsheet. Based on 100 cycles of spreadsheet calculations, the statistical data for simulated values of τ are summarized as follows:

Table 1

Simulation Statistics for τ:	
Min τ =	**30.338142**
Mean τ =	**56.147007**
Max τ =	**90.620057**
Variance τ =	**176.33634**
Standard Deviation τ =	**13.279169**
***COV* τ =**	**0.2365072**

Table 2

Row #	u_1	u_2	u_3	V	Q	I	t	τ
1	0.55212	0.586682	0.015792	907.6087	65.40171323	395.5087	2	75.04154
2	0.759178	0.096958	0.440198	988.5428	55.6858942	523.3761	2	52.58922
3	0.761098	0.956379	0.813879	989.4545	74.94484469	590.0704	2	62.83531
4	0.955204	0.268564	0.780015	1146.521	60.05017365	582.3927	2	59.10856
5	0.666986	0.708399	0.178138	949.2294	67.51176277	473.998	2	67.59959
6	0.277034	0.623772	0.075696	814.8559	66.01858347	441.2409	2	60.95948
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
98	0.113093	0.893548	0.750619	743.0339	71.97197572	576.2649	2	46.40021
99	0.03635	0.205658	0.975836	680.9982	58.74188371	659.2875	2	30.33814
100	0.897365	0.232361	0.22652	1075.15	59.32099173	485.0072	2	65.75054